M A Mackenzie
Trinity College Toronto

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

VOL. XXIX.

LONDON:
CHARLES AND EDWIN LAYTON,
FARRINGDON STREET.
PARIS: L. WARNIER, LIBRAIRIE DES ASSURANCES, 48, RUE LAFFITTE.
HAMBURG: W. MAUKE SÖHNE. LA HAGUE: MARTINUS NIJHOFF.

1892.

LONDON:
PRINTED BY CHARLES AND EDWIN LAYTON,
FARRINGDON STREET.

CONTENTS OF VOL. XXIX.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

Opening Address by the President, BENJAMIN NEWBATT, ESQ.

[Delivered 24 November 1890.]

THE custom of commencing each session with a Presidential address, originated by Mr. Bailey in 1880, and since observed by each succeeding President, is not unlikely, I think, to impose a heavy burden on the future occupants of this chair. Indeed, I have myself already become sensible both of the burden and of its weight. Unlike the President of the British Association, for example, who has the whole field of scientific research in which to range, or even the President of a single scientific society—and there are many such—which yields annually a fruitful harvest of facts and discoveries for record and comment, your President has to move within narrow bounds, the area of our society being limited, as are the events in its history and the products of the research of its members.

Having lately, for my sins, had occasion to take the waters at a German Bad, I determined, as a not unfitting holiday task, to read consecutively and connectedly the several Presidential addresses delivered before this Institute, and also the similar addresses of the successive Presidents of the Actuarial Society of Edinburgh. The task is one which, under happier conditions

than mine, I would strongly advise any gentleman to undertake who desires to obtain a comprehensive idea of what the leaders of our profession have had to say of its dignity and mission, and of the qualities which those must possess who seek to emulate their example and share their fame. To the younger men of the present generation, and still more to those who are to come after, these addresses will, moreover, have a special biographical value, as serving to show what manner of men these leaders were: what their individual idiosyncracies and what their distinctive mental adornments; but, above all, what the qualities of high thought they had in common, and how in each alike there burned

> " . . . the desire of fame,
> And love of truth, and all that makes a man."

If, as may well have been in the golden days of their youth,

> "They slept, and dreamed that life was Beauty",

we know how soon, like all strong men for whom great work is waiting,

> "They woke, and found that life was Duty."

But in my own case, while giving rise to deep respect for their authors, these addresses left behind them a feeling akin to despair, in fit unison with my condition of depression. Like the action of the waters on my animal economy, their effect on my mental condition was wholly "resolvent." Under their influence the ideas which had been vaguely floating in my mind as fit matter for this address were dissipated and dissolved, leaving behind only the conviction that what I had thought of saying had been mostly already said, and better said. Nevertheless, hard as the task may be, the duty of saying a few words to you remains, as, with the honour, I accepted the obligations which accompany the occupancy of this chair.

Though the events of the past year which interest or closely concern us as an Institute have not been numerous, one or two of them deserve more than passing notice, while one, which may possibly come hereafter to be regarded as of first-rate importance—I mean the alteration in the Bye-Laws which regulate the qualifications for and mode of election to the fellowship—will, I

think, furnish matter on which we may not unprofitably take counsel together. I would, however, first pass under review some events of lesser importance, and of these I give precedence to one which at the time engaged our sympathy, and still holds a high place in our historic memory. I mean the official visit to us, in numbers so considerable as to constitute it the greater compliment, of the Council of the Faculty of Actuaries in Scotland. The immediate effect of this visit, following, as it did, so closely on a similar visit paid by members of our Council to the Metropolis of Scotland, was to consolidate and extend the feelings of reciprocal regard which might be expected to be inherent in two such bodies, having such common interests, as the Faculty and the Institute, but which our knowledge of life teaches us it needs personal friendship—that

> " . . . mysterious cement of the soul!
> Sweetener of life! and solder of Society!"—

to bring into perfect development. Though not doing so at once, it may not unlikely be that this visit may affect hereafter, if not the structure, at least the functions of the Institute itself. Our Scottish friends do not, I believe, desire fusion, believing, perhaps rightly, that the two bodies can work to most advantage on separate, though fairly parallel lines; but the age is big with the idea of federation—the complement and corollary to that prevailing spirit of co-operation of which my predecessor spoke last year; and federation, if not yet capable of realization on the extended scale which some of our Antipodean friends have already hinted at, might possibly be brought about between the Institute and the Faculty, with advantages of which one will be sufficiently obvious, that the diplomas of the two bodies might be made to rest on a common qualification.

The mention, in relation to the idea of federation, of the two great Chartered Actuarial Societies, reminds me of what, personally, I think presses on us, the older and larger body, if not as a duty, yet as a thing greatly to be desired—the recognition, by service if it may be, but at least by the expression of our earnest sympathy, of the Provincial Institutes which have sprung up of late years, to the advantage, not merely socially, but, which is our chief concern, educationally, of their members. If I may do so without being thought to be invidious, I would specially commend the work of the Birmingham Institute, where lectures by prominent members of our body have recently been, and

will, I understand, continue to be, delivered to men who, for good or ill, are exercising a potent influence both on the progress and the character of assurance business, and to whom sound knowledge, which, besides power, will bring discretion and a sense of responsibility, is so necessary to enable them to do their duty to their offices, to their clients, and to one another.

Another notable event of the year, which, though external to our concerns, will not be foreign to our sympathies, has been the establishment in Paris of "L'Institut des Actuaires Français", with the "Statuts" and "Reglement Intérieur" of which, as well as with the "Compte Rendu" of the two General Meetings, held respectively in May and June last, at which these Laws and Regulations were adopted, I have been favoured. It occasioned no surprise when, last year, my predecessor found himself able to announce that the great English-speaking people on the other side of the Atlantic, many of whose actuaries were already so distinguished for excellent and laborious work, had at last, after, I believe, more than one failure, established a society—"the Actuarial Society of America"—which, though differing somewhat in title, was avowedly founded on the basis and was intended to follow with some closeness the line of operation of our Institute, the American actuaries recognizing, as fully as we have so consistently done, the need for drawing "the line sharply" —I am now quoting their own words—"between ourselves as men "of business and ourselves as individual members of a profession "where there is much to learn", and being desirous, as anxiously as we have ever been, to avoid "anything that looks like a criticism of individual companies or individual plans." But we were not, I think, so entirely prepared to find our French neighbours not only following their example, but paying this Institute the double compliment of closely copying both its title and its constitution. The objects of the Institute of French Actuaries are declared to be the encouragement and development of the study of what, with some felicity, is termed "mathématiques financières"; the furnishing to its members the means of increasing their professional knowledge; and the placing of competent actuaries at the disposition of "Associations de Prévoyance" and of "Sociétés financières ou industrielles de toute nature." In addition to honorary and corresponding members, the members are of two kinds—"Agrégés" (graduates or fellows) and "Stagiaires" (undergraduates or associates). Only to the latter class is examination

a condition of admission. For the fellowship, the principal qualifications are

1st. That the candidate be of French nationality and not less than 30 years of age:

2nd. That he exercise or have exercised the profession of actuary; and

3rd. That in evidence of his professional knowledge, he submit an original thesis on some technical subject relating to the profession of actuary to be chosen by him and approved by the governing body (Bureau), and subsequently sustain the thesis in argument before a jury of the Institute, to consist of the President and four members elected by the general meeting.

There is in these qualifications a recognition, singularly practical, of the facts of life, worthy, I think, of much consideration from us.

Before passing from this brief retrospect, it will be seemly to place on record our regret at the losses among the Fellows which the Institute has sustained by death during the past year. These, unhappily, have been somewhat numerous. Besides Mr. Vaughan, the actuarial adviser of the Board of Trade, Mr. Edward Butler, and our esteemed foreign Fellow, Herr Wilhelm Lazarus, of Hamburg, they include three names, long familiar to the assurance world, each of which represents a distinct type. The senior of these was Mr. Samuel Ingall, who so lately died at an almost patriarchal age. At one time a prominent member of the small but notable band of capable actuaries whose sense of duty—mistaken, as we have come to think it—or dim perception of the drift and needs of their time, kept them aloof from the early struggles, and strangers to all but the latest achievements, of the Institute, Mr. Ingall was personally known but to few of us, and was, indeed, but little more than a name to the present generation. The other two—Mr. Cutbush and Mr. Morrice A. Black—died in harness. The former was an excellent representative of a large and characteristic class of Englishmen, who do diligently and conscientiously the work which their hands find to do, who lead useful and honoured lives, but who do not seek to pass out of the beaten track of established custom, nor by enterprise and adventure to

leave a distinctive mark on the times in which they live. Their records and their memories belong to their friends and the domestic circle, and they are, perhaps, like a nation, to be accounted happy because they have no history. Of Mr. Morrice Black other language must be used. As in physical distance, so in his public aims and methods, Mr. Black was removed from Mr. Cutbush as far as pole from pole. I would remark of him, first, that he was not an examined Fellow; but I shall do no party, nor school of thought, nor individual among us the wrong of thinking that, tested by whatever standard, he would have been regarded in any quarter as other than a worthy and welcome colleague. Our tests, which the Institute now rightly imposes, and to which our members wisely submit, point only to prospective and potential success; the tests which Mr. Black surmounted were actual and practical, and his achievements now stand out in imperishable retrospect. As to all intents and purposes the organizer, though not the founder, of a great democratic association, his name would in any case have endured; but his fame—the true fame which is cradled in honour—will rest on a surer basis than that of mere vulgar success, for he has furnished proof that to accomplish great ends or to build up lofty superstructures, it is not necessary to degrade service or to palter with the right. Far more than most vigorous and aggressive men, he possessed remarkable powers of well-ordered precision. Too much immersed, however, in the present to have a large prevision, and too sanguine of belief in men and things to exercise a wise foresight, he somewhat forgot that democracies are as fickle as they are proverbially ungrateful. He has gone, however, in the fulness of his powers, if not altogether in the plenitude of his authority, and, though but 56 years of age, who among us that cherish the adage that no man should be counted happy until he be dead, will do other than take comfort that a rich and, we may well believe, a completed life has fallen with honour.

In an age of unrest, when natural evolution marches with such haste as sometimes almost to take step with revolution, it was neither to be expected nor desired that the Institute should escape change. Such a change has, I think, been brought about by the alteration in the Bye-Laws relating to the qualification and

election of Fellows to which I have already referred, and with which you are all familiar. Though the outcome of impatient forces seeking a hasty, if natural, expansion, and marred, too, in its origin by a somewhat narrow eclecticism, I regard the change, in the more catholic shape it has finally taken, as moderate and essentially constitutional—likely, I believe, to uphold the value of the fellowship, without deflecting the fundamental principles of the Institute. Before, however, dealing with this aspect of the question, I should like to refer in some detail to another form of development which, though more insidious and less marked, is probably of even greater personal concern to a large portion of those who now hear me. I mean the change which increase of numbers has made in the *personnel* of the Institute, and, by consequence, in the prospects of its individual members.

Mr. Bailey, in the address delivered by him on the 29 Nov. 1880, records how the Institute, after the convulsions which followed its difficult birth, started on its career with 131 members, of whom only 37 were associates. At the close of the year preceding the grant of the Charter, there were 372 members, of whom 237 were associates; while at the last annual meeting—only six years later—the Council reported a total membership of 588, of whom no fewer than 415 were either associates or students, 207 being of the one class and 208 of the other. That is to say, while the Fellows had slowly increased in number from 94 in 1848, to 135 in 1884, and to 173 in 1890, the number of what I may term the undergraduates—the associates and students combined—had risen so rapidly, that in 1890 they were eleven times more numerous than in 1848—415 as compared with 37. Though I fully share the satisfaction expressed by the Council at the proof which these figures afford "that the younger members of the actuarial profession value and "appreciate the facilities which the Institute offers for securing "efficiency in their professional attainments", I am nevertheless oppressed by the consideration of what they signify to the individual members of this great body. The case is the reverse of that where "the harvest truly is plenteous, but the labourers are few." I shall hope, indeed, before I have done, to suggest considerations which may leave us free to regard these numbers without misgiving, and even to rejoice in their further expansion; but the force of such suggestions will depend on a frank recognition of the fact that what we can do for our younger members,

as well as what they must expect the Institute to do for them, has necessarily and essentially changed.

But we had better first look in the face the obverse side of the question—what, with a restricted area of work, will be the effect on the individual of this great increase of competitors; for that, in plain English, is the problem which disturbs me, as it has disturbed others besides me.

The first object of the Institute, as originally declared, and that which remains its first object still, is "to elevate the attain-"ments and status and to promote the general efficiency of "all . . . connected with the pursuits of an actuary." That the elevation of status here spoken of was concrete as well as abstract, personal as well as corporate, is well known. It is equally well known that one of the earliest efforts of the Institute was directed practically to this end. By means of examinations, the Institute sought to bring to light the talents of its younger members, and to create an avenue for reaching distinction, followed by an almost certainty of reward, for those of its small band of probationers who sought and were worthy of it.

A great step towards the personal advancement of its members—and here will be found the key to the problem the consideration of which I have for the moment postponed—was taken by the Institute when, in 1871, it determined to educate as well as to examine its members. It is a matter of no little satisfaction to me that, as a then member of Council, I was closely associated with those who initiated and matured this project. As a mere autobiographical fact, this is an incident of the smallest consequence; but I recall it in order to establish what is, perhaps, of some importance in the present connection—namely, the fact of my long and early sympathy with the effort to make the Institute for each succeeding generation of its members something of an Alma Mater. In their report for 1871, the Council announced that "observing the continued increase in the class of associates "and in the number of candidates presenting themselves for "examination", they "were considering whether . . . the "Institute could not render some assistance to . . . candi-"dates in their studies, either in the form of lectures or class "instruction or otherwise." The result of such consideration is within the knowledge of you all. Though between the first examination and the first effort at direct education there had been an interval of 21 years, the idea and the wish to combine these excellent works had long been in many minds. As early

as 1852 a scheme had been propounded (*J.I.A.*, iii, 272) for establishing a mathematical professorship in connection with the Institute, the duties attached to which, besides the delivery of a course of not less than twelve lectures during each session and the conduct of "an instruction class for such associates as might be preparing for their examinations", were to include—and this is worth special note—the contribution "of not less than two papers per annum to the *Journal of the Institute*." But the germ of this idea is to be found at an earlier date still. On page 195 of vol. ii of our transactions there is to be seen an editorial suggestion, since matured and embodied in the educational system of the Institute, for the establishment of "even two "professorships sufficiently well endowed to command the services "of able men", the declared object of which was not only generally to elevate the status and character of the profession, but specifically to raise to "a higher degree of public consideration" successful aspirants to the honours of the Institute, to the end and with the result that "their reward will be the greater." I have gone into this matter with some elaborateness, because I wish it to come out clearly that the purpose as well as the natural consequence of the Institute examinations was to promote the personal advancement of the successful candidates.

In the earlier days of the Institute, and for many years afterwards, success in the examination room was the almost certain precursor to success in official life, and rightly so. For not only was the number of men who so distinguished themselves comparatively small, and in no way disproportionate to the chief places to be filled, but in the toilsome process—the more toilsome because almost unaided—by which distinction was then attained, there had to be shown, besides a cultivated intelligence, discipline, devotion, tenacity—those evidences of force and individuality which, as by a sort of natural selection, marked out the successful men as the strong spirits destined to take the lead among their fellows. Year after year, however, in spite of the ever-growing severity of the tests, the number of these successful men has been getting larger, while the official positions of importance which they seek to fill have not become more numerous; and as the result, there are now some 30 examined Fellows for whom opportunities for official distinction have not yet been found. What will be the result, from this point of view, of the recent determination of the Council to make obligatory, instead of optional, the subjects needing some knowledge of the higher mathematics, it is,

perhaps, hard to say. It has not the intention, and it would be deplorable if it should have the result, of seriously diminishing the number of future Fellows. In its aim it is purely educational, and only follows the educational law everywhere observable, as for example in the medical curriculum, in which the new special subjects have of late become so numerous that the General Medical Council have recently suggested, as the only means of overtaking them, the addition of a fifth year to the already long term of pupilage.

If, then, this be the position in regard to the most successful men—the Fellows—what is the outlook for the Associates, of whom some, and for the Students, of whom many, will of necessity fail to pass into the highest grade? In putting the question thus sharply, I am by no means preaching the gospel of despair. While I see, and ask them to see, that as compared with early days there is now a lessened certainty of high reward, even for those who may become the distinguished among them; that as the ascent to the hill-top becomes ever more and more steep, the chance of being classed among the distinguished is lessening too, so that an ever-increasing number must be content with a lower place, I also see, and ask them to see, that the Institute may still be not the less a true Alma Mater to them. In support of this assertion, I do not depend on mere abstractions, such, for example, as the value of knowledge for its own sake, but I rely on material considerations both pertinent and personal. I have already said that in dealing with this problem we must recognize the fact that what the Institute can do for its younger members, as well as what they must expect the Institute to do for them, has changed, and that the key to the solution of the problem is to be found in the further fact that the Institute, in taking up, so truly as it has done, the functions of an educational body, has become a necessity to its younger members, not only to those who are able to use it as a road to high fortune, but to all who desire to occupy worthily, or even to be permitted to occupy at all, any place of consequence in the assurance system. Offices now not uncommonly require, and in time to come will more and more insist, that even their lowest places shall be filled only by those who have a certain amount of technical education, and it is as a school of technical training—training which, as in other walks of life, must be undergone after ordinary school life is over—that the Institute appeals to all, and affords benefit to all, engaged

in life assurance business. I do not know what higher merit the Institute can possess in the eyes of those of whom I am now speaking, or what greater claim it can prefer to their adhesion and gratitude, than this—that to every man among them who desires it, each according to his capacity, it puts into his hands on the threshold of his career such tools, and imparts to him such skill in the use of them, as will be necessary to him in the practical business of life.

These considerations will, no doubt, suffice for the associates; but that the Institute may not only claim but effectually secure the adhesion and gratitude of its student-members, something more, it appears to me, will have to be done for them. With those students who decline to submit to any examination after their admission, we need not greatly concern ourselves. They will, for the most part, pass away, to be replaced, we may reasonably anticipate, by new entrants. I am dealing, rather, with those students, of whom there will be an increasing number, who, whether from choice, or from that compulsion on the part of their employers of which I have just spoken, will pass the first part of the examination for admission to the class of associates, but will then stop, some from lack of perseverance or ambition, and some because their mental endowments fall short of the highest. Will such students, after their student days are over, be content to remain in a class with which their age and associations must needs put them out of sympathy? The answer is obvious, but so also, I think, should be our want of satisfaction with the answer. Do we, in fact, desire to part company with a body of men who, as our valued associates in our daily work, cannot be unworthy to associate with us in this Institute? I cannot doubt that you will think with me that if by any proper means these gentlemen can be retained, they should be retained. As a contribution to this end, I will venture to make a suggestion, for which, as for all else in this address, I am, as you well understand, alone responsible. Let, then, such men be offered an easier mode of attainment to the associateship. At present the second part of the examination for admission to the class of associates is, by hypothesis, beyond their reach, and it is also beyond what is needed in the daily routine of even superior office work. On the other hand, the final examination—that for admission to the class of fellows—relates in no inconsiderable degree to practical subjects with which not a few ordinary members of an office staff may have, or may readily acquire, a very ample acquaintance. Why, then, might not the

less advanced parts of each of these two examinations be combined and made the basis of a modified standard, which, while entitling those who attain to it to the honourable rank of associates, should not give them the right to make such rank the starting point for proceeding to the class of fellows. At most, by a plan of this sort, we should merely be doing for our own students what, under Bye-law 44, may now be done for any person, "whether a student or not", who may approve himself to not less than one-half of the Council as a person "of experience in matters relating to the profession of Actuary." But it would be done with this difference, that in the one case we should be sure that the requisite experience existed, whereas in the other its existence has to be largely taken for granted.

But having done so much for its alumni, and having thus equipped them for the inevitable struggle, the Institute can do but little more. The training of the Institute—the attainment, even, of its fellowship—is not the end, but the means to an end. The reward of high position, or of rich endowment, no more comes as a matter of course to even a brilliant Fellow of this Institute than to a brilliant honour man of one of our old Universities. In both cases alike there is the potentiality and rich promise of greatness, but nothing more. Indeed, I may be permitted to say that, with the attainment of his fellowship, the real work of every true man is only begun. He has acquired, indeed, a great opportunity, but the old question remains: what will he do with it? We have, of course, always known that, besides a due acquaintance with actuarial science, other acquirements are needed for practical success; but of late we have been made painfully aware that for preferment to high official positions there is a tendency to give the preference to business qualifications rather than to professional. Unless an actuary is to be content with a subordinate place, he must, as Mr. Sprague pointed out in one of his presidential addresses, be able to deal not only with figures, but with men. It is not to our credit, as members of this Institute, that any of the high positions in the assurance world should pass into other hands than those of our own graduates; and I will go a step further and say that, while we respect the powers and esteem the persons of many of those who have become managers without first being actuaries, it is not for the good of assurance business itself that power and knowledge should be thus disassociated. There is nothing in the training of an actuary, any more than in the training of a high wrangler, which can render him unfit

for practical work. A man who is faithful in little is more likely rather than less likely to be faithful also in much. Educational attainment—the development of intellectual power—is not a disability but a qualification, and when found in an actuary should constitute an additional presumption that he, rather than the man who has not had his training, will become a good manager. Why, then, does this *a priori* expectation often lack fulfilment? Is it that the actuary finds in his old studies a fatal tendency to indulge in minute subtleties? If so, let such a man take to heart a story which is told of Coleridge, who, on hearing of the intention of a young solicitor to commence practice in a cathedral city, advised him to take a house in the cathedral close, so that the perpetual presence to his sight of a majestic and inspiring edifice might counteract the narrowing influence of the petty affairs of his professional work. Or is it that, forgetting that youth is the time for adventure, he is restrained by the bonds of an undue timidity? If so, to such a man I would commend a story told of Canning, when prime minister. Listening, it is said, one day in the House of Commons to a speech by a man of promise, who was disappointing expectation by doleing out a dreary string of the baldest, and what Mr. Froude has lately termed "perfunctory", platitudes, his growing impatience at last burst bounds, and, turning round to his nearest companion on the Treasury Bench, he exclaimed, with angry petulance, "why on earth doesn't that young man risk something." To both such men I would say attempt some large enterprize and risk something! Strength is not only tested but increased by effort, and for most men early failures are the necessary prelude to final success.

What, from an educational point of view, the Institute has now to do is, it seems to me, to differentiate its Fellows. By its new regulation that successful candidates are to be placed in three classes, according to merit, it has already made a movement in this direction; but it might, I think, do more. Let it require of candidates for the fellowship what, as I have already shown, it would have required of the holder of its mathematical professorship, had the scheme for its establishment come to maturity, and what the French Institute now requires of candidates for its fellowship—the writing of a thesis, for which, if of marked excellence, a prize might be given, on some question of what the French call *mathématiques financières,* sufficiently large to test not so much the technical precision as the practical and philosophic

breadth of their minds. For the rest, for the prominence and precedence which great gifts worthily exercised will give to their possessors, the Fellows must look to themselves. I am not competent to point the way to brilliant discoveries, and, indeed, genius is its own best inspirer; but I may be permitted to point out that there are still many subjects, adapted to various orders of minds, and affording scope for the exercise of both mathematical and practical ability, on which papers will be welcomed, and should, I think, be forthcoming. Of these, I would mention the following, for the suggestion of some of which I am indebted to friends around me, one of whom, a man of specially proved actuarial capacity, says emphatically that "we want more popular papers."

Of purely professional subjects may be mentioned,

1°. The question of extra rating:

(*a*) When on account of inferiority of life, with special reference to the plan of graduating the amount payable at death, according to the number of years lived out of the number originally "expected."

(*b*) When on account of foreign residence, or occupation, dealing specially with the assessment as it should be applied to various kinds of policies and as it affects the equitable interests of the whole of the assured.

2°. The old and much debated, but by no means exhausted, subject of surrender-values, looking at the subject more particularly in its relation to other than whole-term policies, and to policies on which the premiums have been paid, or are payable, by other than equal annual amounts.

3°. The important and pressing subject of endowment assurances—the principles on which they should be valued for reserve, on which bonuses should be allotted to them as compared with ordinary policies, and (unless dealt with under the previous head) on which they should be valued for surrender.

4°. The interesting question of Industrial Assurance, so far as concerns the actuarial principles and processes obtaining in its administration, about which very little is known.

Of semi-professional questions, may be enumerated,

1°. One which has wide interest both for us and for the general community, namely, the responsibility and its limits of consulting actuaries when advising companies with which they are not connected in any other capacity.

2°. The mode of valuing, at periodical actuarial investigations, the investments of a company—a subject on which, notwithstanding its increased importance, little or nothing has been offered to the Institute of late years.

3°. The important question, of which it is not unlikely a good deal will be heard hereafter, of the true relations of Government towards assurance companies—a question which would necessarily take note of Colonial legislation, and might be usefully viewed in the light of the experience of the United States.

4°. The question, of such growing importance, of the general reduction in the rate of interest, and the effect of such reduction upon the future of life assurance, and particularly upon the estimates published by American and other companies.

5°. The series of questions which group themselves round the Census and the periodical returns of the Registrar-General—such as, *e.g.*, the improved rate of mortality among the general population, of which, from the assurance point of view, much more, probably, has been made than is warranted; how it is distributed among the centres of population, and how it affects or is likely to affect the death-strain of assurance companies. Or, again, how a more perfect system than now exists can be adopted for watching the population both locally and in the aggregate, either by more frequent Censuses or by registration. Or, lastly, how, by some approximate method, life tables may be formed from meagre data and the mean duration of life determined from the existing population and death returns.

And then, finally, there are the numerous miscellaneous questions of law, finance (State and domestic),

banking, &c., which so nearly touch the interests of assurance companies, and to which the trained minds of our members might address themselves with so much advantage.

If I have made myself at all clear, it will be seen that, as I understand the position, there is still dignity and reward, differing indeed in degree, and needing, it may be, more strenuous effort in the attainment, for all who, to diligent use of natural gifts, add determination and conduct—conduct, perhaps, above all, for there is a profound truth embodied in the apothegm that "character is "more than talent, especially in the high places and great moments "of the world."

Having thus dealt, I fear at somewhat wearisome length, with the Institute in its relation to its individual members, I will now deal with it, I hope with greater brevity, in its corporate aspect; and here the recent change in the Bye-laws will come to my aid, and will serve as a text, or at least as an excuse, for a brief re-examination of its objects and of the bases on which it rests.

We may sometimes get the best idea of what an institution is or should be, by considering what it is not or should not be. I have seen the Institute assailed on the ground that, on its own showing, it ought to be held answerable for all the errors of principle and vices of practice that occur in life assurance business, while in the same breath the inability of the Institute to exercise, in its corporate capacity, any influence on such business, is asserted as the necessary consequence of its being for the most part composed of assurance officials. By one writer it is laid down that the actuary should be wholly professional and wholly independent, which he cannot be whilst he is the officer of a company, and that as a first act of necessary reform the Institute should carry out the somewhat drastic resolve of expelling from its ranks the high officials of the chief offices—why only the "high" officials, and why only those of "chief" offices, is not explained. By another critic, on the other hand, it is declared, with equal emphasis, that the Institute ought to exercise disciplinary authority over its members—an idea which becomes ludicrous if the former pronouncement be the true one, and the members cease to hold

official positions in life assurance companies, where all the evils to be remedied originate.

It is not necessary that I should waste much time in pointing out how mutually destructive these criticisms and these suggestions are, and I refer to them chiefly in order that, by recognizing the impossible, we may get a clearer idea of what are the only possible bases on which such an institution as the Institute can exist. The conflicting ideas which still perplex many of us are as old as the Institute itself. As in the domestic circle there are sometimes disputes over the name to be given to an expected child, even before its sex can be known, so the foundation of the Institute was attended by such differences of opinion in regard to its prospective functions as detached from its cause many of the leading spirits of the time, with results which are felt to this day. I need not recall at length how the original idea to form a business organization, nor the modified idea to hold meetings which might tend "to establish uniformity in dealing with points of practice", suffered *bouleversement*, as Mr. Bailey terms it, and was superseded by a resolution to establish "a scientific and practical association"—scientific, to again quote Mr. Bailey, "because on a scientific basis alone could its investigations be properly conducted", and practical, because it was "to concern itself with the monetary "affairs of actual life, and at the same time to carefully avoid "interference with the conduct of any existing institutions." This abstention from dealing with points of practice and avoidance of all interference with the conduct of existing institutions, which thus at the very genesis of the Institute was made a fundamental basis, has remained a fundamental basis to this day; and any attempt to change it would, in my judgment, be fatal to the usefulness and even to the existence of the Institute. As a private society, representing an open profession, it has, too, disclaimed all disciplinary powers over its members. Some have at times thought, and I have myself been of the number, that this constitutes a weakness, and that a higher standard of professional duty might have resulted from wholesome supervision. None of us will doubt that to the Life Assurance Companies Act can be traced greater precision of statement and even improvement of method, or, to use another illustration, that to professional audits may be traced more exact book-keeping and a minuter control over small expenditures. And will it be denied, in regard to the individual, that the fiercer the light in which he walks, the more will he be inclined, and even constrained, to tread delicately?

It is nevertheless due to the Institute, and to those who have controlled its destinies, to say that the principle of non-interference, though often challenged, has always, after anxious consideration, been re-affirmed—not from unworthy motives, but under a controlling sense of the law of self-preservation.

It is matter of interest and most pertinent to my present purpose to notice, that while in the resolution to establish "a scientific and practical" Association, already in part quoted, it was considered desirable to include in the Association the "actuaries, "secretaries, and managers of the life assurance societies of Great "Britain", the first constitution prepared in pursuance of this resolution put actuaries into the superior grade of Fellows, and relegated "managers, managing or resident directors, secretaries, or other chief officers" to the second and inferior grade of Official Associates. I point this out with so much circumstance, because I wish to admit frankly that, though there was a period, commencing in 1864, and continuing down to the date of the Charter, during which the principle was suspended, the principle that Fellows should be actuaries in the strict sense was, nevertheless, the original cardinal principle of the Institute; that though re-affirmed by the Charter, it was not a creation of the Charter; and that, consequently, the action of those to whose initiative the recent change in the Bye-laws is due did not violate the spirit of our constitution. Nor can I deny that, as to the victors belong the spoils, it was consistent with human nature that those who through toil and long endurance had reached the goal of the fellowship by the appointed way, should have wished to compel others to travel by the same road that they have trod. But all the same, to have affirmed such a principle as one admitting of no exception, to have flown in the face of accomplished facts and have denied the fellowship to men of undoubted competence who, without the *cachet* of the Institute, had attained to the position of actuary by other and, perhaps, not less arduous paths, would, I am convinced, have been a fatal mistake; and I do not hesitate, therefore, to say that those who, in accepting what was in fact an honourable compromise, subordinated their convictions to the recommendations of the Council, will come hereafter to see that what was intended only as a graceful manifestation of loyalty to their leaders, was also an act of the highest wisdom.

It is probably a safe thing to say of any mere private society, having no legislative recognition, still less legislative authority, but assuming, withal, large functions closely resembling public

duties, that as between a policy of exclusion and a policy of inclusion no wise man would hesitate. In our case, especially, unless we desire to degrade ourselves to the level of a *dilletanti* or *doctrinaire* body, it is incumbent on us to seize hold of any fit element of strength which brings us into touch with authority. There are already, as I have previously hinted at, two forces working in opposite directions which may hereafter affect the dignity of the Institute as the authoritative exponent of all the intellectual aspects of assurance business, whether in its theory or its practice—I mean the increasing number of fellows who, because of their number, do not hold official positions as actuaries, and the increasing number of high official places which are filled by men having no claim, and putting forth no pretensions, to be regarded as actuaries. It is matter of common knowledge that there are among us some—not I think a large number—whose attainments and services entitle them to the most respectful hearing, who, stimulated, no doubt, by this latter consideration, hold that even the compromise of which I have just spoken was in the wrong direction, believing it to be in the highest degree desirable that every chief officer of an assurance society, even though not an actuary, should be eligible for the fellowship.

The desire to make the Institute supreme in every department of assurance life—practical and personal, as well as scientific—which lies at the bottom of this suggestion, is an idea with which a good many of us have at some time or another been fascinated. Some months ago I received, and read with great sympathy and a prepared readiness of response, a long and powerful memorandum emanating from one of the most philosophic minds to be found in our ranks, urgently appealing to me to do my best to bring about, if it were possible, such a combination of the scientific and practical functions in assurance work as will admit of both being performed by one society, under one roof. This dream, as I think our friend knew, had long been mine too, and, indeed, in quite recent times, I had endeavoured unsuccessfully to give practical even if only partial effect to it by a plan of affiliation; so that I shall be acquitted of exaggeration when I say that the idea and the desire to see a mode of realizing it have never for any lengthened period been wholly absent from my mind. I have, however, come to the conclusion, reluctantly but clearly, that by no device of which I am cognizant can practical effect be given to the notion. The difficulty of realizing it, if at the outset of the Institute felt to be insuperable, has been daily growing more

and more difficult since. The 94 fellows at the foundation of the Institute—most of them heads of offices—absolutely dominated the 37 members of inferior grade with whom they were associated, whereas now the 173 fellows—many of whom are not heads of offices—are obviously open to be controlled by the 415 associates and students, or, leaving the students out of the account, as by our regulations we must do for this purpose, by the 207 associates alone. When we remember that even the associates are composed chiefly of gentlemen not yet in responsible positions, the impossibility of submitting to them delicate pieces of what I may call legislation becomes obvious, while, if left out of the reckoning, they would naturally be wounded, and would soon become resentful. I might also urge, though on this I do not place great stress, that as a society of individuals in voluntary association for clearly defined purposes, it is incumbent on us to expend our funds on objects of professional value and common interest to all our members, and not on objects which, being chiefly of commercial value to business organizations, should be paid for by those for whose benefit the work may be undertaken.

How then do I satisfy myself, and seek to make clear to you, that in thus recognizing the limits within which the Institute can work, I am not condemning it to that narrow and *doctrinaire* condition into which I have previously expressed my anxious desire to prevent its falling? It is not by wishing to restrict, but rather by urging it to extend, its dealings with practical questions. To deny it the power of legislation is not to exclude it from practical affairs. I regard the position of the Institute in reference to business affairs as analogous to that of the press or the platform in reference to State politics. In both cases alike, though they cannot legislate, they can form and develop opinion, which leads to legislation. So far from wishing to limit the operations of the Institute, I desire to see it take larger views of its corporate duty. Though appreciated by the discerning, it has not yet wholly conquered that larger place in public regard we desire for it. To acquire this place it must work on larger lines and deal with broader issues. Its fame has hitherto chiefly rested, as it must no doubt continue to chiefly rest, on the aggregated achievements of its individual members; but this is not corporate fame, resting on corporate work. Its mortality table, and its text book, and possibly also its classes and its examinations, may be held to rank as corporate work; but is there no more such work that it can do?

If, as I do not doubt, I am right in treating the preparation of a mortality table as corporate work, then I know that in many minds a fitting task is seen to be waiting for the Institute in the compilation of a new table of mortality among assured lives. Without myself regarding such a work as of pressing need, I acknowledge both its professional value and its national usefulness, and I am prepared individually to further it by all means in my power. As to what such a new table will teach us, there are diversities of opinion. Some think the shifting and evanescent nature of much of our modern business will bring out less favourable results than before, while others are impressed with the idea that the large degree in which endowment assurance has in recent years superseded ordinary life assurance, has introduced an element of self-selection which must needs bring out more favourable results. I am myself chiefly anxious about another matter. I want to know more about the later years of assured life, and if it were possible to take up the work of 1869 and to continue the observation of the lives comprised in the present Institute tables, this would, in my judgment, be the best of all possible services in this direction. On all these points, and on the more particular subject of the methods to be employed in the new investigation, preliminary discussion is needed, and papers on them will be warmly welcomed.

But I want something done of even more direct interest to society and the State. Are we all of opinion, for example, that the present Life Assurance Companies Act tells the world all that it would be well for it to know of the condition of the offices which make returns under it? If the answer, as it must, be in the negative, shall we not be wise in our own interests, as well as in a high degree patriotic, if we anticipate outside pressure and take this matter in hand for ourselves?

Among the numerous socialistic ideas now seething in many philanthropic and not a few wise minds, is that of State provision for old age; and schemes are, I know, being devised which, though not wholly devoid of scientific guidance, would be all the better for being examined in the light of the larger knowledge of this Institute. In the great German Empire, which less, perhaps, than any European country is given to chimeras, there is at work a large scheme of State assurance against sickness, old age and death. What do the English people or its legislators, or what even does the Institute of Actuaries, know of this great scheme? Cannot we do something in regard to this great question, which, while aiding our senators, will earn for our Institute that place in the esteem

of our nation of which I have already spoken? The work to be done is almost beyond individual effort, and needs the aid, intellectual and pecuniary, of the Institute itself. This is true corporate work, the doing of which would at once lift up the Institute to a higher level, and extend the intellectual horizon of its members.* Other subjects will occur to you, similar, if not identical in nature—questions involving scientific finance; questions, too, of health, population and thrift, so closely concerning the body-politic—which, if too weighty for individual effort, the Institute might make its own, and in dealing with which might become the guide as well as the servant of the State. But I have said enough.

It will have been manifest from the whole tenor of what has preceded that I regard the functions of the actuary and the business of life assurance as almost identical. Of the Institute of Actuaries I have on a former occasion said that it is the scientific handmaid of life assurance business; and this, I think, defines with reasonable precision their true relations. In saying this, I am not unmindful of those who would create for the actuary a larger sphere of operation. While sympathizing with the desire, I cannot, as a practical man, see much chance of its being realized. There may and I trust there will be in the future some greater occasion for, as from recent disclosures there would certainly seem to be the utmost need of, the services and guidance of actuaries in connection with friendly societies; but friendly societies being among the institutions analogous to life assurance societies of which our Charter and Bye-laws already take cognizance, their introduction into the actuary's field of operation does not militate against my argument. Outside this sphere of operation there is, indeed, work of the highest class to be done, but it is so limited in amount as to be incapable of sustaining the calling of an actuary as a profession. The existence

* Since this was written, I have learnt from our Vice-President, Mr. Young, that, thanks to the courteous assistance of the British embassy at Berlin, he has become possessed of much valuable matter relating to this subject, both statistical, mathematical and legislative; and I am not without hope that, notwithstanding his many engagements, he may himself be able to deal with it. Failing this, however, he has kindly promised to place his materials—much of which he has had translated at his own cost—at the service of the Institute, to be utilized in the manner indicated in the text, or for the purposes of, say, a Brown prize essay.

of life assurance societies is the *raison d'être,* and practically the only *raison d'être,* of the actuary and of the Institute of Actuaries. If this be so, it would seem to follow in natural sequence that I should say something of those great interests amid which we, as actuaries, move and have our being. But large as is this subject, and interesting as the treatment of some of its aspects might be made, I remember that this is not an occasion on which I, at least, am free to deal with what may easily become burning questions. In the little I propose to say, therefore, I shall restrict myself to generalities which I hope will wound no individual susceptibilities.

It used to be matter of complaint that life assurance societies failed in their duty to their assured. In the present day it is sometimes asserted that they fail in their duty towards one another. To what extent are these allegations well-founded? Like most other general propositions, each is partly true and partly untrue. Let us for a moment or two look at each in turn. If we compare the present attitude of life assurance companies towards their *clientèle* with that which prevailed in the earlier stages of their history, and assume that every modern development is evidence of an ancient wrong; if, for example, we contrast our present delicate adjustment of premiums to risks, and even to fractional divisions of age, with the rough rule of the Amicable Society in 1706, of charging £1. 11*s.* a quarter for each £100 of assurance whatever might be the age of the assured, and rashly conclude that our equity is evidence of their inequity, we must admit the first allegation to be well-founded. But though comparisons of this kind not unfairly illustrate the shallow criticism to which those commonly resort who seek to decry the past, it were surely as unwise to talk in such sort as it would be to seek to rob Gutenberg and Fust of their fame, because, though they may have invented printing, they did not also foresee the discovery of shorthand and the type-writer, nor that still more wonderful phonograph, which is already superseding stenography, and which even threatens to make sight give way to sound. The truth is, that every generation must be judged by the light in which it has to walk; and while I admit, and rejoice in the admission, that owing in part to increase of professional knowledge radiating largely from this Institute, but still more to new conditions of life and movement which have influenced and developed not the business of life assurance merely, but arts and sciences and handicrafts—all, in fact, that touches the social, sanitary, and economic life of man—

there has been never-ceasing improvement both in the doctrine and the practice of assurance, I have never known the time when among the better class of offices the desire to do their duty towards their assured did not exist. Some may not have kept equal step with the others, and some, indeed, may have lagged behind their fellows; but I am satisfied—and I speak from an experience fast approaching to 40 years—that on all the points which have given rise to controversy, whether reserves, or equitable distribution of profits, or surrender-values, or limits of residence, or "conditions" of assurance, as most of us are still content to call them, the wish to act with fairness, in the interests of the assured and of the assurers alike, has, in the main, always prevailed. I might go a step further and say that, in regard to some of the more recent developments of assurance practice, I am not so sure that the permanent interests of the assured have been as well considered as they were in days so often unfairly reviled. The pressure of competition, which I suppose is only another name for the "necessity" of the proverb, has been a fruitful mother of not always wise invention. As one illustration of my meaning, which will serve as well as another, I will take that modern form of endowment assurance which, under the delusive idea that it furnishes a profitable investment, is so largely superseding the unselfish forms of life assurance, and ministering to the selfish instincts of mankind. I am not quarrelling with the original idea of endowment assurance, which, when it postpones the endowment benefit till late life, combines adequate protection to the family of the assured with provision for the old age of the assured himself. Nor will I deny that when adopted by those for whom originally it seemed almost exclusively suitable—for the man, that is, of slender means, whose small savings could not be readily accumulated to advantage—it furnishes a fair investment, as well as that provision in case of early death which is the true function of life assurance. But I cannot think that endowment assurances of the modern type, which mature at short dates, which carry assurance only for a period correspondingly short, which may be realized and dissipated in middle life without providing either that protection to the family or that provision for old age which is their professed office, are to be commended, or should receive the undue encouragement at our hands which is sometimes given to them. I am not suggesting that attempts should be made to run counter to public demands. That would, indeed, be to act after the foolish manner of Mrs. Partington, and to attempt

to stem an inflowing tide. But I do suggest that public demands should not be stimulated by fallacious representations; and can any man doubt that such representations are abundantly rife? We in England have, I fear, fallen of late into mistaken practices which used to be confined to our trans-Atlantic cousins. In saying this I shall not, I hope, be misunderstood. I have no jealousy of our American competitors. More than 20 years ago, in an appreciative article on a great American company to be found in our *Journal* (*J.I.A.*, xiv, 322), I allowed myself to use these words. "One reflection is forced on us "by what has preceded. How long will it be before the insurance "tide which has so long set westward shall be rolled back on our "own shores? When it comes, if it should come, how shall we be "able to withstand it? We can do nothing for our policyholders "that will bear a moment's comparison with the results which this "company has accomplished and, we cannot doubt, will continue "for many years to come to accomplish for theirs. At present we "are safe in the merited distrust which hangs about the public "credit of the country. But old stains, if not renewed, will "assuredly be worn out with time. Let its public credit be once "established beyond reach of doubt or cavil, and America must "become the savings bank of Europe, and certainly not least so in "respect of such savings as take the form of assurance premiums." This prophecy has had, I think, a fairly reasonable amount of fulfilment. But by the nature of things a prophecy of this nature, based on shifting and probably transient economic conditions, could not have, and was not intended to have, perpetual application; and, in my judgment, it has for some time past ceased to have application. I was dealing with a company which during the 25 years it had then been in existence had earned interest at the average rate of 7 per-cent. But with changing conditions, there must of necessity be a change in the results; and my complaint is that the promise of results has been maintained—I am not now referring to any American office in particular—long after the conditions on which the results depended, of which economy and large interest earnings are the chief, have materially altered. Speaking, at all events, for ourselves in England, where do we find the elements of profit which justify the estimates which are too often put forth? Does a man, able to pay, say, £120 a year as premium for an endowment assurance, and capable therefore of being his own investor, ever consider what becomes of his money—ever consider the extent to which toll is

taken of him? In the case of one of the short-term endowment assurances which I am considering, not more than £20 out of this £120 may at most be needed for the assurance, and as to that I say nothing, for however large a charge of 10 or 15 per-cent—a common English rate—or of 20 or 25 per-cent—a not uncommon American rate—may seem, the assured has no alternative but to pay it, or something like it, as that part of his business can be managed in no other way. In the words of a recent Edinburgh reviewer, "by no other process known to mankind can he accomplish this"—the "this" referred to being the "making instantaneous provision for wife and family", even though death should occur "the very instant after paying the first premium." With regard, however, to the larger portion of his premium—the £100 not wanted for assurance—the case is quite different. Not only is the man who can afford to pay such a premium capable of being his own investor, but he can probably invest to greater advantage than can the assurance company; and yet, assuming him to be aware of what he is doing, he is satisfied to allow an English office to take £10 or £15, or an American office £20 or £25 out of his £100, for doing work which he can better do, deluding himself, or permitting himself to be deluded, with the notion that, owing to some magical power on the part of the companies which exists only in his uninformed or unreflecting mind, £85 a year invested at, say, $4\frac{1}{4}$ per-cent in England, or £75 a year invested at, say, 5 per-cent in America, is to produce more in the hands of the companies than the whole £100 invested at a not less high, and probably at a higher rate, as it might be invested if kept in his own hands. Holding, as I do, that the proper business of life assurance companies is life assurance, I deprecate proceedings which must ultimately tend to lower the repute of assurance business.

On the second point—whether the charge be true that in these days life assurance companies sometimes fail in their duty towards one another—I must be even more reserved. Such an enquiry might easily lead to a discussion of the general question of the ethics of assurance, of which it is a particular case—a tempting subject, on which others might be heard in this room with advantage, but, because tempting, needing in one in my position only the greater restraint. My words, therefore, must be few, and they shall be chiefly such as are required to constitute a declaration of my own faith. Briefly, then, I am no trades-unionist, but a believer in free labour, and free invention, and

free competition. Upon competition, even upon its excesses and its new and perilous forms, I would put no other restraint than that which is involved in a due recognition of the eternal truth, from which, however much we may forget it in the stress and strain of life, we cannot escape, that if one member of the body suffer all the members suffer with it. Beyond this, indeed, I hold that the one thing of all others which differentiates a life assurance company, so weighted, as it is, with obligations akin to trusts, from an ordinary trader, is the duty imposed on it of practising strict equity towards all who trust its public declarations. An ordinary trader, choosing at times to forgo some of his own profit, may, I am willing to believe, have two prices, without violating the moral canon; but for a life assurance company to have two prices, applicable to either of two persons in no way differing from one another in their value or their services to the company itself, is to offend after the manner of those who disturb the just balances. Can it, indeed, be possible that we may come to so poor a pass as this? I know it is said that the business of life assurance, notwithstanding its lofty aims and beneficent purposes, is, after all, only a branch of commerce, and, as such, must be carried on on commercial principles. I am not concerned to contest the statement; for, even if this be so, I am still left free to urge that it at least belongs to those higher walks of commerce, where the culture, the professional knowledge, and—may I not add?—the honour that we claim to possess, may find fit employment, and in the purer air of which, to use the quaint but lofty language of George Herbert, men "dare to be true" and "nothing can need a lie."

A vote of thanks to the President for his address was moved by Mr. A. H. Bailey and supported by Mr. M. N. Adler and Mr. T. E. Young, and was heartily accorded by the meeting.

The President, in acknowledging the vote, referred to the fact that since the last meeting an addition had been made to the "counterfeit presentments" of the worthies of the Institute. His friend Mr. Archibald Day had been desirous that his portrait should not appear on the walls until he had ceased from active participation in the work of the Institute, but he was glad to say that the force of custom and the solicitation of friends had prevailed over Mr. Day's objections. The portrait was an admirable likeness of a distinguished and kindly man, and would convey to future members of the Institute an instinctive knowledge of those qualities in Mr. Day which had endeared him to the present generation.

English Life Tables Nos. III *and* IV.

THE following tables are extracted from the supplement to the forty-fifth annual report of the Registrar-General of Births, Deaths and Marriages in England, and give for each age and for males and females separately the values of l_x and $\mathring{e}_x$, according to the English Life Tables Nos. III and IV.

It may be convenient to place on record in the pages of the *Journal* a brief statement of the bases of these tables and of the earlier tables founded upon the census returns and death registers for England and Wales.

ENGLISH LIFE TABLE NO. I.

This table was prepared by Dr. Farr from the census returns and the recorded deaths for the year 1841, and will be found in the 5th report of the Registrar-General, published in 1843. The population estimated to the middle of the year was 15,927,867, and the deaths were 343,847.*

ENGLISH LIFE TABLE NO. II.

The population returns of 1841 were employed in the formation of this table as in the case of the previous one, with some corrections, but the deaths were taken for the extended period of 7 years, 1838-44. The mean population was 15,929,834, and the deaths in the 7 years were 2,436,648. The table is contained in the 12th report of the Registrar-General, published in 1853.

ENGLISH LIFE TABLE NO. III.

This table, together with extensive tables of monetary values founded thereon, is to be found in Dr. Farr's well-known volume published by the authority of the Registrar-General in 1864. The table is also given in the 28th report of the Registrar-General published in 1867. The data employed in the construction of the table were as follows:

Mean Population 1841	15,929,492
,, ,, 1851	17,982,849
Deaths 1838-54 (17 years)	6,470,720

For the values of l_x and e_x according to this table, *see* pages 30, 31 (columns (1), (3), (5) and (7)).

* Mr. F. G. P. Neison, in his *Contributions to Vital Statistics* published in 1845, gives a life table founded upon the census of 1841 and the deaths during the $3\frac{1}{2}$ years July 1838 and December 1841.

English Life Table (Healthy Districts).

The "Healthy English" table was based by Dr. Farr upon the census returns for 1851 and the deaths during the 5 years 1849-53, in 63 districts of England and Wales wherein the annual rate of mortality among the general population did not exceed 17 per 1,000 for the 10 years 1841-50. Dr. Farr gave a full account of the construction of the table in a paper submitted to the Royal Society in 1860 (reprinted *J.I.A.*, ix, 121, 188 from the *Philosophical Transactions*, 1860). The number of lives dealt with was 996,773, and the deaths during the 5 years, 87,345.

English Life Table No. IV.

This is the most recent life table formed from English census returns. It is based upon the censuses of 1871 and 1881, and the deaths during the 10 years 1871-80. The table was prepared by Dr. William Ogle, who gave an account of it in the supplement to the 45th annual report of the Registrar-General, published in 1885. The mean population during the period over which the observations extended was as follows:

Males	11,849,414
Females	12,493,934
	24,343,348

and the deaths during the 10 years were—

Males	2,679,416
Females	2,498,895
	5,178,311

On pages 30, 31 (columns (2), (4), (6) and (8)) will be found the values of l_x and e_x according to the English Life Table No. IV.*

* The tables given by Mr. Noel A. Humphreys in his paper read before the Statistical Society in 1883 (reprinted *J.I.A.*, xxvii, 186), took into account the deaths during 1876-80 only, and were prepared before the results of the 1881 census were fully known. They must therefore be looked upon as only an approximation to Dr. Ogle's tables.

English Life Tables, *Nos.* III *and* IV, *based respectively upon the Mortality in* 1838–54, *and in* 1871–80.

Age x	Males l_x Eng. Life III (1)	Males l_x Eng. Life IV (2)	Males $\mathring{e}_x$ Eng. Life III (3)	Males $\mathring{e}_x$ Eng. Life IV (4)	Females l_x Eng. Life III (5)	Females l_x Eng. Life IV (6)	Females $\mathring{e}_x$ Eng. Life III (7)	Females $\mathring{e}_x$ Eng. Life IV (8)	Age x
0	1,000,000	1,000,000	39·91	41·35	1,000,000	1,000,000	41·85	44·62	0
1	836,405	841,417	46·65	48·05	865,288	871,266	47·31	50·44	1
2	782,626	790,201	48·83	50·14	811,711	820,480	49·40	52·22	2
3	754,849	763,737	49·61	50·86	782,990	793,359	50·20	52·99	3
4	736,845	746,587	49·84	51·01	764,060	775,427	50·43	53·20	4
5	723,716	734,068	49·71	50·87	750,550	762,622	50·33	53·08	5
6	713,881	726,815	49·39	50·38	740,584	755,713	50·00	52·56	6
7	706,156	721,103	48·92	49·77	732,771	750,276	49·53	51·94	7
8	699,688	716,309	48·37	49·10	726,116	745,631	48·98	51·26	8
9	694,346	712,337	47·74	48·37	720,537	741,727	48·35	50·53	9
10	689,857	708,990	47·05	47·60	715,769	738,382	47·67	49·76	10
11	685,982	706,146	46·31	46·79	711,581	735,405	46·95	48·96	11
12	682,512	703,595	45·54	45·96	707,770	732,697	46·20	48·13	12
13	679,256	701,200	44·76	45·11	704,155	730,122	45·44	47·30	13
14	676,057	698,840	43·97	44·26	700,581	727,571	44·66	46·47	14
15	672,776	696,419	43·18	43·41	696,917	724,956	43·90	45·63	15
16	669,296	693,695	42·40	42·58	693,050	722,084	43·14	44·81	16
17	665,529	690,746	41·64	41·76	688,894	718,993	42·40	44·00	17
18	661,402	687,507	40·90	40·96	684,378	715,622	41·67	43·21	18
19	656,868	683,941	40·17	40·17	679,463	711,946	40·97	42·43	19
20	651,903	680,033	39·48	39·40	674,119	707,949	40·29	41·66	20
21	646,502	675,769	38·80	38·64	668,345	703,646	39·63	40·92	21
22	641,028	671,344	38·13	37·89	662,474	699,141	38·98	40·18	22
23	635,486	666,754	37·46	37·15	656,509	694,521	38·33	39·44	23
24	629,882	661,997	36·79	36·41	650,463	689,759	37·68	38·71	24
25	624,221	657,077	36·12	35·68	644,342	684,858	37·04	37·98	25
26	618,503	651,998	35·44	34·96	638,148	679,822	36·39	37·26	26
27	612,731	646,757	34·77	34·24	631,891	674,661	35·75	36·54	27
28	606,906	641,353	34·10	33·52	625,575	669,372	35·10	35·83	28
29	601,026	635,778	33·43	32·81	619,201	663,959	34·46	35·11	29
30	595,089	630,038	32·76	32·10	612,774	658,418	33·81	34·41	30
31	589,094	624,124	32·09	31·40	606,296	652,747	33·17	33·70	31
32	583,036	618,056	31·42	30·71	599,769	646,957	32·53	33·00	32
33	576,912	611,827	30·74	30·01	593,196	641,045	31·88	32·30	33
34	570,716	605,430	30·07	29·33	586,575	635,003	31·23	31·60	34
35	564,441	598,860	29·40	28·64	579,908	628,842	30·59	30·90	35
36	558,083	592,107	28·73	27·96	573,192	622,554	29·94	30·21	36
37	551,634	585,167	28·06	27·29	566,431	616,144	29·29	29·52	37
38	545,084	578,019	27·39	26·62	559,619	609,599	28·64	28·83	38
39	538,428	570,656	26·72	25·96	552,758	602,924	27·99	28·15	39
40	531,657	563,077	26·06	25·30	545,844	596,113	27·34	27·46	40
41	524,761	555,254	25·39	24·65	538,876	589,167	26·69	26·78	41
42	517,734	547,288	24·73	24·00	531,849	582,104	26·03	26·10	42
43	510,567	539,161	24·07	23·35	524,765	574,919	25·38	25·42	43
44	503,247	530,858	23·41	22·71	517,617	567,612	24·72	24·74	44
45	495,770	522,374	22·76	22·07	510,403	560,174	24·06	24·06	45
46	488,126	513,702	22·11	21·44	503,122	552,602	23·40	23·38	46
47	480,308	504,836	21·46	20·80	495,768	544,892	22·74	22·71	47
48	472,306	495,761	20·82	20·18	488,339	537,013	22·08	22·03	48
49	464,114	486,479	20·17	19·55	480,833	529,048	21·42	21·36	49

ENGLISH LIFE TABLES, *Nos.* III *and* IV, *based respectively upon the Mortality in* 1838–54, *and in* 1871–80—(continued).

Age x	Males l_x Eng. Life III (1)	Males l_x Eng. Life IV (2)	Males $\mathring{e}_x$ Eng. Life III (3)	Males $\mathring{e}_x$ Eng. Life IV (4)	Females l_x Eng. Life III (5)	Females l_x Eng. Life IV (6)	Females $\mathring{e}_x$ Eng. Life III (7)	Females $\mathring{e}_x$ Eng. Life IV (8)	Age x
50	455,727	476,980	19·54	18·93	473,245	520,901	20·75	20·68	50
51	447,139	467,254	18·90	18·31	465,572	512,607	20·09	20·01	51
52	438,099	457,022	18·28	17·71	457,814	504,188	19·42	19·34	52
53	428,801	446,510	17·67	17·12	449,966	495,645	18·75	18·66	53
54	419,256	435,729	17·06	16·53	442,027	486,973	18·08	17·98	54
55	409,460	424,677	16·45	15·95	433,331	477,440	17·43	17·33	55
56	399,498	413,351	15·86	15·37	424,239	467,413	16·79	16·69	56
57	389,088	401,710	15·26	14·80	414,761	456,992	16·17	16·06	57
58	378,481	389,827	14·68	14·24	404,895	446,079	15·55	15·45	58
59	367,570	377,591	14·10	13·68	394,636	434,695	14·94	14·84	59
60	356,330	365,011	13·53	13·14	383,971	422,835	14·34	14·24	60
61	344,711	352,071	12·96	12·60	372,895	410,477	13·75	13·65	61
62	332,789	338,820	12·41	12·07	361,387	397,644	13·17	13·08	62
63	320,451	325,256	11·87	11·56	349,436	384,319	12·60	12·51	63
64	307,720	311,368	11·34	11·05	337,031	370,495	12·05	11·96	64
65	294,588	297,156	10·82	10·55	324,165	356,165	11·51	11·42	65
66	281,054	282,638	10·32	10·07	310,833	341,326	10·98	10·90	66
67	267,160	267,829	9·83	9·60	297,048	325,988	10·47	10·39	67
68	252,901	252,763	9·36	9·14	282,819	310,170	9·97	9·89	68
69	238,328	237,487	8·90	8·70	268,177	293,899	9·48	9·41	69
70	223,490	222,056	8·45	8·27	253,161	277,225	9·02	8·95	70
71	208,453	206,539	8·03	7·85	237,822	260,207	8·57	8·50	71
72	193,297	190,971	7·62	7·45	222,230	242,934	8·13	8·07	72
73	178,114	175,419	7·22	7·07	206,461	225,497	7·71	7·65	73
74	163,003	160,074	6·85	6·70	190,620	208,003	7·31	7·25	74
75	148,076	144,960	6·49	6·34	174,890	190,566	6·93	6·87	75
76	133,453	130,227	6·15	6·00	159,126	173,316	6·56	6·51	76
77	119,251	115,986	5·82	5·68	143,722	156,392	6·21	6·16	77
78	105,592	102,359	5·51	5·37	128,711	139,927	5·88	5·82	78
79	92,587	89,449	5·21	5·07	114,229	124,065	5·56	5·50	79
80	80,343	77,354	4·93	4·79	100,394	108,935	5·26	5·20	80
81	68,946	66,153	4·66	4·51	87,323	94,662	4·98	4·90	81
82	58,471	55,812	4·41	4·26	75,119	81,305	4·71	4·63	82
83	48,970	46,489	4·17	4·01	63,862	68,966	4·45	4·37	83
84	40,171	38,132	3·95	3·58	53,615	57,723	4·21	4·12	84
85	32,979	30,785	3·73	3·56	44,419	47,631	3·98	3·88	85
86	26,476	24,136	3·53	3·36	36,284	38,710	3·76	3·66	86
87	20,926	19,054	3·34	3·17	29,202	30,958	3·56	3·46	87
88	16,268	14,576	3·16	2·99	23,135	24,338	3·36	3·26	88
89	12,428	10,926	3·00	2·82	18,027	18,788	3·18	3·08	89
90	9,321	8,015	2·84	2·66	13,802	14,225	3·01	2·90	90
91	6,859	5,748	2·69	2·51	10,376	10,553	2·85	2·74	91
92	4,946	4,025	2·55	2·37	7,650	7,658	2·70	2·58	92
93	3,492	2,749	2·41	2·24	5,526	5,429	2·55	2·44	93
94	2,411	1,828	2·29	2·12	3,908	3,756	2·42	2·30	94
95	1,628	1,183	2·17	2·01	2,704	2,533	2·29	2·17	95
96	1,071	742	2·06	1·90	1,827	1,661	2·17	2·11	96
97	688	452	1·95	1·81	1,204	1,057	2·06	2·03	97
98	430	266	1·85	1·72	771	653	1·96	1·83	98
99	262	151	1·76	1·65	483	389	1·86	1·73	99
100	154	82	1·68	1·61	295	225	1·76	1·62	100

The Western Australian Life Assurance Companies Act, 1889.

WE have received, through the kindness of Mr. Richard Teece, a copy of "The Western Australian Life Assurance Companies Act, 1889," and now give the text of it in full. It will be found to agree in its main provisions with the Act for South Australia, passed in 1882, and which is given in *J.I.A.*, xxvi, 24.

With respect to *Section* 33, Mr. Teece points out that the provision for the protection of policies against the claims of creditors exists in all the Australian Acts, although the amount varies in almost every case. *Section* 47, as originally introduced, provided that all offices should make returns of the amounts allowed as surrender-values, and should use such amounts as a fund to be drawn upon for the payment of overdue premiums. Owing, however, to opposition on the part of one company, the section was altered to the form in which it now stands.

Regarding *Section* 63, Mr. Teece takes exception to the retention of the form of assignment by endorsement appearing in several of the Australian Acts. He says that the fact that the assignment can be made without expense commends it to the public, who seem to be careless of the circumstance that it leads them to make an absolute assignment when they intend only to give a mortgage.

Mr. Teece does not explain *Sections* 68–71, but states that they are rendered necessary by local circumstances.

WESTERN AUSTRALIA.

ANNO QUINQUAGESIMO TERTIO VICTORIÆ REGINÆ.

No. XII.

An Act to regulate Life Assurance. [Assented to 4 December 1889.]

Preamble.

WHEREAS it is expedient to encourage the practice of life assurance, and to protect persons assured: Be it therefore enacted by His Excellency the Governor of Western Australia and its Dependencies, by and with the advice and consent of the Legislative Council thereof, as follows:

Division.

1. This Act is divided into three parts, as follows:

PART I.—Preliminary.
PART II.—Provisions for Security of Assured.
PART III.—Application of Companies Act, Procedure, and Miscellaneous.

Part I.—Preliminary.

2. This Act may be cited as "The Life Assurance Companies Act, 1889." Short title.

3. In this Act the following terms have the following meanings, unless the context requires a different construction— Interpretation.

"Company" means any persons, corporate or unincorporate, who grant assurances, endowments, or annuities upon human life within Western Australia:

"Chairman" means the person for the time being presiding over the board of directors, committee of management, or other managing body in Western Australia of the company:

"Policy" means any contract for assurance, endowment, or annuity on human life:

"Life assurance business" means the granting of policies:

"Financial year" means each period of twelve months, at the end of which the balance of the accounts of the company is struck, or if no such balance is struck, then each period of twelve months ending with the thirty-first day of December:

"Court" means the Supreme Court of Western Australia:

"Registrar" means the Registrar of Joint Stock Companies under "The Joint Stock Companies Ordinance, 1858":

"Local company" means a company having its head office in Western Australia:

"Foreign company" means a company not having its head office in Western Australia.

Part II.—Provisions for Security of Assured.

4. Every company which shall commence or carry on the business of life assurance within the colony shall deposit with the Colonial Treasurer securities to the value of ten thousand pounds, being mortgages of freehold real estate in Western Australia, on which the money advanced does not exceed two-thirds of the value of the estate mortgaged, or title deeds or certificates of real estate, or bonds, debentures, treasury bills, or other securities issued by the Government, or by any municipal corporation in the colony, duly authorized in that behalf, or the receipt of some incorporated or chartered joint stock bank carrying on business in Western Australia, Companies to deposit securities with Treasurer.

and approved by the Colonial Treasurer, for moneys placed on fixed deposit at the said bank in the name of the Colonial Treasurer, the income arising from such deposit being received by the company: Provided always, that any local company shall not be required to deposit more than fifty per-centum on the amounts of the premiums actually received until the deposit shall amount to ten thousand pounds.

Time for making deposits.

5. As regards any company which now carries on the business of life assurance in Western Australia, the deposit may be made at any time not later than one year after this Act shall come into operation; and as regards any company formed after this Act, the deposit may be made at any time within six months of its incorporation or registration: and no company shall be deemed to carry on the business of life assurance by reason only of receiving premiums in respect of policies issued before this Act shall come into operation.

Further deposits to be made out of receipts.

6. Every company which shall carry on the business of life assurance in Western Australia shall send in annually to the Colonial Treasurer, on or before the first day of June in each year, returns, verified by the certificate of the agent or principal officer of the company in Western Australia, of the amounts received and paid by the company during the year ending the thirty-first day of December then last, or on such other date as the financial year of each company shall determine, on account of policies issued in Western Australia by the company, whether before or after the commencement of this Act, and shall, until the total amount deposited by the company under section 4 and this section shall amount to the sum of twenty thousand pounds, deposit with the Colonial Treasurer securities similar to those mentioned in section 4 to the value of twenty-five per-centum of the excess of the receipts over the disbursements appearing from such returns.

Provision in case of loss of securities.

7. If any securities deposited under this Act are, whilst so deposited, lost, stolen, destroyed, or damaged, the injury occasioned to all persons interested shall be made good out of moneys to be appropriated for the purpose by an Act of the Legislative Council.

Further deposits may be made.

8. Any company may from to time deposit with the Colonial Treasurer any securities of any kind, and to any amount, besides and beyond the securities hereby required to be deposited.

Income and withdrawal of deposits.

9. Any company depositing any securities under this Act shall be entitled to receive the income therefrom, and securities deposited may be withdrawn on timely notice, and, where the deposit is compulsory, on the substitution of similar securities of equal value, and the decision of the Colonial Treasurer shall be conclusive in all matters relating to the value of securities under this Act.

10. All securities deposited with the Colonial Treasurer under this Act shall be primarily charged with the payment and satisfaction of all the liabilities in Western Australia of the depositing company, whether arising in respect of policies issued before or after the commencement of this Act or otherwise howsoever, and no part of such securities shall be applied in payment of any liabilities other than those so charged as aforesaid until the whole of such last-mentioned liabilities shall be paid in full.

Securities deposited to be charged with liabilities in Western Australia.

11. Every foreign company shall keep a separate account of all the life assurance business transacted in Western Australia, and of the entire assets of the company in Western Australia: and in the event of the company becoming bankrupt, or being ordered to be wound up, the entire assets of the company in Western Australia shall be applied, so far as the same will extend, in or towards satisfaction of the liabilities of the company in Western Australia, and no part of such assets shall be applied in payment of any liabilities of the company incurred elsewhere than in Western Australia until the whole of the liabilities incurred in Western Australia shall have been paid in full.

Discharge of Western Australian liabilities of foreign companies.

12. If any foreign company is adjudged bankrupt, or ordered to be wound up, elsewhere than in Western Australia, such company, so far only as regards its assets and liabilities in Western Australia, may, upon the application of any policyholder or shareholder, be ordered to be wound up in Western Australia in like manner as if such company were registered under "The Joint Stock Companies Ordinance, 1858", and proof of such company having become adjudged bankrupt, or ordered to be wound up, shall be conclusive evidence that it is unable to pay its debts.

Mode of distribution of assets of insolvent foreign company.

13. Any director, agent, officer, or servant of any company wilfully committing or assisting in the commission of any breach of the provisions of section 11 shall be deemed guilty of a breach of trust, and be held liable to replace the amount applied contrary to the said section, and shall also be deemed guilty of a misdemeanour punishable on conviction, at the discretion of the Court, by imprisonment for any term not exceeding three years, or by a fine not exceeding five hundred pounds.

Penalty for infringement of Act.

14. Every company transacting other business besides life assurance business shall keep a separate account of all receipts after the passing of this Act in respect of the life assurance business of the company, and the said receipts shall be carried to and form a separate fund to be called the "Life Assurance Fund" of the company, and such fund, however invested, shall be as absolutely the security of the life assurance policyholders as though it belonged to a company carrying on no other than life assurance business, and shall not be liable for any contracts of the

Separation of life assurance and other business.

company for which it would not have been liable had the business of the company been only life assurance business.

Application of preceding section to existing companies.

15. In respect of all existing companies the exemption of the life assurance fund from liability for other obligations than to its life assurance policyholders shall have reference only to the contracts entered into after the passing of this Act, unless by the constitution of the company such exemption already exists; but this and the preceding section shall not apply to any contracts made by any existing company by the terms of whose deed of settlement or articles of association the whole of the profits of all the business are paid exclusively to the life and endowment policy or annuity holders, and on the face of which contracts the liability of the insurers distinctly appears: Provided always, that this Act shall not diminish the liability of the life assurance fund for any contracts of the company entered into before the passing of this Act.

Accounts to be rendered by life assurance companies.

16. Every company transacting life assurance business only shall, at the expiration of each financial year or half-year of such company, prepare a statement of its revenue account for such year or half-year, and of its balance sheet at the close of such year or half-year, in the forms respectively contained in the First and Second Schedules to this Act.

Accounts to be rendered by companies carrying on life assurance and other business.

17. Every company which, concurrently with the transaction of life assurance business, transacts any other kind of assurance or other business, shall, at the expiration of each financial year of such company, prepare a statement of its revenue account for such year, and of its balance sheet at the close of such year, in the forms respectively contained in the Third and Fourth Schedules to this Act.

Accounts to be rendered by foreign companies.

18. Every foreign company shall, at the expiration of each financial year of such company, prepare, in addition to all other statements required by this Act, a statement of all its policies in force at the close of such year, in the form contained in the Fifth Schedule to this Act.

Actuarial report and abstract.

19. Every company shall, once in five years, or at such shorter intervals as may be prescribed by the instrument constituting the company, or by its articles of association, regulations, or bye-laws, cause an investigation to be made into its financial condition by an actuary, and shall cause an abstract of the report of such actuary to be made in the form prescribed in the Sixth Schedule to this Act.

Statement of life annuity business.

20. Every company shall, on or before the thirty-first day of December, one thousand eight hundred and ninety, and thereafter within nine months after the date of each such investigation as aforesaid into its financial condition, prepare a statement of its life assurance and annuity

business, in the form contained in the Seventh Schedule to this Act, each of such statements to be made up as at the date of the last investigation, whether such investigation be made previously or subsequently to the passing of this Act: Provided as follows:

1. If the next financial investigation, after the passing of this Act, of any company fall during the year one thousand eight hundred and ninety-one, the said statement of such company shall be prepared within nine months after the date of such investigation instead of on or before the thirty-first day of December, one thousand eight hundred and ninety.
2. If such investigation be made annually by any company, such company may prepare such statement at any time, so that it be made at least once in every three years.

The expression "date of each such investigation", in this section, shall mean the date to which the accounts of each company are made up for the purposes of each such investigation.

21. Any company whose head office or principal place of business is not in Western Australia may, in lieu of the statement and abstract mentioned in the preceding two sections, deposit at the office of the Registrar a copy, certified in accordance with the 17th section of the Act of the Imperial Parliament known as the "Life Assurance Companies Act, 1870", of the last preceding statement or abstract deposited with the Board of Trade in conformity with the provisions of the 10th section of the said Act.

Forms authorized by Imperial Statute, "Life Assurance Companies Act, 1870," may be used in certain cases.

22. The Governor may alter the forms contained in the schedules to this Act, for the purpose of adapting them to the circumstances of any company, or of better carrying into effect the objects of this Act.

Forms may be altered.

23. Every statement or abstract hereinbefore required to be made shall be signed by the chairman and two of the directors or committee of management, or by the agent of the company in Western Australia, and by the principal officer or agent managing the life assurance business of the company in Western Australia, and if the company has a managing director in Western Australia, by such managing director, and shall be printed; and the original so signed as aforesaid, together with three printed copies thereof, shall be deposited at the office of the Registrar within three months of the dates respectively hereinbefore prescribed as the dates at which the same are to be prepared; and every annual statement so deposited after the first investigation, after the passing of this Act, shall be accompanied by a printed copy of the abstract required to be made as aforesaid.

Statements, &c., to be signed and printed and deposited with Registrar.

24. A printed copy of the last deposited statement, abstract, or other document, by this Act required to be

Copies to be furnished.

printed, shall be forwarded by the company, by post or otherwise, to every shareholder, member, and policyholder of the company in Western Australia.

Agent to be appointed for foreign companies.

25. Every foreign company shall, within six calendar months after the coming into operation of this section, or before doing business in Western Australia, in writing, appoint a person resident therein as general agent, upon whom all lawful processes against the company may be served with like effect as if the company existed in this province; and the said writing or power of attorney shall stipulate and agree, on the part of the company making the same, that any lawful process against the said company which is served on the said general agent shall be of the same legal force and validity as if served on the said company.

Copy of writing to be filed in office of Registrar.

26. A copy of the writing, duly certified and authenticated, shall be filed in the office of the Registrar, and copies certified by him shall be sufficient evidence in all Courts of law and in all judicial proceedings.

Agency to continue as long as any liability outstanding in the colony.

27. The said agency shall continue while any liability remains outstanding against the company in Western Australia, and the power shall not be revoked until the same power is given to another such agent and a like copy filed as aforesaid.

Service of process, &c., on agent sufficient.

28. Service of any process, notice, or otherwise upon the said agent shall be deemed sufficient service upon the principal.

Penalty for non-compliance.

29. No person shall act either as general or other agent of a foreign company until he has complied with all the requirements of this Act; and every person so acting without such compliance, or who knowingly procures payment, or any obligation for the payment, of any premium for insurance or endowment, or for sale of an annuity by fraudulent representations, shall be liable to a penalty not exceeding two hundred and fifty pounds for each offence.

When contracts valid.

30. Every contract for life assurance made by any foreign company without complying with the provisions of this Act contained in the sections numbered 25 to 29, both inclusive, shall be valid and binding on the company; but the agent making the contract shall be liable to the penalty provided in section 31 of this Act; and any such company which neglects to appoint and keep appointed a general agent agreeably to the provisions of this Act shall not recover any premium or other payment on any contract of life assurance with a person resident in Western Australia.

Companies may be prohibited from transacting business in certain cases.

31. Every company which makes default in complying with the provisions of this Act shall not only be liable to the penalties set forth in the 14th section, but may also, if it be made to appear to the Governor that such default has continued for a period of three months, be prohibited

by the Governor from transacting business within the province, either absolutely or for a time, as the Governor may think fit.

Prohibition to be published in *Government Gazette*.

32. Such absolute or temporary prohibition shall be published in the *Government Gazette* of Western Australia; and if any such company, or any person as agent for such company, or otherwise for or on behalf of any such company, shall, after such absolute prohibition, or during any such temporary prohibition, receive any applications for any life assurance, or accept any premium for any life assurance, or otherwise carry on the business of life assurance within Western Australia, such company and person shall respectively be liable to a penalty of two hundred and fifty pounds.

Interest of Assured not liable under certain circumstances.

33. The property and interest of every policyholder in any policy or policies, or in the moneys payable under or in respect of such policy or policies (including every sum payable by way of bonus or profit), shall be exempt from liability to any law now or hereafter in force relating to bankruptcy or insolvency, or from liability to be seized or levied upon by the process of any Court whatever. Provided that no policy for a life assurance or endowment shall be so protected until it shall have endured for at least two years, but that after an endurance of two years such protection shall be afforded to the extent of two hundred pounds of assurance or endowment, and to the contributions made towards the same; and after an endurance of five years, to the extent of five hundred pounds; and after an endurance of seven years, to the extent of one thousand pounds; and after an endurance of ten years, to the extent of two thousand pounds; and that no policy for providing an annuity, nor the contributions made towards the same, shall be protected until the payments made on behalf of such annuity shall have extended over a period of six or more years, or unless it shall have been purchased at a date more than six years prior to the commencement of the annuity, and that such annuity shall not exceed the sum of one hundred and four pounds per annum. Provided also, that the protection hereby afforded shall, in the case of an annuity, accrue only to the benefit of the policyholder himself, and only to such part thereof as shall be payable after he shall have attained the age of fifty years; and, in case of an endowment, for the benefit of the nominee only; and, in the case of a life assurance, for the benefit of the personal representatives only of the policyholder, and in no case for any assignee of the policyholder.

In case any policyholder, or in case of the death of any policyholder his personal representative, entitled to protection under the preceding section has an interest in a policy or policies to an amount greater in the whole than the sums thereunder protected, he shall be entitled

after execution has issued, or a bankruptcy petition or order has been presented and granted, to elect by a writing under his hand, notice of which shall be given to the company, and also to the Sheriff in case of execution issued, and to the trustees of the estate in case of bankruptcy, which of such policies or what part of such policies, up to the limit of the value specified in the preceding clause, shall be so protected. Provided that if he should fail or from any cause be unable so to elect within ten days after being called upon so to do by notice in writing signed by the Sheriff or by such trustee as the case may require, the Sheriff or such trustee may then proceed to elect in like manner as the policyholder could have done, and to have his title to such policy or policies, being in excess of value over the sums protected by the preceding section, registered in due form by the company.

PART III.—APPLICATION OF COMPANIES ACT, PROCEDURE, AND MISCELLANEOUS.

Application of Companies Act to life assurance companies.

34. The provisions of "The Joint Stock Companies Ordinance, 1858", and of any Act amending the same, shall apply to life assurance companies, except in so far as such provisions are modified by this Act, and except also that no life assurance company shall be required to hold more than one general meeting in the year.

List of shareholders.

35. Every proprietary company shall provide a book, to be called "The Shareholders' Address Book," in which the company shall cause to be entered from time to time, in alphabetical order, the corporate names and places of business of the several shareholders of the company being corporations, and the surnames of the several other shareholders, with their respective christian names, places of abode, and descriptions, so far as the same shall be known to the company: and every policyholder or shareholder, or if such shareholder or policyholder be a corporation, the clerk or agent of such corporations, may at all convenient times peruse such book gratis, and the company shall furnish on application, to every shareholder and policyholder of the company, a copy of such book, or of any part thereof, on payment of a sum not exceeding sixpence for every hundred words to be copied for such purpose.

Deed of settlement to be printed.

36. Every company which is not registered under "The Joint Stock Companies Ordinance, 1858", shall cause a sufficient number of copies of its deed of settlement, Act, or charter of incorporation, or other instrument regulating the constitution of the company, to be printed, and shall furnish on application to every shareholder and policyholder of the company a copy thereof, on payment of a sum not exceeding two shillings and sixpence.

37. Where it is intended to amalgamate two or more companies, or to transfer the life assurance business of one company to another, the directors of any one or more of such companies may apply to the Court by petition to sanction the proposed arrangement, fourteen days' previous notice of such application being published in the *Government Gazette* of Western Australia, and the Court, after hearing the directors and other persons whom it considers entitled to be heard upon the petition, may confirm the same if it is satisfied that no sufficient objection to the arrangement has been established. Amalgamation or transfer.

38. Before any such application is made to the Court, notice of such application, together with a statement of the nature of the amalgamation or transfer, as the case may be, and an abstract containing the material facts embodied in the agreement or deed under which such amalgamation or transfer is proposed to be effected, and copies of the actuarial or other reports upon which such agreement or deed is founded, shall be forwarded to each policyholder of both companies in cases of amalgamation, or to each policyholder of the transferred company in case of transfer, by the same being transmitted through the post, directed according to the registered or other known address of such policyholder, within such period as to admit of its being delivered in the due course of delivery fourteen days at least before the day named for the hearing of such application; and in proving such service it shall be sufficient to prove that such notice was properly addressed and put into the post office; and the agreement or deed under which such amalgamation or transfer is effected shall be open for the inspection of the policyholders and shareholders at the office or offices of the company or companies for a period of fifteen days after the issuing of the abstract herein provided. Procedure.

39. The Court shall not sanction any amalgamation or transfer in any case in which it appears to the Court that policyholders representing one-fifth or more of the total amount assured in any company which it is proposed to amalgamate, or in any company the business of which it is proposed to transfer, dissent from such amalgamation or transfer. Conditions.

40. No company shall amalgamate with another, or transfer its business to another, unless such amalgamation or transfer is confirmed by the Court in accordance with this section: Provided always, that this section shall not apply in any case in which the business of any company which is sought to be amalgamated or transferred does not comprise life assurance business. Confirmation.

41. When an amalgamation takes place between any companies, or when the business of one company is transferred to another company, the combined company or the purchasing company, as the case may be, shall, Statements in case of amalgamation or transfer.

within ten days from the date of the completion of the amalgamation or transfer, deposit at the office of the Registrar certified copies of statements of the assets and liabilities of the companies concerned in such amalgamation or transfer, together with a statement of the nature and terms of the amalgamation or transfer, and a certified copy of the agreement or deed under which such amalgamation or transfer is effected, and certified copies of the actuarial or other reports upon which such agreement or deed is founded, and the statement and agreement or deed of amalgamation or transfer shall be accompanied by a declaration under the hand of the chairman of each company, and the principal managing officers of each company, that, to the best of their belief, every payment made, or to be made, to any person whatsoever, on account of the said amalgamation or transfer, is therein fully set forth, and that no other payments beyond those set forth have been made or are to be made either in money, policies, bonds, valuable securities, or other property, by or with the knowledge of any parties to the said amalgamation or transfer.

Regulation as to novations by policyholders.

42. Where a company, either before or after the passing of this Act, has transferred its business to, or been amalgamated with, another company, no policyholder in the first-mentioned company who shall pay to the other company the premiums accruing due in respect of his policy shall, by reason of any such payment made after the passing of this Act, or by reason of any other act done after the passing of this Act, be deemed to have abandoned any claim which he would have had against the first-mentioned company on due payment of premiums to such company, or to have accepted, in lieu thereof, the liability of the other company, unless such abandonment and acceptance have been signified by some writing signed by him, or by his agent lawfully authorized.

Inspection of deposited documents.

43. Any person may, on payment of such fees as the Governor may direct, inspect at the office of the Registrar any printed or other document required by this Act to be deposited at such office, and procure copies thereof.

Documents to be received in evidence.

44. Every statement, abstract, or other document deposited with the Registrar under this Act shall be receivable in evidence, and every document purporting to be certified by the Registrar to be such deposited document, and every document purporting to be similarly certified to be a copy of such deposited document, shall, if produced out of the custody of the Registrar, be deemed to be such deposited document as aforesaid, or a copy thereof, and shall be received in evidence as if it were the original document, unless some variation between it and the original document shall be proved.

45. Every company which makes default in complying with the requirements of this Act, and shall continue in such default for seven days after notice by the Registrar, or any person interested in the matter of such default, shall be liable to a penalty not exceeding fifty pounds for every day during which the default continues: and in the case of a foreign company the general agent shall be liable to such penalty as well as the company: and in the case of companies registered under "The Joint Stock Companies Ordinance, 1858", if default continues for a period of three months after notice of default by the Colonial Treasurer, which notice shall be published in one or more newspapers as the Colonial Treasurer may direct, the Court may order the winding-up of the company, in accordance with the said Act, upon the application of one or more policyholders or shareholders.

Penalty for non-compliance with Act.

46. If any statement, abstract, or other document required by this Act is false in any particular to the knowledge of any person who signs the same, such person shall be guilty of a misdemeanour, and being convicted thereof shall be liable, at the discretion of the Court, to be imprisoned for any term not exceeding three years, or to a penalty not exceeding five hundred pounds.

Penalty for falsifying statements.

47. Every life assurance society shall declare the surrender-value at which the said society becomes bound to accept their policies.

Surrender-value to be declared.

48. Every penalty imposed by this Act shall be recovered and applied in the same manner as penalties imposed by "The Joint Stock Companies Ordinance, 1858", are recoverable and applicable.

Recovery and application of penalties.

49. The Court may order the winding-up of any company in accordance with "The Joint Stock Companies Ordinance, 1858", on the petition of five or more policyholders or shareholders, upon its being proved to the satisfaction of the Court that the company is insolvent; and in determining whether or not the company is insolvent the Court shall take into account its contingent or prospective liability, under policies and annuity and other existing contracts.

Winding-up of company.

50. The Court shall not give a hearing to the petition until security for costs, for such amount as a judge shall think reasonable, shall be given, and until a *primâ facie* case shall also be established to the satisfaction of the judge.

Security for costs.

51. In the case of a proprietary company having an uncalled capital of an amount sufficient, with the future premiums receivable by the company, to make up the actual invested assets equal to the amount of the estimated liability, the Court shall suspend further proceedings on the petition for a reasonable time (in the discretion of the

Proprietary company.

Court) to enable the uncalled capital, or a sufficient part thereof, to be called up, and if, at the end of the original or any extended time for which the proceedings have been suspended, such an amount shall not have been realized by means of calls as with the already invested assets shall be equal to the liabilities, an order shall be made on the petition as if the company had been proved insolvent.

Winding-up of subsidiary company.

52. Where the business or any part of the business of a company has, either before or after the passing of this Act, been transferred to another company, under an arrangement in pursuance of which such first-mentioned company (in this Act called the subsidiary company), or the creditors thereof, has or have claims against the company to which such transfer was made (in this Act called the principal company), then, if such principal company is being wound up by or under the supervision of the Court, either at or after the passing of this Act, the Court shall (subject as hereinafter mentioned) order the subsidiary company to be wound up in conjunction with the principal company, and may, by the same or any subsequent order, appoint the same person to be liquidator for the two companies, and make provisions for such other matters as may seem to the Court necessary, with a view to such companies being wound up as if they were one company, and the commencement of the winding-up of the principal company shall, save as otherwise ordered by the Court, be the commencement of the winding-up of the subsidiary company. The Court, nevertheless, shall have regard, in adjusting the rights and liabilities of the members of the several companies between themselves, to the constitution of such companies, and to the arrangements entered into between the said companies, in the same manner as the Court has regard to the rights and liabilities of different classes of contributories in the case of the winding up of a single company, or as near thereto as circumstances admit.

Contracts of insolvent company may be reduced.

53. The Court, in the case of a company which has been proved to be insolvent, may, if it thinks fit, reduce the amount of the contracts of the company or society, upon such terms and subject to such conditions as the Court thinks fit, in place of making a winding-up order.

Provision in case of subsidiary company not being in process of winding-up.

54. Where any subsidiary company or company alleged to be subsidiary, is not in process of being wound up at the same time as the principal company to which it is subsidiary, the Court shall not direct such subsidiary company to be wound up unless, after hearing all objections (if any) that may be urged by or on behalf of such company against its being wound up, the Court is of opinion that such company is subsidiary to the

principal company, and that the winding-up of such company in conjunction with the principal company is just and equitable.

55. An application may be made in relation to the winding up of any subsidiary company in conjunction with a principal company by any creditor or policyholder of, or person interested in, such principal or subsidiary company.

Application for winding-up subsidiary company.

56. Where a company stands in the relation of a principal company to one company, and in the relation of a subsidiary company to some other company, or where there are several companies standing in the relation of subsidiary companies to one principal company, the Court may deal with any number of such companies, together or in separate groups, as it thinks most expedient, upon the principles laid down in this section.

Grouping of companies.

57. Where a company is being wound up by the Court, or subject to the supervision of the Court, or voluntarily, the value of every life annuity and life policy requiring to be valued shall be estimated in manner provided by the Eighth Schedule.

Valuation of policies.

58. When an assurance company is being wound up by the Court, or subject to the supervision of the Court, the official liquidator, in case of all persons appearing by the books of the company to be entitled to or interested in policies granted by such company for life assurance, endowment, annuity, or other payment, is to ascertain the values of such policies, and give notice of such value to such persons; and any person to whom notice is so given shall be bound by the value so ascertained, unless he give notice of his intention to dispute such value in manner and within a time to be prescribed by a rule or order of the Court.

Notice of valuation to be given.

59. Any notice which is by this Act required to be sent to any policyholder may be addressed and sent to the person to whom notices respecting such policy are usually sent: and any notice so addressed and sent shall be deemed and taken to be notice to the holders of such policy.

Notices to policyholders.

60. There shall be laid annually before the Legislative Council the statements and abstracts of reports deposited with the Registrar under this Act during the preceding year.

Statements to be laid before Legislative Council.

61. Upon the death of any holder of a policy upon his own life for a sum not exceeding two hundred pounds, if no probate of his will or letters of administration to his estate be taken out within three months after his death, the company may pay the amount of such policy to his widow, or any adult child of his, and the receipt of such widow or child shall be a valid discharge, both at law and in equity, for the same.

Probate or administration may be dispensed with in certain cases.

Receipts of executors or administrators valid discharges.

62. The receipt of the executor or administrator of any deceased policyholder shall be a valid discharge, both at law and in equity, for any moneys payable under the policy held by him at the time of his death.

Assignment of policies.

63. Every assignment made after the first day of January, one thousand eight hundred and ninety, of a policy, may be made by memorandum of transfer indorsed upon such policy, in the form in the Tenth Schedule, and no such assignment shall be of any validity until registered as hereinafter provided.

Notice and registration.

64. Notice of every such assignment shall be given to the company for the time being liable upon the policy assigned, and such assignment shall be registered in a book to be provided by the company for that purpose, and the date of such registration shall be inserted in the memorandum of transfer, which shall be also signed by the principal officer managing the life assurance business of the company in Western Australia, or his deputy, and thereafter the assignee may sue, as well at law as in equity, in his own name, on the policy assigned, and the receipt of such assignee shall be a valid discharge, both at law and in equity, for all moneys payable thereunder. Every such memorandum of transfer signed as last aforesaid shall be conclusive evidence of the registration thereof, and of the date of such registration.

No notice of mortgage or trusts.

65. If any policy is assigned by way of mortgage, or upon any trust, such mortgage or trust shall be effected by way of defeasance or declaration of trust by some separate instrument, and no notice of any such mortgage or trust shall be entered on the memorandum of transfer or indorsed on the policy, and the company shall not be affected either by express, implied, or constructive notice of any such mortgage or trust, nor be bound or concerned to see to the application of any moneys payable under such policy.

Duties of Colonial Treasurer.

66. It shall be the duty of the Colonial Treasurer to secure the due observance by every company of the provisions of this Act; and the Governor may, by regulation to be published in the *Government Gazette* of Western Australia, declare what fees shall be payable by companies in respect of the duties to be performed hereunder by the Colonial Treasurer.

Registration in case policy lost.

67. In case any policy or instrument required to be registered, or the production of which is in any way essential to any registration required to be made under this Act, shall be lost or destroyed, the manager upon such evidence, and subject to such conditions as he shall think fit, shall issue a certified copy of such policy, and he may, upon such conditions as he shall see fit, effect any registration, notwithstanding the loss or destruction of such instrument as aforesaid.

68. All policies for the time being on the Western Australian register shall be treated as Western Australian assets and liabilities of the company on whose register they are, and be subject in every respect to the laws of such colony.

Policies on Western Australian register subject to laws of colony.

69. Any policy issued in Western Australia by a company, or transferred to the register of that company in Western Australia, may, at the request in writing of the policyholder and with the consent of the company, be transferred to the register of that company in any other colony or country, and shall thereupon cease to be subject to the laws of this colony.

Transfers.

70. Any policy issued outside Western Australia by a company, or transferred from the Western Australia register of that company to its register in any other colony or country, may, at the request in writing of the policyholder and with the consent of the company, be transferred to the register of that company in Western Australia.

Transfers.

71. The provisions of the two preceding sections shall be retrospective as well as prospective, and shall apply to transfers of policies already made, as well as to those which shall be made after the passing of this Act.

Operation of two preceding sections.

In the name and on behalf of the Queen I hereby assent to this Act.

F. NAPIER BROOME,

Governor.

SCHEDULES.

FIRST SCHEDULE.

Revenue Account *for the Year ending*

	£ s. d.		£ s. d.
18 (Date)		18 (Date)	
Amount of Funds at beginning of the year . . .		Claims under Policies (after deduction of Sums Re-assured)	
Renewal Premiums after deduction of Re-assurance Premiums		Surrenders	
New Premiums (after deduction of Re-assurance Premiums) on new Policies, assuring £ . and yielding an Annual Revenue of £ , after deduction of Re-assurances		Annuities	
Consideration for Annuities granted		Commission on New Premiums	
Interest and Dividends		" Renewals	
Other Receipts (Accounts to be specified) . . .		Expenses of Management	
		Dividends and Bonuses to Shareholders (if any) . .	
		Other Payments (Accounts to be specified) . . .	
		Amount of Funds at the end of the year, as per Second or Fourth Schedule	
£		£	

Note 1.—Companies having separate accounts for annuities to return particulars of their annuity business in a separate statement.

Note 2.—Items in this and in the accounts in the Third and Fifth Schedules should be the net amounts, after the deduction of the amounts paid and received in respect of re-assurances.

SECOND SCHEDULE.

Balance Sheet of the on the 18 .

Liabilities.	£ s. d.	£ s. d.
Shareholders' Capital paid up (if any)		
Assurance Fund		
Annuity Fund (if any)		
Other Funds (if any, to be specified) . . .		
Total Funds, as per Third Schedule		
Claims admitted but not paid*		
Other Sums owing to the Company (Accounts to be specified)*		
	£	

Assets.	£ s. d.	£ s. d.
In Western Australia:		
Western Australian Government Securities . .		
Other Government Securities (particulars to be specified)		
Mortgages		
Loans on the Company's Policies		
Loans upon Personal Security		
Railway and other Debentures and Debenture Stocks		
Railway Shares (Preferential and Ordinary) . .		
House Property		
Other Investments (to be specified) . . .		
Agents' Balances		
Outstanding Premiums		
Outstanding Interest		
Cash On Deposit		
In hand and on Current Account . . .		
Other Assets (to be specified)		
Total Assets in Western Australia . .		
Elsewhere than Western Australia:		
British Government Securities		
Indian and Colonial Government Securities . .		
Foreign Government Securities		
Mortgages (stating where)		
Loans on the Company's Policies		
Loans upon Personal Security		
Railway and other Debentures and Debenture Stocks		
Railway Shares (Preferential and Ordinary) . .		
House Property (stating where)		
Other Investments (to be specified)		
Agents' Balances		
Outstanding Premiums		
Outstanding Interest		
Cash On Deposit		
In hand and on Current Account . . .		
Other Assets (to be specified)		
Total Assets elsewhere than in Western Australia .		
Total Assets	£	

* These items are included in the corresponding items in the First Schedule.

THIRD SCHEDULE.

Revenue Account of the *for the Year ending*

No. 1.—LIFE ASSURANCE ACCOUNT.

(Date)	£ s. d.	(Date)	£ s. d.
Amount of Life Assurance Fund at the beginning of the year		Claims under Life Policies, after deduction of Sums Re-assured	
Renewal Premiums, after deduction of Re-assurance Premiums		Surrenders	
New Premiums (after deduction of Re-assurance Premiums) on New Policies assuring £ , and yielding an Annual Income of £ , after deduction of Re-assurances .		Annuities	
Consideration for Annuities granted		Commission	
Interest and Dividends		Expenses of Management	
Other Receipts (Accounts to be specified)		Other Payments (Accounts to be specified)	
		Amount of Life Assurance Fund at the end of the year, as per Fourth Schedule	
£		£	

Note.—Companies having separate Accounts for Annuities to return the particulars of their Annuity business in a separate Statement.

No. 2.—FIRE ACCOUNT.

(Date)	£ s. d.	(Date)	£ s. d.
Amount of Fire Insurance Fund at the beginning of the year		Losses by Fire, after deduction of Re-assurance . . .	
Premiums received, after deduction of Re-assurances . .		Expenses of Management	
Other Receipts (to be specified)		Commission	
		Other Payments (to be specified)	
		Amount of Fire Insurance Fund at the end of the year, as per Fourth Schedule	
£		£	

Note.—When Marine or any other branch of business is carried on, the Income and Expenditure thereof to be in like manner stated in a separate Account.

No. 3.—PROFIT AND LOSS ACCOUNT.

(Date)	£ s. d.	(Date)	£ s. d.
Balance of last year's Accounts		Dividends and Bonuses to Shareholders	
Interest and Dividends not carried to other Accounts . .		Expenses not charged to other Accounts	
Profits realized (Accounts to be specified)		Loss realized (Accounts to be specified)	
Other Receipts		Other Payments	
		Balance as per Fourth Schedule	
£		£	

Note.—This account is not required if the items have been incorporated in the other accounts of this schedule.

FOURTH SCHEDULE.

Balance Sheet of the on the 18 .

LIABILITIES.	£ s. d.	ASSETS.	£ s. d.	£ s. d.
Shareholders' Capital		*In Western Australia:*		
General Reserve Fund (if any)		Western Australian Government Securities . .		
Life Assurance Fund*		Other Government Securities (particulars to be specified)		
Annuity Fund (if any)*		Mortgages		
Fire Fund		Loans on Company's Policies		
Marine Fund		Loans upon Personal Security		
Profit and Loss (if any)		Railway and other Debentures and Debenture Stocks		
Other Funds (if any, to be specified)		Railway Shares (Preferential and Ordinary) . .		
£		House Property		
		Other Investments (to be specified) . . .		
Claims under Life Policies admitted but not yet paid* . .		Agents' Balances		
Outstanding Fire Losses		Outstanding Premiums		
Outstanding Marine Losses		Outstanding Interest		
Other Sums owing by the Company (Accounts to be specified)		Cash—On Deposit		
		In hand and on Current Account . . .		
		Other Assets (to be specified)		
		Total Assets in Western Australia . .		
		Elsewhere than in Western Australia:		
		British Government Securities		
		Indian and Colonial Government Securities . .		
		Foreign Government Securities		
		Mortgages (stating where)		
		Loans on the Company's Policies		
		Loans upon Personal Security		
		Railway and other Debentures and Debenture Stocks		
		Railway Shares (Preferential and Ordinary) . .		
		House Property (stating where)		
		Other Investments (to be specified) . . .		
		Agents' Balances		
		Outstanding Premiums		
		Outstanding Interest		
		Cash—On Deposit		
		In hand and on current Account . . .		
		Other Assets (to be specified)		
		Total Assets elsewhere than in Western Australia .		
£		Total Assets . . . . £		

* If the life assurance fund is in accordance with section 20 of this Act, a separate trust fund for the sole security of the life policyholders, or a separate balance sheet for the life branch, may be given in the form contained in the Second Schedule. In other respects the company is to observe the above form. See also note to Second Schedule.

FIFTH SCHEDULE.

1. New policies issued by the during the year ending

	In West Australia				Elsewhere			
	No. of Policies	Sum Assured	Single Premiums	Annual Premiums	No. of Policies	Sum Assured	Single Premiums	Annual Premiums
Assurance . . .								
Endowment . .								
Annuity . . .		(per annum)				(per annum)		
Totals . . .								

2. Policies discontinued in the during the year ending

How discontinued	Assurance Policies			Endowment Policies			Annuity Policies		
	No.	Sum Assured	Annual Premiums	No.	Sum Assured	Annual Premiums	No.	Annuity per annum	Annual Premiums
		£ s. d.	£ s. d.		£ s. d.	£ s. d.		£ s. d.	£ s. d.
By Death or Maturity									
By Surrender . .									
By Forfeiture . .									
Total . .									

3. Policies of the existing on the

	In New South Wales		In Victoria		In New Zealand		In South Australia		In Queensland		In Tasmania		In West Australia		Elsewhere	
	No. of Poli-cies	Sums Assured	No. of Poli-cies	Sums Assured	No. of Poli-cies	Sums Assured	No. of Poli-cies	Sums Assured	No. of Poli-cies	Sums Assured	No. of Poli-cies	Sums Assured	No. of Poli-cies	Sums Assured	No. of Poli-cies	Sums Assured
Assurance . .																
Endowment .																
Annuity . .		per ann.		per ann.		per ann.		per ann.		per ann.		per ann.		per ann.		per ann.
Totals . .																

4. Progress of the Life Assurance business of the since its establishment to

	No. of Policies	Sums Assured	Annuity per annum	Annual Premiums
Total Issued				
Total Discontinued . .		£ s. d.	£ s. d.	£ s. d.
Existing . . .				
Annual Income from Interest				
Total Annual Income				

SIXTH SCHEDULE.

Statement respecting the Valuation of the Liabilities under Life Policies and Annuities of the to be made by the Actuary.

[The answers should be numbered to accord with the numbers of the corresponding questions.]

1. The date up to which the valuation was made?

2. The principles upon which the valuation was made, and whether these principles were determined by the instrument constituting the company or by its regulations, or by bye-laws or otherwise?

   (*a*) Were the policies valued individually or in classes?

   (*b*) If in classes, how was the valuation age determined?

   (*c*) What portion (if any) of a year's premium was assumed to be due?

   (*d*) Were lives assured at increased rates assumed to be of the age at entry corresponding to the premium charged? If not, how were they dealt with?

3. The table or tables of mortality used in the valuation?

4. The rate or rates of interest assumed in the calculations?

5. By what table of mortality, and according to what rate of interest, have the net premiums valued been computed? Give specimens of such premiums for a whole-life and an endowment-assurance policy of £100, for ages at entry, 20, 25, 30, 35, 40, 45, 50, 55, 60.

6. The proportion of the annual premium income (if any) reserved as a provision for future expenses and profits? [*If none, state how this provision is made.*] In cases were assurances have been effected by single or limited premiums, state what provision has been made for expenses when the premiums cease to be payable.

7. The consolidated revenue account since the last valuation, or, in case of a company which has made no valuation, since the commencement of the business? [*This return must be made in the form annexed.*]

8. The liabilities of the company under life policies and annuities at the date of the valuation, showing the number of policies, the amount assured, and the amount of premiums payable annually under each class of policies, both with and without participation in profits, and also the net liabilities and assets of the company with the amount of surplus or deficiency? [*These returns must be made in the forms annexed.*]

9. The principles upon which the distributions of profits are made?

10. The time during which a policy must be in force to entitle it to share in the profits?

11. The results of the valuation, showing:—

   (*a*) The total amount of profit made by the company.

   (*b*) The amount of profit divided among the policyholders, and the number and amount of policies which participated.

   (*c*) Specimens of bonuses allotted to policies for £100, effected at the respective ages of 20, 30, 40, 50, and having been respectively in force for five years, ten years, and upwards, at intervals of five years, together with the amount appropriated under the various modes in which the bonus might be received.

[FORM referred to under heading No. 7 in Sixth Schedule.]

Consolidated Revenue Account of the *for* *years, commencing* *and ending*

	£ s. d.		£ s. d.
Amount of Funds on 18 , the beginning of		Claims under Policies, after deduction of Sums Re-assured . .	
Premiums, after deduction of Re-assurance Premiums . . .		Surrenders	
Consideration for annuities granted		Annuities	
Interest and Dividends . . .		Commission	
Other Receipts (Accounts to be specified)		Expenses of Management . .	
		Dividends and Bonuses to Shareholders (if any) . . .	
		Other Payments (Accounts to be specified)	
		Amount of Funds on 18 , the end of the period, as per First (or Third) Schedule .	
£		£	

[FORM referred to under heading No. 8, in Sixth Schedule.]

Summary and Valuation of the Policies of the as at

Description of Transactions	Particulars of the Policies for Valuation						Valuation — Value by the Table, Interest per-cent					
	No. of Policies	Sum Assured	Bonus Addition	Office Yearly Premiums	Net Yearly Premiums	Loading	Sum Assured	Bonus Addition	Office Yearly Premium	Net Yearly Premium	Loading	Net Liability
Assurances—												
1. With participation in Profits—												
For Whole Term of Life . .												
Other Classes (to be specified) .												
Extra Premiums payable . .												
Total Assurances with Profits												
2. Without participation in Profits—												
For the Whole Term of Life .												
Other Classes (to be specified) .												
Extra Premiums payable . .												
Total Assurances without Profits												
Total Assurances												
Deduct Re-assurances . . .												
Net Amount of Assurances . .												
Adjustments (if any) . . .												
Annuities—												
Immediate												
Other Classes (to be specified) . .												
Total of the Results . .												

The item "Extra Premium", in this Act shall be taken to mean the charge for any risk not provided for in the minimum contract premium. If policies are issued in or for any country, at rates of premium deduced from tables other than the European mortality tables adopted by the company, separate schedules, similar in form to the above, must be furnished.

[FORM referred to under heading No. 8 in Sixth Schedule.]

Valuation Balance Sheet of *as at* 18

DR.	£ s. d.	CR.	£ s. d.
To net Liability under Assurance and Annuity Transactions (as per Summary Statement provided in Seventh Schedule) .		By Life Assurance and Annuity Funds (as per Balance Sheet under Third or Fifth Schedule)	
To Surplus (if any) . . .		By Deficiency (if any) . .	
£		£	

SEVENTH SCHEDULE.

Statement of the Life Assurance and Annuity business of the *on the*

[The answers should be numbered to accord with the numbers of the corresponding questions. Statements of re-assurances, corresponding to the statements in respect of assurances under headings 2, 3, 4, 5, 6, and 7, are to be given.]

1. The published table or tables of premiums for assurances for the whole term of life which were in use at the date above mentioned?

2. The total number of policies and amounts assured on lives for the whole term of life which were in existence at the date above mentioned, distinguishing the portions assured with and without profits, stating separately the total reversionary bonuses, and specifying the sums assured for each year of life, from the youngest to the oldest ages?

3. The amount of premiums receivable annually for each year of life after deducting the abatements made by the application of bonuses in respect of the respective assurances mentioned under heading No. 2, distinguishing ordinary from extra premiums?

4. The total amount of premiums which have been received from the commencement upon all policies mentioned under heading No. 2, which were in force at the above-mentioned date?

5. The total number of policies and amounts assured under classes of assurance business other than for the whole term of life, distinguishing the number of policies and the sums assured under each class, and stating separately the amount assured, with and without profits, and the total amount of reversionary bonuses?

6. The amount of premiums receivable annually in respect of each such special class of assurances mentioned under heading No. 5, distinguishing ordinary from extra premiums?

7. The total amount of premiums which have been received from the commencement upon all policies under each special class mentioned under heading No. 5, which were in force at the date above mentioned?

8. The total amount of immediate annuities on lives, distinguishing the amount for each year of life?

9. The amount of all annuities other than those specified under heading No. 8, distinguishing the amount payable under each class, the amount of premiums annually receivable, and the amount of consideration money received in respect of each such class, and the total amount of premiums received from the commencement upon all deferred annuities?

10. The average rate of interest at which the life assurance fund of the company was invested at the close of each year during the period since the last investigation, together with a statement of the manner in which such average has been computed?

11. A table of minimum values (if any) allowed for the surrender of policies for the whole term of life, and for endowments, and endowment assurances, or a statement of the method pursued in calculating such surrender-values, with instances of its application to policies of different standing, and taken out at various interval ages, from the youngest to the oldest?

12. Separate statements to be furnished for business at other than European rates, together with a statement of the manner in which policies on unhealthy lives are dealt with.

EIGHTH SCHEDULE.

Rule for Valuing an Annuity.

An annuity shall be valued according to the tables used by the company which granted such annuity at the time of granting the same; and where such tables cannot be ascertained or adopted to the satisfaction of the Court, then according to the table known in England as the Government Annuities Experience Table, interest being reckoned at the rate of four pounds per-centum per annum.

Rule for Valuing a Policy.

The value of the policy is to be the difference between the present value of the reversion in the sum assured on the decease of the life, including any bonus or addition thereto made before the

commencement of the winding-up, and the present value of the future annual premiums.

In calculating such present values the rate of interest is to be assumed as being four pounds per-centum per annum, and the rate of mortality as that of the tables known in England as the Institute of Actuaries H^M Life Tables.

The premium to be calculated is to be such a premium as, according to the said rate of interest and rate of mortality, is sufficient to provide for the risk incurred by the office in issuing the policy, exclusive of any addition thereto for office expenses and other charges.

NINTH SCHEDULE.

Memorandum of Transfer.

Date of transfer, and state whether transfer absolute or partial, and if partial to what amount transferred	Signature of Transferor	Witness	Transferee: Name in full	Transferee: Address	Transferee: Occupation	Signature of Transferee	Witness	Date of Registration of Transfer	Signature of principal Officer of Company

---

Further Remarks on Mr. Woolhouse's Method of Graduation; with a new Adjustment of the $H^{M(5)}$ Table by the Graphic Method. *By* T. B. Sprague, M.A., F.I.A., &c.

Mr. Woolhouse must have been very angry and excited when he wrote his *Observations* (*J.I.A.*, xxvi, 424) on my paper describing the Graphic Method of Graduation; or he would certainly not have used the very strong language he has indulged in. Besides speaking in the first paragraph of his paper of a "diatribe of erroneous statements", "gratuitous and inconsiderate statements", and "grossly misleading" statements, he employs many other expressions in the course of his paper, which seem to me to be quite unsuitable in the discussion of a scientific

question: see, especially, the unmannerly sneer on p. 425, *top*. I regret that my criticisms of his method should have ruffled his temper; but in a matter of this kind I must be guided more by regard for scientific truth, than by any personal considerations; and at the risk, therefore, of again arousing his wrath, I wish to say deliberately that further consideration has not led me to alter in any way the opinions I exprest in my paper on the Graphic Method (*J.I.A.*, xxvi, 77). I trust, however, that the time which has elapst since the appearance of Mr. Woolhouse's *Observations* (more than three years) will have been sufficient to allow temporary feelings of irritation on either side to cool down, and that any further discussion of the points at issue may be carried on in the spirit of calm scientific enquiry.

I wish to assure Mr. Woolhouse in the first place that my remarks on his method were certainly not, as he says, "inconsiderate"; but were, on the contrary, slowly formed, maturely considered, and carefully exprest. I believe that the points in dispute between Mr. Woolhouse and myself are not mere matters of opinion, but are matters of fact, that admit of being settled by the use of appropriate arguments. I have said nothing in disparagement of his method without giving at the same time what I considered was a sufficient reason for being dissatisfied with his reasoning or conclusions; and my criticisms cannot be disposed of by characterizing them as "simply amusing", or by saying, without proof, that my statements are erroneous.

I will now make such replies to Mr. Woolhouse's observations as seem desirable, without repeating what I said in my above-mentioned paper; and my readers will then be in a position to judge for themselves whether it is Mr. Woolhouse's statements, or mine, that are erroneous.

Mr. Woolhouse tells us (p. 422) that there is nothing of an arbitrary nature in his process; but I say that his selection of quinquennial values in the series to be adjusted, is entirely arbitrary. If he had taken every third, or fourth, or sixth, or seventh value, instead of every fifth, he would have got a different formula; and I cannot find that he has given any reason why the formula resulting from quinquennial values should be preferred to all the others.

Mr. Woolhouse also tells us (p. 424) that his method stands alone, as systematically based on reasonable principles directed

primarily to the elimination of error; but, notwithstanding the learned discourse he has given about probable errors, I fail to see that he has said anything that should lead us to expect that his formula is at all likely to give us better results than would be given by any one of a dozen others which might easily be framed; such, for instance, as Mr. Higham's. In fact, the general impression left on my mind after a fresh perusal of Mr. Woolhouse's various remarks, is that they contain much positive assertion as to the merits of his process, but very little, if anything, in the way of proof of those merits. I therefore see no sufficient reason for distinguishing between his formula and others of a like kind; and I include them all in one general condemnation.

Mr. Woolhouse states (p. 422) that a repetition of his process would be objectionable, because it would introduce terms of the series that are too remote to be admitted as having any legitimate influence on the result. In my opinion the same objection applies to his process, as handled by himself; for I cannot admit that, when we seek for an adjusted value of l, the values of l_{-7}, l_{-6}, l_{+6}, l_{+7}, necessarily have always a legitimate influence on the result.

In Mr. Woolhouse's original description of his process (*Institute of Actuaries' Life Tables*, p. xciii), he says that, if the original points, taken in groups, were to range in curves of the third order, they would in such case not be subject to any alteration whatever by the operation of adjustment made by his process. *This statement, thus made without any qualification, is incorrect.* If one of the groups supposed by Mr. Woolhouse contains fifteen terms, the central term of the group will certainly remain unaltered by his adjustment, but *all the others will be altered.* If the group contains more terms, then more of them will be left unaltered, but the seven first and the seven last in the group, will always be altered. Now it is so well known as to require no demonstration, that no series of numbers connected with a mortality table, proceeds by constant third differences *throughout the whole extent* of the table; and it is therefore of no avail for Mr. Woolhouse to argue, as he does on page 425, that the constant multiplier is precisely unity for all algebraic functions of the third degree, and that this is all that is required to establish the practical accuracy of his method. He tells us that "the assumption of constant fourth differences will necessarily

take a series of numbers very far out of the range of all mortality tables." I think, however, that it should have been proved, and not assumed, that constant fourth differences would give worse results than constant third differences; and until this is proved, I am entitled to say that fourth differences would give a better approximation to the truth than third differences; and that, if we were to take constant fifth and sixth differences we should get still more accurate results, none of them, however, being quite satisfactory. To sum up this branch of my argument, the fact is undeniable, that the numbers living in a mortality table do not form a series with constant third differences; and I have proved, and Mr. Woolhouse virtually admits, that the application of his method to any series that is not of that character, will, to a greater or less extent, alter the law of the series.

The objections I urge against Mr. Woolhouse's method and all similar methods of graduation, are, firstly, that they do not get rid of the irregularities which the original series of numbers presents; secondly, that they have a tendency to disturb the law of the series. The first of these objections Mr. Woolhouse admits, telling us (p. 424) that his method regards smoothness of progression only as an adjunct: and that, when his method has been applied, there may still remain traces of irregularity which are to be amended by inspection of the differences (p. 422). I believe that, if Mr. Woolhouse had so plainly stated all this in the first instance, his method would not have been so greatly esteemed, and so popular, as it was for a time. I maintain, on the contrary, that the true law of mortality can show no irregularities as we pass from age to age; and, accordingly, in my method of graduation I lay it down as a fundamental principle, that the irregularities must be got rid of. In a passage referred to by Mr. Woolhouse (p. 425, *top*), I prove that his formula and all kindred ones must necessarily fail to eliminate the irregularities of the original observations; and I now repeat what I said before, that this argument, when fully considered, seems to be conclusive against all such methods. The difference of opinion between Mr. Woolhouse and myself on this point arises from the fact that he is satisfied with reducing the irregularities, or rather distributing them over other parts of the curve; whereas I say that a method of graduation, in order to be satisfactory, must altogether get rid of the irregularities.

My second objection to Mr. Woolhouse's method and all

similar methods, namely, that they have a tendency to distort the true law, he meets with a flat denial. He does not attempt to throw any doubt on the validity of the mathematical demonstration by which I prove beyond all question that his process must distort any law except that of constant third differences; but he contents himself with a plausible argument (p. 423) as to the effect of the process of interpolation. Having carefully examined this argument, I have come to the conclusion that it is so vague that, taking it by itself, nobody can possibly say whether it is entitled to any weight. Nevertheless, I feel no hesitation in saying that, since his argument leads to a conclusion which I have proved to be false, the argument itself is fallacious.

The real question at issue being—Which is the better method of graduation, Mr. Woolhouse's or mine? it is clear that a satisfactory answer can only be obtained by comparing two graduations of the same mortality table made by the two methods. Mr. Woolhouse has suggested that I should apply his method to graduate the experience of the Female Government Annuitants; but I have thought it better for various reasons to apply my own method to a table previously graduated by him, and I have selected for this purpose the $H^{M(5)}$ Table. I cannot find that the unadjusted facts of that table have been ever printed, and consequently the first step I had to take was to calculate the numbers at risk, and the deaths at each age, from the original figures given in the "Mortality Experience" publisht by the Institute in the year 1869. These fundamental data are given in the following table:

Age	Numbers at Risk	Deaths	Age	Numbers at Risk	Deaths
5	24	...	52	20,295·5	368
6	38	...	53	19,456	350
7	46	...	54	18,664	389
8	51	...	55	17,890·5	421
9	62·5	...	56	16,959	410
10	81	...	57	16,013·5	405
11	101·5	...	58	15,092·5	396
12	139	1	59	14,195·5	369
13	179	1	60	13,230	421
14	230	2	61	12,245	409
15	260	...	62	11,381	404
16	292·5	...	63	10,561·5	411
17	331	1	64	9,713·5	395
18	405	3	65	8,844·5	399
19	497·5	5	66	8,046·5	385
20	586·5	3	67	7,269	370
21	670	6	68	6,559·5	372
22	812·5	9	69	5,873·5	358
23	997	17	70	5,232	294
24	1,304	6	71	4,651	290
25	1,790	19	72	4,152	336
26	2,681·5	30	73	3,590·5	291
27	3,835	41	74	3,086	324
28	5,108	42	75	2,587·5	242
29	6,731	63	76	2,185	239
30	8,410	88	77	1,793	199
31	10,188	93	78	1,497·5	188
32	12,097	101	79	1,229·5	166
33	13,916·5	124	80	974·5	140
34	15,938	165	81	767	125
35	17,479	152	82	598·5	101
36	19,035	205	83	457·5	95
37	20,308	227	84	335	59
38	21,570	247	85	252·5	55
39	22,618·5	255	86	182	40
40	23,455·5	268	87	127	28
41	23,789	258	88	90	26
42	24,263	278	89	56	11
43	24,377·5	281	90	43	10
44	24,591·5	317	91	32	10
45	24,410·5	318	92	20	9
46	24,078·5	304	93	10	7
47	23,634·5	363	94	3	...
48	22,976·5	332	95	3	1
49	22,488·5	367	96	2	2
50	21,878	394			
51	21,072·5	403			
					16,109

The graduated probabilities of dying in a year, as obtained by the graphic method, are given in column (2) of the following table:

Table A.

Probability of Dying in a Year, according to the H^{M} Table, as adjusted by Mr. Sprague.

Age x (1)	Probability of Dying in a Year q_x (2)	Differences $\Delta q_x \times 10^4$ (3)	Expected Deaths (4)	Excess of Expected over Actual Deaths (5)	Progressive Totals of Numbers in (5) (6)
5	·0020	1	·0	·0	·0
6	·0021	2	·1	·1	·1
7	·0023	3	·1	·1	·2
8	·0026	3	·1	·1	·3
9	·0029	3	·2	·2	·5
10	·0032	3	·3	·3	·8
11	·0035	3	·4	·4	1·2
12	·0038	4	·5	− ·5	·7
13	·0042	4	·8	− ·2	·5
14	·0046	4	1·1	− ·9	− ·4
15	·0050	4	1·3	1·3	·9
16	·0054	5	1·6	1·6	2·5
17	·0059	6	2·0	1·0	3·5
18	·0065	7	2·6	− ·4	3·1
19	·0072	8	3·6	− 1·4	1·7
20	·0080	10	4·7	1·7	3·4
21	·0090	8	6·0	0·0	3·4
22	·0098	6	8·0	− 1·0	2·4
23	·0104	3	10·4	− 6·6	− 4·2
24	·0107	1	14·0	8·0	3·8
25	·0108	0	19·3	·3	4·1
26	·0108	−1	29·0	− 1·0	3·1
27	·0107	−6	41·0	0·0	3·1
28	·0101	−6	51·6	9·6	12·7
29	·0095	−4	63·9	·9	13·6
30	·0091	−3	76·5	−11·5	2·1
31	·0088	−1	89·7	− 3·3	− 1·2
32	·0087	2	105·2	4·2	3·0
33	·0089	4	123·9	− ·1	2·9
34	·0093	6	148·2	−16·8	−13·9
35	·0099	6	173·0	21·0	7·1
36	·0105	5	199·9	− 5·1	2·0
37	·0110	3	223·4	− 3·6	− 1·6
38	·0113	1	243·7	− 3·3	− 4·9
39	·0114	·5	257·9	2·9	− 2·0
40	·01145	·5	268·6	·6	− 1·4
41	·0115	1	273·6	15·6	14·2
42	·0116	1·5	281·5	3·5	17·7
43	·01175	2·5	286·4	5·4	23·1
44	·0120	5	295·1	−21·9	1·2
45	·0125	7	305·1	−12·9	−11·7
46	·0132	9	317·8	13·8	2·1
47	·0141	10	333·2	−29·8	−27·7
48	·0151	10	346·9	14·9	−12·8
49	·0161	10	362·0	− 5·0	−17·8
50	·0171	10	374·1	−19·9	−37·7
51	·0181	10	381·4	−21·6	−59·3
52	·0191	10	387·7	19·7	−39·6
53	·0201	11	391·1	41·1	1·5
54	·0212	13	395·7	6·7	8·2

TABLE A—(*continued*).

Age x (1)	Probability of Dying in a Year q_x (2)	Differences $\Delta q_x \times 10^4$ (3)	Expected Deaths (4)	Excess of Expected over Actual Deaths (5)	Progressive Totals of Numbers in (5) (6)
55	·0225	14	402·5	−18·5	−10·3
56	·0239	15	405·4	− 4·6	−14·9
57	·0254	16	406·7	1·7	−13·2
58	·0270	18	407·5	11·5	− 1·7
59	·0288	20	408·8	39·8	38·1
60	·0308	22	407·5	−13·5	24·6
61	·0330	24	404·1	− 4·9	19·7
62	·0354	26	402·8	− 1·2	18·5
63	·0380	28	401·3	− 9·7	8·8
64	·0408	30	396·3	1·3	10·1
65	·0438	33	387·4	−11·6	− 1·5
66	·0471	36	379·0	− 6·0	− 7·5
67	·0507	39	368·5	− 1·5	− 9·0
68	·0546	43	358·2	−13·8	−22·8
69	·0589	47	345·9	−12·1	−34·9
70	·0636	53	332·8	38·8	3·9
71	·0689	59	320·5	30·5	34·4
72	·0748	66	310·6	−25·4	9·0
73	·0814	73	292·3	1·3	10·3
74	·0887	79	273·7	−50·3	−40·0
75	·0966	86	250·0	8·0	−32·0
76	·1052	95	229·9	− 9·1	−41·1
77	·1147	103	205·7	6·7	−34·4
78	·1250	113	187·2	− ·8	−35·2
79	·1363	122	167·6	1·6	−33·6
80	·1485	132	144·7	4·7	−28·9
81	·1617	141	124·0	− 1·0	−29·9
82	·1758	151	105·2	4·2	−25·7
83	·1909	161	87·3	− 7·7	−33·4
84	·2070	171	69·4	10·4	−23·0
85	·2241	182	56·6	1·6	−21·4
86	·2423	193	44·1	4·1	−17·3
87	·2616	205	33·2	5·2	−12·1
88	·2821	218	25·4	− ·6	−12·7
89	·3039	237	17·0	6·0	− 6·7
90	·3276	263	14·1	4·1	− 2·6
91	·3539	301	11·3	1·3	− 1·3
92	·3840	560	7·7	− 1·3	− 2·6
93	·4400	1,200	4·4	− 2·6	− 5·2
94	·5600	1,800	1·7	1·7	− 3·5
95	·7400	2,600	2·2	1·2	− 2·3
96	1·0000	...	2·0	0·0	− 2·3

In order to assist the reader in judging of the smoothness of the progression, the differences of the probabilities are given in column (3). A simple inspection of these differences is, I think, sufficient to show that throughout the table the graduation leaves nothing to be desired in this respect. We must next, in order to satisfy ourselves that the original facts are followed with sufficient

accuracy, compare the expected deaths according to the adjusted mortality, with the actual deaths. For this purpose I have given in column (4) the expected deaths at each age; and in column (5) the difference between the expected and the actual deaths. The number opposite any age in column (6), is the total of the numbers in column (5), opposite that and all younger ages. These numbers are the differences between the total expected and total actual deaths up to each age; and they show in the clearest possible way what alterations the adjustment has made in the incidence of the mortality. This method is greatly preferable to the mere comparison of the adjusted with the actual mortality, and to Mr. Woolhouse's method of comparing the numbers living in the adjusted mortality table with the corresponding numbers in the unadjusted table. The numbers in column (6) enable us to determine the points at which the agreement between the total actual and the total expected deaths, is closest; and by selecting these points we obtain the following comparison between the actual and expected deaths:

TABLE B.

$H^{M(5)}$ *Table: Mr. Sprague's Adjustment.—Comparison of the Actual with the Expected Deaths.*

Ages	Actual Deaths	Expected Deaths	Difference	Progressive Totals of the Differences
5–19	13	14·7	1·7	...
20–22	18	18·7	·7	2·4
23–30	306	305·7	− ·3	2·1
31–36	840	839·9	− ·1	2·0
37–44	2,131	2,130·2	− ·8	1·2
45, 46	622	622·9	·9	2·1
47–53	2,577	2,576·4	− ·6	1·5
54–58	2,021	2,027·8	6·8	8·3
59–65	2,808	2,798·2	− 9·8	− 1·5
66–70	1,779	1,784·4	5·4	3·9
71–73	917	923·4	6·4	10·3
74–80	1,498	1,458·8	−39·2	−28·9
81–86	475	486·6	11·6	−17·3
87–96	104	119·0	15·0	− 2·3
	16,109	16,106·7	− 2·3	...

We have now to treat Mr. Woolhouse's adjustment in a similar manner. In the following table column (2) contain Mr. Woolhouse's adjusted probabilities of dying in a year, cut down

to the same number of decimal places as mine; and column (3) shows their differences. Column (4) contains the expected deaths according to Mr. Woolhouse's adjustment, these being calculated, not from the probabilities in column (2), but from those given by Mr. Woolhouse himself. In column (5) are given the differences between the expected and the actual deaths, and in column (6) the totals of these differences as above explained.

TABLE C.

$H^{M(5)}$ *Table.—Probability of Dying in a Year, and Expected Deaths, according to Mr. Woolhouse's Adjustment.*

Age x (1)	Probability of Dying in a Year q_x (2)	Differences $\Delta q_x \times 10^4$ (3)	Expected Deaths (4)	Excess of Expected over Actual Deaths (5)	Progressive Totals of Numbers in (5) (6)
5	...	...	...	...	...
6	...	...	...	...	...
7	...	...	...	...	...
8	...	...	...	...	...
9	...	...	...	...	...
10	·0040	−6	0·3	·3	·3
11	·0034	−4	0·3	·3	·6
12	·0030	−1	0·4	− ·6	·0
13	·0029	1	0·5	− ·5	− ·5
14	·0030	3	0·7	− 1·3	− 1·8
15	·0033	5	0·8	·8	− 1·0
16	·0038	6	1·1	1·1	·1
17	·0044	12	1·5	·5	·6
18	·0056	11	2·2	− ·8	− ·2
19	·0067	16	3·3	− 1·7	− 1·9
20	·0083	14	4·9	1·9	·0
21	·0097	6	6·5	·5	·5
22	·0103	4	8·4	− ·6	− ·1
23	·0107	1	10·7	− 6·3	− 6·4
24	·0108	−3	14·1	8·1	1·7
25	·0105	−2	18·8	− ·2	1·5
26	·0101	−2	27·0	− 3·0	− 1·5
27	·0099	−2	38·1	− 2·9	− 4·4
28	·0097	−2	49·6	7·6	3·2
29	·0095	−3	63·7	·7	3·9
30	·0092	0	77·4	−10·6	− 6·7
31	·0092	1	93·4	·4	− 6·3
32	·0093	−1	112·0	11·0	4·7
33	·0092	2	128·4	4·4	9·1
34	·0094	6	150·3	−14·7	− 5·6
35	·0100	3	174·8	22·8	17·2
36	·0103	4	197·0	− 8·0	9·2
37	·0107	4	217·3	− 9·7	− ·5
38	·0111	1	238·7	− 8·3	− 8·8
39	·0112	1	253·1	− 1·9	−10·7
40	·0113	0	265·4	− 2·6	−13·3
41	·0113	3	269·2	11·2	− 2·1
42	·0116	2	280·9	2·9	·8
43	·0118	5	288·7	7·7	8·5

TABLE C—(*continued*).

Age x (1)	Probability of Dying in a Year q_x (2)	Differences $\Delta q_x \times 10^4$ (3)	Expected Deaths (4)	Excess of Expected over Actual Deaths (5)	Progressive Totals of Numbers in (5) (6)
44	·0123	6	301·3	−15·7	− 7·2
45	·0129	8	315·9	− 2·1	− 9·3
46	·0137	7	328·9	24·9	15·6
47	·0144	9	340·4	−22·6	− 7·0
48	·0153	10	351·9	19·9	12·9
49	·0163	8	365·8	− 1·2	11·7
50	·0171	9	374·5	−19·5	− 7·8
51	·0180	8	379·4	−23·6	−31·4
52	·0188	11	381·3	13·3	−18·1
53	·0199	10	387·4	37·4	19·3
54	·0209	13	390·8	1·8	21·1
55	·0222	13	396·9	−24·1	− 3·0
56	·0235	18	398·6	−11·4	−14·4
57	·0251	15	401·5	− 3·5	−17·9
58	·0266	18	401·1	5·1	−12·8
59	·0284	22	402·6	33·6	20·8
60	·0306	23	405·3	−15·7	5·1
61	·0329	27	403·1	− 5·9	− ·8
62	·0356	29	405·0	1·0	·2
63	·0385	32	406·6	− 4·4	− 4·2
64	·0417	29	405·2	10·2	6·0
65	·0446	32	394·6	− 4·4	1·6
66	·0478	32	384·9	− ·1	1·5
67	·0510	34	370·4	·4	1·9
68	·0544	37	357·2	−14·8	−12·9
69	·0581	47	344·4	−16·6	−29·5
70	·0628	58	328·8	34·8	5·3
71	·0686	70	318·9	28·9	34·2
72	·0756	79	313·7	−22·3	11·9
73	·0835	87	299·7	8·7	20·6
74	·0922	73	284·6	−39·4	−18·8
75	·0995	86	257·4	15·4	− 3·4
76	·1081	74	236·3	− 2·7	− 6·1
77	·1155	91	207·1	8·1	2·0
78	·1246	103	186·6	− 1·4	·6
79	·1349	109	165·9	− ·1	·5
80	·1458	129	142·1	2·1	2·6
81	·1587	136	121·7	− 3·3	− ·7
82	·1723	135	103·1	2·1	1·4
83	·1858	132	85·0	−10·0	− 8·6
84	·1990	111	66·7	7·7	− ·9
85	·2101	98	53·1	− 1·9	− 2·8
86	·2199	129	40·0	0·0	− 2·8
87	·2328	108	29·6	1·6	− 1·2
88	·2436	163	21·9	− 4·1	− 5·3
89	·2599	225	14·6	3·6	− 1·7
90	·2824	367	12·1	2·1	·4
91	·3191	403	10·2	·2	·6
92	·3594	796	7·2	− 1·8	− 1·2
93	·4390	827	4·4	− 2·6	− 3·8
94	·5217	1,147	1·6	1·6	− 2·2
95	·6364	3,636	1·9	·9	− 1·3
96	1·0000	...	2·0	0·0	− 1·3

By selecting, as before, the points at which the agreement between the actual and expected deaths is closest, we get the following table:

Table D.

$H^{M(5)}$ *Table: Mr. Woolhouse's Adjustment.—Comparison of the Actual with the Expected Deaths.*

Ages	Actual Deaths	Expected Deaths	Difference	Progressive Totals of the Differences
10–16	4	4·1	·1	...
17–20	12	11·9	– ·1	0
21, 22	15	14·9	– ·1	– ·1
23–26	72	70·6	– 1·4	–1·5
27–32	428	434·2	6·2	4·7
33–37	873	867·8	– 5·2	– ·5
38–42	1,306	1,307·3	1·3	+ ·8
43–47	1,583	1,575·2	– 7·8	–7·0
48–55	3,024	3,028·0	4·0	–3·0
56–62	2,814	2,817·2	3·2	·2
63–67	1,960	1,961·7	1·7	1·9
68–72	1,650	1,660·0	10·0	11·9
73–79	1,649	1,637·6	–11·4	·5
80–84	520	518·6	– 1·4	– ·9
85–90	170	171·3	1·3	·4
91–96	29	27·3	– 1·7	–1·3
	16,109	16,107·7	– 1·3	...

Inspection of Table C shows that, in Mr. Woolhouse's adjustment, the differences of q_x do not proceed at all regularly; in fact, that in some parts of the table—for instance, from the age of 70 to the end—the differences proceed so irregularly that we might almost as well have no adjustment at all. A study of these differences shows conclusively that Mr. Woolhouse's process cannot be relied on to give a good graduation.

It may be thought that, although Mr. Woolhouse's adjustment fails as regards regularity, it will at all events keep closer to the original facts than a graduation which produces more regularity; but, on comparing Tables B and D, it will be seen that, even as regards adherence to the original facts, my table is, up to about the age of 53, decidedly superior to Mr. Woolhouse's. Above the age of 73 my adjustment does not agree so closely with the facts as could be desired; but I do not think it would be possible to follow the facts more closely, without sacrificing the regular increase in the rate of mortality which we have every reason to believe must exist from the age of 70 to the end of life.

The actual death-rate at the age of 74 is exceptionally heavy; and the heavy mortality at this age is by my process distributed over the subsequent years of life, in such a way as to make the rate of mortality steadily and regularly increase. In this way, I believe, we approach more nearly to the true law of mortality, than we should do by following the facts more closely within short groups of years.

Some Statistics of Female Assured Lives. *By* G. H. Ryan, F.I.A.

SINCE the late Mr. Cornelius Walford's essay on the subject of "Female as contrasted with Male Lives" (*J.I.A.*, xix, 174), no contribution to the *Journal* has attempted to settle the vexed question whether female lives should be considered as favourably as male lives for insurance purposes, or, if not, how they ought properly to be dealt with. And as a period of fifteen years has elapsed, it will not be thought premature, at any rate, to revive the discussion by bringing together certain statistics that have been produced in the interval. Further justification for such a course could be found in the increased attention which women receive in regard to everything that affects their status as independent agents in the affairs of the world, and (for readers of the *Journal* especially) in the greater interest which the female community now-a-days takes in life assurance. In the discussion following Mr. Walford's paper, Mr. Harben hazarded the opinion that the Married Women's Property Act would have an important influence on the business of life offices, especially those with extensive agency connections. This forecast has no doubt been realized, although the degree in which female insurances have increased cannot be measured accurately.

Before introducing recent statistics, it may be well to point out a fact the recollection of which time may have effaced. In the early days, it would appear (*vide* Mr. C. Ansell's evidence before the Select Parliamentary Committee on Joint Stock Companies, 1843, referred to by Mr. C. Walford, *J.I.A.*, xix, 186) that insurance companies either charged *the same premium for females as for males, or less.* Different views now prevail, and the interesting information collected by the Actuaries'

Club* last year shows broadly, that rather more than half the companies charge the same rates, and the remainder increased rates of premium. This change of front is probably due to the Scottish life offices, which, in 1874 or 1875, decided to impose on female lives an additional charge of 5*s.* per-cent per annum up to age 50, with a further single extra of 10*s.* per-cent when the proposer is pregnant for the first time at the date of insurance. The report of the committee appointed to consider the question is given in the *Journal* (xix, 209).

In the present divided state of opinion, adherents of both views will, it is reasonable to assume, be glad of the latest available information. First, I will refer to a paper contributed by M. L. Massé to the *Moniteur des Assurances* (about four years ago), on the Mortality of Female Lives. The author points out the difference between the H^M and H^F pure premiums as evidence of the greater mortality among females between the ages of 20 and 45. The same feature, he shows, is observable in the English [Institute Experience], German [source not stated], and American [Thirty Offices' Experience] statistics. From these facts he deduces the following conclusions:

1. That up to about age 45 the mortality of males is lower than that of females.
2. That after age 45 the mortality of females is lower than that of males.
3. From 25 to 45 the mortality of females is *practically constant.*

Generally, he considers that female risks undertaken before 45 are unprofitable when a uniform table is used for both sexes.

M. Massé adds to these facts (which are available to the public) some particulars placed at his disposal by M. Fouret privately, dealing with the experience of a French company from 1 January 1873 to 31 December 1879. Once more the marked inferiority of female lives under age 45 is shown, and the results agree closely with those already examined by M. Massé. The remainder of his article is devoted to an examination of the causes of the increased mortality among female lives in the period mentioned, which the author, on the strength of the figures he has collected, regards as a fact no longer open to question. This excess

* The courtesy of the Actuaries' Club enables us to place these facts before our readers: see p. 75.—*Ed. J.I.A.*

mortality he mainly accounts for by reference to the risks and diseases peculiar to females; but the American statistics, which give the causes of death, prove that other influences are at work. Phthisis and anœmia are the diseases to which he is disposed to attribute the further mortality. M. Massé closes his essay with a reiteration of his opinion that female risks under age 45 should not be accepted at the normal premiums, bearing in mind that cancelled contracts deprive the companies of the advantage of the more favourable mortality which females eventually enjoy; but a proper extra premium should be charged in all such cases. He mentions finally that certain German companies add three years to the age on female lives assuring before 45, and remove the addition on that age being reached.

At the time when M. Massé wrote the contribution to which the foregoing references have been made, he was correct in saying that no statistics of French experience were available to compare with those of other countries. This reproach can no longer be sustained. Four of the leading French offices have lately combined their experience, and submitted the results to the *Exposition Universelle de* 1889 (Economie Sociale—Section vii). Their names and dates of establishment are: Assurances Générales (1819), Union (1829), Nationale (1830), and Phénix (1844), and the observations extend from 1819 to 1888. From the pamphlet which records their results the following figures relating to the mortality of male and female assured lives have been prepared (by Crelle's three-figure tables).

Full particulars of the German companies' experience are likewise obtainable from the mortality experience of 23 German companies published in 1883,* in the preparation of which our esteemed fellow, the late Herr Wilhelm Lazarus, bore an important part. The companies include most of the well-established offices, with the notable exception of the Gotha. The observations are classified into four groups, the first—the only one with which I shall deal—comprising lives assured at the normal rate of premium, after strict medical examination. The rates of mortality given below are taken from pages 102–4 and 200–2 of the volume.

* *Deutsche Sterblichkeits-Tafeln, aus den Erfahrungen von dreiundzwanzig Lebensversicherungs-Gesellschaften, veröffentlicht im Auftrage des Collegiums für Lebensversicherungs-Wissenschaft zu Berlin.* E. S. Mittler und Sohn, 1883.

Rates of Mortality among Male and Female Assured Lives respectively, from 20 to 45, according to the Experience of 4 French Companies and 23 German Companies.

Completed Age	4 French Companies $100q_x$			23 German Companies $100q_x$		
	Males	Females	Δ (+ or −)	Males	Females	Δ (+ or −)
20	·927	·881	−·046	·639	·813	+·174
21	·656	·800	+·144	·782	1·119	·337
22	·687	1·123	+·436	·549	1·334	·785
23	·497	·684	+·187	·604	1·160	·556
24	·650	·651	+·001	·610	1·190	·580
25	·566	·560	−·006	·709	1·161	·452
26	·558	·851	+·293	·645	1·143	·498
27	·616	·699	+·083	·729	1·065	·336
28	·596	·765	+·169	·657	1·145	·488
29	·658	·634	−·024	·758	1·059	·301
30	·662	·837	+·175	·738	1·281	·543
31	·718	·792	+·074	·807	1·130	·323
32	·686	·848	+·162	·841	1·180	·339
33	·695	·755	+·060	·887	1·113	·226
34	·746	·779	+·033	·885	1·168	·283
35	·832	·896	+·064	·914	1·238	·324
36	·878	·867	−·011	·961	1·376	·415
37	·930	·767	−·163	1·000	1·168	·168
38	·920	·752	−·168	1·124	1·147	·023
39	·951	·879	−·072	1·065	1·298	·233
40	1·028	·809	−·219	1·106	1·151	·045
41	1·096	·958	−·138	1·245	1·411	·196
42	1·085	1·059	−·026	1·266	1·257	−·009
43	1·162	·768	−·394	1·388	1·209	−·179
44	1·186	·955	−·231	1·473	1·264	−·209
			+1·881			+7·625
			−1·498			− ·397
			25) ·383			25) 7·228
		Average Annual Excess	·015			·289

Examining these figures, the first thing to arrest attention is the much higher rate of mortality, in both sexes, shown by the German when compared with the French companies. To attempt to explain this discrepancy would lead me away from the immediate object of this article; but there seems no reason in the facts themselves why the *difference* between male and female mortality should be greater in one case than in the other, unless we consider that the prevailing birth-rates among the two nations affect the

question. These certainly do vary considerably, as the following figures attest:

**Mean Annual Birth-Rates* (1868 *to* 1881) *per* 1,000 *of Population.*

France	25·6
Germany	39·7
England and Wales	35·1
Scotland	35·0
Ireland	26·4

I have added the rates for the United Kingdom, because oddly enough a very similar difference is seen to exist between those for Great Britain, on the one part, and Ireland on the other, to that exhibited by the rates of Germany and France, although the variation is not so great. I doubt, however, whether the birth-rate among the general population seriously influences the extra hazard of female assured lives; and on the whole I fear we must conclude that the results of the two experiences are hopelessly irreconcilable, and mark as great a distinction of experience as the custom of British companies shows distinction of practice. Practically, the German figures confirm the views entertained by those who would charge 5*s.* per-cent per annum up to middle life (the mean annual excess of mortality from 20 to 44 being ·00289); while the French figures lend support to those who are disposed to consider female lives as average risks.

Bearing in mind, then, that offices might almost be evenly divided into those which charge extra for such risks and those which do not, the statistics now produced can only be held to prove that the matter is a very open one, upon which individual judgment and observation must incline the balance to one side or the other. A point of minor importance which both bodies of facts tend to establish is that after age 45 no extra whatever is required.

The Practice of Life Offices in regard to Assurances on Female Lives.

THE Actuaries' Club have kindly permitted us to place before our readers the following memorandum regarding the practice of British and Irish life offices in dealing with assurances on female lives.

Circulars, embodying the questions printed below in italics,

* Victorian Year Book, 1883-4 (p. 213).

were addressed to the various offices, and a careful summary was made of the replies received.—ED. *J.I.A.*

ANALYSIS *of the Replies received by the Treasurer of the* ACTUARIES' CLUB *to the Circular issued on the* 11*th November*, 1889, *to British and Irish Life Assurance Companies.*

Eighty circulars were issued, replies to which were kindly sent by the companies addressed.

In several cases the writers enter upon a close and interesting discussion of the problem which prompted the issue of the above circular; but, in the following analysis, the information collected is treated in a purely statistical manner.

Of the 80 companies, 2 stated that no fixed rules had been adopted in their practice. They are accordingly omitted in the following abstract, which shows the results appertaining to the remaining 78 offices.

I. *What additional premium (if any) does your company charge?*

(*a*) *For unmarried women:*

(*a*) 1 company accepts no female lives.

46 companies charge no extra (but 2 stipulate for 20*s.* per-cent on marriage, and 1 for 10*s.* per-cent on first confinement.)

22 charge 5*s.* per-cent per annum:
- 15 discontinue such charge at age 50.
- 1 discontinues such charge at age 45.
- 6 continue charge throughout whole duration.

2 charge 10 per-cent of the annual premium throughout the whole duration.

6 charge single premiums:
- 5 20*s.* per-cent.
- 1 10*s.* to 20*s.* per-cent.

1 uses the H^F Table throughout.

Total 78

(*b*) *For married women who have borne no children:*

(*b*) 1 company accepts no female lives.

41 charge no extra:
- 1 would charge 5*s.* per-cent per annum if proposer under 30.
- 1 stipulates for 10*s.* per-cent (single) on first pregnancy.
- 1 would charge 20*s.* per-cent if proposer in first year of married life.

21 charge 5*s.* per-cent per annum:
- 15 discontinue such charge at age 50.
- 6 continue such charge throughout whole duration.

2 charge 10 per-cent of the annual premium throughout the whole duration.

12 charge single premiums { 10*s*. to 20*s*. per-cent, according to duration of marriage, age, &c.

1 uses the H^F Table.

Total 78

(*c*) *For married women who have borne children:*

(*c*) 1 company accepts no female lives.

52 charge no extra (but 1 would charge 5*s*. per-cent per annum, if proposer under 30).

21 charge 5*s*. per-cent per annum. { 15 discontinue such charge at age 50.
6 continue charge throughout whole duration (1 would charge a single premium of 20*s*. per-cent if proposed above 35 and below 45, in lieu of annual extra).

2 charge 10 per-cent of the annual premium, as above.

1 charges a single premium of 20*s*. per-cent if proposer under 50.

1 uses H^F Table throughout.

Total 78

II. *Is the additional premium, so charged, payable during the term of the assurance, or only until the attainment of a specified age? In the latter case please state the age.*

Particulars given in connection with I (*a*), (*b*), and (*c*).

III. *What additional premium (if any) would your company charge if the person proposed were pregnant at the time of examination?*

1 company accepts no female lives.

1	Postpone or charge	£3. 3*s*. per-cent single premium.	
2		£1. 1*s*. ,, ,, ,,	
27		£1 ,, ,, ,,	(1 says 20*s*. or 30*s*.).
3		15*s*. ,, ,, ,,	(2 say 15*s*. or 10*s*.).
3		10*s*. 6*d*. ,, ,, ,,	
9		10*s*. ,, ,, ,,	

2 make no extra charge (1 uses H^F Table).

25 postpone altogether.

5 return indefinite replies, or omit to answer the question.

Total 78

The above particulars seem to relate to cases of first pregnancy. Less than a dozen companies volunteer information concerning their practice in regard to subsequent confinements.

IV. *May I further enquire whether your practice is unchanged if the assurances should be for the whole of life, for short terms, or on the endowment assurance basis?*

Nearly all companies adhere to the foregoing regulations as regards endowment assurances, as well as whole-life assurances; but 3 would charge no extra on short-term endowment assurances (5*s.* per-cent being their annual charge on whole-life cases); 1 would "modify" its extra (5*s.* per-cent per annum); and 1 would increase its single extra (10*s.* or 15*s.* per-cent).

Respecting term assurances, several companies state that they would deal exceptionally with any proposals on female lives upon this basis.

February 1890.

THE INSTITUTE OF ACTUARIES.

EXAMINATION OF THE INSTITUTE, OCTOBER 1890.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE (PART I).

Examiners—B. A. BERRY, Esq.; F. E. COLENSO, Esq., M.A.; J. E. FAULKS, Esq.; G. KING, Esq.

First Paper.

1. If 7 horses eat 16 acres of grass in 20 days, how many sheep will be required to eat 24 acres in 7 days, assuming that 9 sheep eat as much in 8 days as 12 horses in 4 days?

2. An army on the march is advancing at the rate of 13 miles a day. A detachment, 50 miles in the rear, is ordered to join it. How long will it take to do so, supposing that, from increasing impediments on the road, its advance is limited to 25 miles on the first day, 24 the 2nd, 23 the 3rd, and so on?

3. Define the characteristic and the mantissa of a logarithm, and find the characteristic of log 5 when the base is 3, and of $\log \frac{1}{3}$ when the base is 5.

Having given
$$\begin{aligned}\log 3{,}796 &= 3{\cdot}5793262\\ \log 2{,}984 &= 3{\cdot}4747988\\ \log 90{,}714 &= 4{\cdot}9576743\\ \log 90{,}715 &= 4{\cdot}9576791\end{aligned}$$

calculate to 7 places of decimals $\sqrt[5]{\dfrac{({\cdot}3796)^3}{({\cdot}2984)^2}}$.

4. S is the sum of n terms of a series in Arithmetical Progression of which the first term is unity and the common difference k. Find a value of k such that $S+3$ is divisible by $2k$ for all odd values of n.

5. Assuming the usual expansion of $\log_e(1+h)$ in ascending powers of h, prove that

$$\log_e\left(1+\frac{n}{x}\right)=2\left\{\frac{n}{2x+n}+\frac{1}{3}\left(\frac{n}{2x+n}\right)^3+\frac{1}{5}\left(\frac{n}{2x+n}\right)^5+\&c.\right\}.$$

6. Demonstrate Lagrange's theorem of interpolation; and, by means of the formula, find the probability that a person aged 53 will live a year, having given

Probability that a person aged			50 will live a year			$= \cdot 98428$
,,	,,	,,	51	,,	,,	$= \cdot 98335$
,,	,,	,,	54	,,	,,	$= \cdot 98008$
,,	,,	,,	55	,,	,,	$= \cdot 97877$

7. Five men speak at a meeting; what is the chance that A speaks immediately before B?

What is the chance, if B agrees not to speak till A has spoken?

8. A dynamically perfect coin having been tossed 10 times in succession has always come up heads. What is the probability that the 11th toss will give a head (*a*) when the coin is known to have an obverse and a reverse, (*b*) when the probability of a coin being made with two sides alike is one in a million?

9.

2 railway trucks travelled for			3 hours at	4 miles an hour,		
3	,,	,,	4	,,	5	,, ,,
4	,,	,,	5	,,	6	,, ,,
&c.			&c.		&c.	
and 28	,,	,,	29	,,	30	,, ,,

What was the mileage run in the aggregate?

10. What is the fundamental principle involved in book-keeping by Double Entry?

Explain the terms, *Debtor*, *Creditor*, *Balance*, *Assets*, *Capital*, *Liabilities*.

11. Divide a given straight line into two parts so that the rectangle contained by the whole and one of the parts shall be equal to the square on the other part.

It is optional on the part of the Candidate to answer the following question; but due weight will be given to answers sent in.

12. Find the equation to each of the three sides of a triangle, the angles of which are α, β, and γ, respectively; the side subtending γ being inclined to the axis of x at an angle of 45°, and being of length $= C$.

Second Paper.

13. Find the number consisting of two digits such that, when it is divided by the difference of its digits the quotient is 21, and when it is divided by the sum of its digits, and the quotient is increased by 17, the digits are transposed.

14. A cube contains 1,953,125 cubic inches: find to 4 places of decimals the difference between the lengths of its edge and its diagonal.

15. Find the six roots of the equation

$$x^6 = a^6.$$

Show by actual multiplication that the roots found are correct.

16. Expand any two of the expressions $(1+2x^2)^{-3}$, $(a+2b)^{\frac{2}{3}}$, and $(ax-x^2)^{-\frac{1}{3}}$, each to five terms, and give their general terms; also find the last term but two of the expansion of $\left(\frac{1}{3x}-2x\right)^{10}$.

17. A person finds that if he invests in 3 per-cent stock at 100, his present net income is greater by £10 than if he invests in $2\frac{3}{4}$ per-cent stock at 96. Find the sum he has to invest, reckoning brokerage at $\frac{1}{8}$ per-cent, and income tax at 6*d.* in the £.

18. Explain what is meant by a Continued Fraction. By means of a Continued Fraction find a series of convergent fractions giving successive approximations to the probability of drawing a white ball at the first trial from an urn containing 1,000 balls, of which 785 are white.

19. Deduce an expression for u_{x+nh} in terms of u_x and its first n leading differences, and hence find the fourth power of 23.

20. Show how "Central Differences" may be derived from ordinary differences.

21. A general orders 2 men drawn by lot out of 150 mutineers to be shot. If there are 12 ringleaders of the mutiny, find the chances that (a) only one, (b) two, (c) none of the ringleaders are chosen. Prove your answers.

22. The crew of an eight-oar (excluding the coxswain) are composed of 3 heavy men, who must row either 4, 5, or 6, counting from the bows, and 5 lighter men, who may change places in the other five seats; but 2 of these latter can only row on the stroke side: with these limitations they try all possible arrangements, and at length settle to one, and train for a race. Supposing that a stranger, who knows nothing about rowing, were to arrange them indiscriminately in the boat, what are the chances, first, that they would be in any one of the orders they had previously tried; second, that they would be in the final order in which they trained?

23. Describe the distinguishing features of a Cash Account, a Revenue Account, and a Balance Sheet, also the process of taking out, and the uses of, a Trial Balance.

24. In a right-angled triangle, if a perpendicular be drawn from the right angle to the base, the triangles on each side of the perpendicular are similar to the whole triangle and to each other.

Find a mean proportional to two given straight lines.

It is optional on the part of the Candidate to answer the following question; but due weight will be given to answers sent in.

25. An ellipse has its foci at the points whose co-ordinates are $(a.b.)$ and $(a'.b.)$ and cuts the axes of x and y at distances $-m$ and n respectively from the origin. Find its equation, and that of the tangent to it at the point $(0.n.)$.

INSTITUTE OF ACTUARIES' TEXT BOOK, PART II—LIST OF ERRATA.

The list of errata in Part II of the *Text Book*, given in *J.I.A.*, xxviii, 160, has now been reprinted, and copies may be obtained by owners of the *Text Book* on application at the Institute.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

An Examination of the Published Experience of several Life Offices and Groups of Life Offices, for the purpose of determining with the greatest accuracy obtainable—(1) the Rate of Mortality among recently-selected Lives: (2) the Rate at which that Mortality increases during the Ten Years after Insurance; and (3) whether such Rate of Increase depends upon the Rate of Discontinuance or not; with a general outline of the best methods to be pursued in the future collection and compilation of any Experience. By James Chatham, F.I.A., F.F.A., *of the Scottish Equitable Life Assurance Society.*

[To this Essay was awarded the First Prize, offered by Mr. Sprague in 1889. (For the syllabus and conditions of the competition, see *J.I.A.*, xxviii, 167.)]

[Read before the Institute, 22 December 1890.]

Before proceeding to describe the mortality experiences made use of in this investigation, I will first of all discuss the various materials that have been used in investigations into the mortality among assured lives, the methods of arranging those materials in order to deduce the rate of mortality, and the method of graduation best adapted for the adjustment of the rate of mortality among recently-selected lives, pointing out the

advantages and disadvantages of each, and stating which I have adopted. This will necessarily partake more or less of the nature of an historical sketch; but I think it will conduce to clearness, and will, I trust, prove interesting as well as instructive. I have also thought it desirable to state somewhat fully the information on which I have based my conclusions, in order that my readers may be in a position to exercise an independent judgment.

I.—Materials.

Three materials have been used in investigations for ascertaining the rate of mortality among assured persons—(1) lives, (2) policies, (3) amounts.

Lives. This was, of course, the material used in the construction of the early tables formed from the bills of mortality in various towns, and it was natural to adopt it in the first investigation made into the mortality among assured lives, namely, that of the Amicable Society, published in 1778, by Mr. Charles Brand, the registrar to the society. The methods of forming these tables are very crude; but when the science of life contingencies had developed, and new and improved methods of treating the observations had been adopted, lives still continued to be the principal material used in investigations into the mortality among assured lives, although in recent years it has been to some extent superseded.

Policies. These were used instead of lives in the first investigation made into the combined experience of a number of offices, namely, the Seventeen Offices' Experience, published in 1843. The following passage, taken from the circular sent to each of the offices that engaged to contribute their experience, explains the reasons of the committee who had charge of the investigation for adopting this material: "The "question of founding the experience from returns of policies "issued, or on lives assured, was fully discussed by the com-"mittee;—to confine the returns to a list of the lives assured in each "office might at first appear desirable, as a means of avoiding the "insertion of the same life more than once, in cases where more "than one policy has been granted thereon; but when it was "considered that, in combining the returns of several offices, it "would be impossible to prevent the repetition of the same life, "as many are insured in several offices, and that, in combining "large numbers where lives represented by duplicate policies are

"subject to the same ratio of mortality as those represented by "single policies, the result cannot be sensibly affected by the "duplication, it was determined by the committee to confine the "lists to a record of *policies issued on single lives.*" In view of the great extension of life insurance business, however, considerable doubts seem to have been subsequently entertained by actuaries as to the advisability of basing experiences upon policies; and when the next investigation into the mortality experience of the combined offices was undertaken by the Institute of Actuaries in 1862, lives were reverted to. The committee who had charge of this investigation stated that the most important point to be attended to was "that the cards should contain the name of the "life assured, so that any person whose life was assured at the "same time in more than one office might be counted once only "during the period under observation. This is of the greater "consequence, as the considerable extension of life assurance "business of late years has led not only to numerous policies "being effected on a single life, but (partly in consequence "thereof or by reason of the large assurances now effected) to "the same life appearing either directly or by re-assurance in the "books of different companies."

Amounts. In a paper by Mr. Ryan, read before the Institute some years ago, it is stated (*J.I.A.*, xxvi, 250) that, "in this country, it has been the invariable custom to construct "tables of experience based upon the observations of lives or "policies, and no investigation appears to have been made in "which the experience is deduced on the basis of the amounts "insured"; and in the discussion which followed the reading of that paper, the credit of first using amounts for that purpose was given to the American actuaries. This, however, is a mistake, as the method was employed in this country many years ago.

The idea of comparing claims paid with sums assured to ascertain the mortality among assured lives, seems to have originated with the late Mr. Spens, manager of the Scottish Amicable Society. In 1850 he read a paper to the Institute, in which he stated his belief that there was no materially greater risk in the assurance for a year of a select life of from 40 to 45 than of a select life of from 20 to 25. Mr. Farren contested this point, and Mr. Spens, in the course of the discussion which followed, asked certain offices to furnish him "with their experience of the capital sums of policies assured which had fallen by death within a year of the assurance." One

of his reasons for asking the information in reference to the sums assured, and not the individuals, was (*J.I.A.*, iv, 4) that he believed the return could be more easily made; and on the whole he was inclined to think that no error of any consequence could have arisen from estimating the rate of mortality in this manner; and he adds, that had the "Experience [Seventeen Offices'] " data been made out in this way, he believes they would have represented the truth with much more accuracy—at all events, it is obvious they would have represented, so to speak, the financial mortality. Amounts were subsequently used by the late Mr. James John Downes, the actuary of the Economic Life Assurance Society, in an investigation he made into the mortality experience of that society, which was published in 1857, but they do not seem to have been much employed by British actuaries. Amounts have latterly, however, found great favour in America, several of the mortality experiences published there in recent years having been based upon them. In the introduction to the Thirty American Offices' Experience, published in 1881, it is stated (page 12B, revised edition, 1886) that "a majority of "the committee held that an experience table based on the "*amounts* would represent the actual business of a life office "more closely than one based upon *lives*, and the different sums "insured found on pages 159, 160, sustained this view; while "the method of lives virtually assumes the sum insured to be "uniform at every age. In separate offices, however, the ratios "of death-claims by ages, although more correct, were more "fluctuating than the ratios from lives. This difficulty was "obviated by resorting to the *larger numbers* of the Thirty "Offices united, which give a very regular curve from the "amounts, as shown in the fundamental table XXVI."

Comparison of Lives, Policies, and Amounts.

In order that these three materials may be compared, and their relative merits ascertained, I give in the following table the results of two mortality experiences, namely, the Economic already mentioned, and the Connecticut Mutual, published in 1884, in which the rates have been deduced for lives, policies, and amounts. The former experience embraces a period of nearly 33 years, and contains 11,945 policies, assuring 9,335 lives, so that about 22 per-cent of the policies are duplicates; and the years of life were 86,499·5. The latter embraces a period of 32 years, the years of life for males (life and endowment assurance policies) being 714,077; and the percentage of duplicate policies appears to be almost exactly the same as in the Economic.

TABLE 1.

Ages	Economic			Connecticut Mutual		
	Lives	Policies	Amounts	Lives	Policies	Amounts
7–20	·00699	·00686	·00774	·00895	·00888	·00827
21–25	·00832	·00852	·00791	·00689	·00671	·00636
26–30	·00667	·00673	·00645	·00676	·00677	·00678
31–35	·00658	·00615	·00551	·00705	·00707	·00705
36–40	·00891	·00882	·00763	·00801	·00798	·00828
41–45	·01143	·01160	·01098	·00880	·00878	·00889
46–50	·01302	·01283	·01072	·01037	·01049	·01079
51–55	·01894	·02021	·01687	·01455	·01454	·01494
56–60	·02741	·02686	·02859	·01919	·01967	·02080
61–65	·04288	·04163	·04307	·02962	·02997	·03138
66–70	·05090	·05488	·05223	·04067	·04270	·04609
71–75	·06344	·06477	·08089	·05949	·06207	·06267
76–80	·12215	·13363	·10622	·10706	·10783	·10284
81–85	·14141	·14199	·11187	·15238	·14159	·15340
86–90	·20409	·21505	·26019	·20000	·19565	·13647
All Ages	·01482	·01502	·01447	·01122	·01137	·01155

The rates for the three classes run somewhat irregularly; but if we take larger groups, we obtain a steadier progression:

TABLE 2.

Ages	Economic			Connecticut Mutual		
	Lives	Policies	Amounts	Lives	Policies	Amounts
7–40	·00761	·00748	·00677	·00737	·00736	·00745
41–65	·01809	·01818	·01727	·01309	·01324	·01359
66–90	·06263	·06580	·06440	·05112	·05270	·05381
All Ages	·01482	·01502	·01447	·01122	·01137	·01155

Mr. Downes, in his introduction to the Economic experience, says: "The results obtained from the experience on lives and on "policies differ so little from each other that it would appear "immaterial whether in large experiences policies or lives are "made the basis of observation, though that on lives must be "regarded theoretically most exact." It is true that, at the earlier periods of life, the difference is small; but it will be observed that the rate for policies in the first group of ages in Table 2 is less than that for lives, in the second higher, and in the last

group very much higher; also that the same features are found in the Connecticut Mutual experience. And this is what was to have been expected. At the early periods of life the policies exposed to risk are continually being increased by some of the lives already insured undergoing a fresh medical examination, and taking out new policies; but afterwards, as the lives get old, this is of less frequent occurrence, and when the policies are practically no longer recruited in this way, the rate becomes higher. The rates, therefore, deduced from policies, will be too low at first and too high afterwards; and the extent to which they are affected will, as a rule, depend upon the number of duplicate policies. In some offices the number of duplicate policies is very much higher than in the two mentioned above. For instance, in the New York Mutual there were 152,149 policies on the lives of 101,967 persons, so that the duplicate policies formed nearly one-third of the whole. Mr. G. F. Hardy, however, justifies (*J.I.A.*, xxiii, 2) the use of policies instead of lives where only the general effect of the mortality upon annuity-values is in question, and where the rate of mortality at individual ages is not wanted.

When we turn to the rates for amounts, however, the experience of the two offices is very different. In the Economic the rates in each of the three large groups in Table 2 are less than those for policies, while in the Connecticut Mutual they are higher. In the former experience, therefore, the mortality among policies for large amounts must have been less than that for policies of small amounts, while in the latter the reverse has been the case—a feature which is apparently common to most American companies. In the introduction to the mortality experience of the Washington Life Insurance Company of New York, it is stated that "where "observations have been made, the percentage of mortality is "greater on amounts of policies than on lives or policies." In Germany, on the other hand, the experience of the Gotha Life Office is the same as that of the Economic. As Mr. Ryan has pointed out in a recent number of the *Journal* (*J.I.A.*, xxviii, 223), two companies exhibiting similar rates of mortality might show very different rates of loss; and I have therefore come to the conclusion that the best material for making the proposed comparisons is lives. They are less liable to fluctuations than amounts, and interesting comparisons may be made with the rates deduced from population returns. Most of the investigations

also into the mortality of assured lives have, as already stated, been based upon lives. Amounts, however, are more suitable for some office calculations, such as a comparison of the expected claims with the actual; and if the American companies find that the rate for amounts is persistently higher than that for policies or lives, then I think they are right in taking this circumstance into account—indeed, it would be unwise for them to neglect it. If duplicate policies are not eliminated, however, amounts are open to the same objection as that urged against policies; while it must be remembered that they are not so easily dealt with as lives or policies, any variation in the sum assured requiring to be taken into account.

objection not app duplicate policies in same

II.—Methods of arranging Materials.

Mortality experiences have been made up in two ways—(1) by calendar years, (2) by policy years.

Calendar years. This method was introduced by Mr. Woolhouse into the Seventeen Offices' Experience already mentioned; and as it has played an important part in the history of life assurance statistics, I will give the description of it in full, in his own words. In his paper "On the Construction of Tables of Mortality", read before the Institute, 30 April 1866 (*J.I.A.*, xiii, 75), he says: "In the year 1839 I had occasion, officially, "to go into an investigation of the mortality in the Indian army. "An abstract of this investigation was published at the time in "the form of a pamphlet.* As discussions of this kind are, as "a matter of course, attended by a somewhat tedious amount of "numerical operations, I was induced, in the first instance, to "examine the mathematical relations with the object of system-"atically reducing them to such a form as would give the utmost "brevity and simplicity to the process of calculation, and, what is "of still greater moment, obviate any possible risk of error. "The resulting method which I then followed was afterwards "approved and expressly adopted by the committee of actuaries in "the computation of the Tables exhibiting the Law of Mortality, "deduced from the Combined Experience of Seventeen Life "Assurance Offices; and the superintendence of those calculations "was eventually entrusted to the author, who was a member of "that committee. . . . .

* *Investigation of Mortality in the Indian Army*, by W. S. B. Woolhouse, F.R.A.S., &c., London, 1839.

"In the Experience of Offices only the calendar years, instead "of the dates of entry and exit, were deemed to be sufficient for the "purpose, it being considered that large numbers promiscuously "spread over any given year when taken in combination may, "on the average, be practically assumed with tolerable correctness "as uniformly dating from the middle of the year. For a like "reason, only the current year of age, or the office age, is required "to be stated. Thus, by assuming the number of lives that "entered in each current year of age to be, on the average, admitted "also at the middle of such year, a convenient coincidence will "thereby subsist between calendar years and years of age; and "the lives, one with another, may be considered as completing "the office age at the end of the calendar year of entry. . . .

"The several entrances and exits which, in the year of age, "have operated in disturbing the number of lives otherwise "than by death, are distributed through the year of age at "indiscriminate intervals, and may, with large numbers, be "practically considered, on the average, as having reference to "the middle of the year, or as having undergone half the chance "of death in one year. The average interval by which the date "of entry precedes the birthday is here assumed to be half a year, "which may be expected to be somewhat in excess, since in some "cases it is natural to surmise a disposition to expedite an "assurance when a coming birthday, in near perspective, is "suggestive of a higher rate of premium. According to actual "experience the average interval is found to be about three-"eighths of a year. The difference is, however, not material. "As the lives are by this means accounted nearly one-eighth "of a year younger than they really are, the ultimate tendency "will be to slightly increase the rate of mortality."

This method of treating the materials was almost universally adopted in all the large experiences published since then until recent years; for instance, in the Institute (or Twenty Offices') Experience published in 1869, the Mutual Life Insurance Company of New York in 1876, the Thirty American Offices in 1881, &c. In the introduction to the last mentioned, a table is given on page 24 which shows concisely the assumptions made in this method, and I reproduce it here with the explanatory remarks which accompanied it. "For greater convenience, as "before mentioned, the new business of different calendar years "is super-imposed, for this investigation, as if all the insured "entered the company uniformly during one initial year. On "the average they enter at the middle of this year, and so the

"average are exposed for only the latter half of the first calendar "year. The exits as well as the entrances of subsequent years, "are likewise assumed to occur uniformly during each separate "calendar year. *Average policy years thus begin and end with "the middle of calendar years;* and the average birthday is taken "at the middle of the initial policy year. The ratio of mortality for the "first year has been a subject of discussion in the *Journal of the "Institute*; but, as there stated, 'it is difficult to devise a better "plan'; and in practice, the regular formula for subsequent years "has been generally applied to the first year, as will be presently "exhibited. . . . .

Calendar Years	Years of Insurance		Existing, Discontinued, Died
Beginning .	...	...	Insurances commence
Middle . .	Beginning .	0	Average date of entrants
End . . .	Middle . .	½	First year's "existing" record closes
Middle . .	End . . .	1	Average date of deaths and discontinued
Middle . .	Beginning .	1	Survivors enter on second policy year
End . . .	Middle . .	1½	Second year's "existing" record closes
Middle . .	End . . .	2	Average date of deaths and discontinued
Middle . .	Beginning .	2	Survivors enter on third policy year
&c.	&c.	2½	Third year's "existing" record closes

" . . . The preceding principles require that, after passing "from deaths to the ratios of mortality, or to the life table, the "current ages on the plan of 'nearest birthday' are to be "diminished by half a year, as if changing the ages from the "middle to the beginning of the year of the ratio. Thus 29½ "current is changed to 29 years. The English custom of age "'next birthday' would require a similar, final change of one "year, reducing the current age 30 to 29, for example, as noted "in the first volume of the Institute Experience, page 18."

The example referred to in the Institute volume is given here to practically illustrate the method, along with an extract from the table of observations for healthy male lives for "current" age at entry 30, that is 30 next birthday, in order that it may be easily followed:

Current Age at Entry, 30.—*Number of Entrants*, 5,791.

Current Age at Exit	Existing	Discontinued	Died
30	319	75	4
31	252	365	28
32	230	220	35
33	235	153	49
34	198	147	51
35, &c.	161	84	36

				Exposed to Risk	
The number of entrants was	5,791,	half of which,	2,895·5		
Discontinued . . .	75	,, ,,	37·5		
				2,858	Age 29, or 1st year.
Entered on the first year .	5,791				
Deduct—Existing . 319					
Discontinued 75					
Died . . 4					
	398				
Entered on the second year	5,393,	less half of the discontinued	182·5		
				5,210·5	Age 30, or 2nd year.
Deduct—Existing . 252					
Discontinued 365					
Died . . 28					
	645				
Entered on the third year .	4,748,	less half of the discontinued	110		
				4,638	Age 31, or 3rd year.
&c.,	&c.				

The annual rate of mortality for those who entered at age 30 next birthday, and were assumed to attain that age at the end of the calendar year of entry, is, therefore, $\frac{4}{2858}=\cdot00140$, which was put opposite age 29. As Mr. Sutton has pointed out (*J.I.A.*, xvi, 75), those who discontinued in the calendar year of entry are not taken into account at all: thus,

$$\tfrac{1}{2}(5791-75)=\tfrac{1}{2}.5716=2858.$$

The following formula, taken from Mr. W. T. Gray's paper "On Average Rates of Mortality" (*J.I.A.*, xxv, 369), with slight modifications, shows how the exposed to risk at any age is deduced from the exposed to risk at the previous age, when the mortality table is formed in the usual way by taking all lives of the same age together, irrespective of the length of time for which they have been insured:

$$E_x=E_{x-1}+\tfrac{1}{2}(n_x+n_{x+1})-\tfrac{1}{2}(w_x+w_{x+1})-(d_x+e_x)$$

where E_x = exposed to risk at age x

n_x = new entrants at age x next birthday

w_x = withdrawals ,, ,,

d_x = deaths ,, ,,

e_x = existing ,, ,,

For instance, according to the Institute Experience, the numbers of healthy males entering at ages 29 and 30 next birthday, are 5,239 and 5,791 respectively; the numbers withdrawing, 1,139 and 1,165; the number dying at age 29 next birthday, 171; and the

number existing, 1,441; and as the number exposed to risk at age 28 is 21,837, we get

$$\begin{aligned}E_{29} &= E_{28} + \tfrac{1}{2}(n_{29}+n_{30}) - \tfrac{1}{2}(w_{29}+w_{30}) - (d_{29}+e_{29})\\ &= 21{,}837 + \tfrac{1}{2}(5{,}239+5{,}791) - \tfrac{1}{2}(1{,}139+1{,}165) - (171+1{,}441)\\ &= 21{,}837 + 5{,}515 - 1{,}152 - 1{,}612\\ &= 24{,}588\end{aligned}$$

among whom 181 deaths took place, so that the unadjusted rate of mortality for age 29 is $\frac{181}{24588} = \cdot 00736$.

It may be mentioned that this formula is not the most convenient for the actual calculation of the exposed to risk, and the same remark will also apply to some of the other formulas given here. The formula used by Mr. Meikle to obtain the exposed to risk in the Ten Scotch Offices who contributed their experience to the Institute in 1862, was

$$\begin{aligned}E_x &= \Sigma n_x - \Sigma(d_x + w_x + e_x) + \tfrac{1}{2}(n_{x+1} - w_{x+1})\\ &= \Sigma n_x - \Sigma f_x + \tfrac{1}{2}(n_{x+1} - w_{x+1})\\ &= \Sigma(n_x - f_x) + \tfrac{1}{2}(n_{x+1} - w_{x+1})\end{aligned}$$

where $\qquad f_x = d_x + w_x + e_x$

and it is agreed to use Σu_x to represent the sum of $u_1 + u_2 + \ldots\ldots + u_x$. The final expression is that given by Mr. Ryan in a paper "On certain Methods of deducing the Number Exposed to Risk" (*J.I.A.*, xxvi, 256), from which I have taken, with slight modifications (which I trust will be considered improvements), one or two of the formulas subsequently given. This formula gives the same results as the other, but is more easily applied. A similar arrangement of the facts is given in the Seventeen Offices' Experience, but the committee state that it will not be found so convenient in practice as the one they actually adopted. For a description of the latter I must refer the reader to the book itself, as it is difficult to explain it apart from the figures.

Mr. Meikle, in his paper "On the Official Publications of the Mortality of Assured Lives" advocates that the lives should be taken from birthday to birthday, and that the fraction of the year during which they are exposed to risk at entry and exit (otherwise than by death) should be taken in months. The mortality experience of the Twenty-three German Offices, published in 1883, is made up in a somewhat similar way, but the exact ages have apparently not been taken. ". . . . The ages

"at entry have proceeded upon a fresh calculation of the "difference between the year of birth and year of entry, and the "ages at exit upon the difference between year of birth and year "of exit." A similar method of estimating the ages was adopted by Messrs. Hardy and Rothery in a formula for obtaining the exposed to risk (*J.I.A.*, xxvii, 165), by which they state all fractions are avoided and the labour of calculation considerably reduced:

"Let n_x = the number entering at *mean age* x.
x = year of entry—year of birth.

w_x = the number discontinuing at *mean age* x.
x = year of discontinuance—year of birth.

e_x = the number existing at the close of the observations at *mean age* x.
x = nearest age at the close of the observations.

d_x = the number dying at assumed age x last birthday, obtained by adding the curtate duration to the assumed age at entry."

Then,
$$\begin{aligned} E_x &= E_{x-1} + n_x - w_x - e_x - d_{x-1} \\ &= \Sigma(n_x - w_x - e_x - d_{x-1}) \\ &= \Sigma(n_x - f_x) \end{aligned}$$
where
$$f_x = w_x + e_x + d_{x-1}.$$

Mr. King has given (*J.I.A.*, xxvii, 218) another formula for deducing the exposed to risk from the records of a life assurance company, the more especial object of his investigation being to obtain a formula which would adapt itself to the valuation classification registers, and so produce the desired results with a minimum of trouble. "Let it be supposed that the mortality "experience of a company is required for the interval between "two valuation epochs, and that the policies at each valuation "are classified according to age *nearest* birthday; so that, for "instance, if the valuations take place as on 31 December, those "policies are brought together which are on the lives of persons "born between 1 July in one year and 30 June in the next, both "days inclusive. Of course the same principle of classification "applies whatever may be the date of closing the books, but it "will be convenient here, for purposes of illustration, to speak "only of 31 December. Then, taking

s_x = survivors at commencement of observations, where x is the age nearest birthday on 31 December, the day before such commencement;

n_x = entrants during period of observation, where x is the age nearest birthday on nearest 31 December to date of entry;

w_x = discontinuants, where x is age nearest birthday on nearest 31 December to date of discontinuance;

d_x = deaths, where x is age nearest birthday on 31 December preceding death;

e_x = remaining on books on 31 December at close of the observations, x being age nearest birthday at that date;

E_x = exposed to risk in year of age x to $x+1$;

we have $$E_x = E_{x-1} + s_x + n_x - w_x - e_x - d_{x-1}$$

and $$q_x = d_x \div E_x.$$

" . . . . . . Although devised more particularly for taking " out the mortality experience of a company in the interval between " two valuation epochs, the formula may be used with equal " facility to obtain the total experience of a company from the " commencement. In such a case there will be no "survivors", " and s_x will be equal to zero at all ages. . . . . If entries " and exits are uniformly distributed throughout the calendar year, " the results, on the average, will be the same by the new formula " as by the Institute Experience method." If we make s_x equal to zero in the above formula we get

$$E_x = E_{x-1} + n_x - w_x - e_x - d_{x-1}$$

the same expression as Messrs. Hardy and Rothery's given above. In giving the formulas for the exposed to risk, I have used the same notation throughout, Messrs. Hardy and Rothery and Mr. King having each adopted a different notation from that employed by Mr. Gray and followed by Mr. Ryan,—a practice much to be deprecated when there is no sufficient reason for it.

It should be mentioned that the mortality experience of the Equitable Society by Mr. Arthur Morgan, published in 1834, was made up by calendar years; but a different method, apparently, of dealing with the first year, which will be discussed later on, was adopted.

Policy years. The next investigation into the mortality among assured lives published in this country after the Equitable last mentioned, was that of the Amicable Society, by Mr. Thomas Galloway, in 1841. It was made up on an entirely different principle, namely, by policy years instead of calendar years; and as it was the first investigation in which this plan was adopted, I will give the description of it in the author's own words. "Having obtained an exact list of the individuals upon "whose lives the observation was to be founded, together with their "ages at admission and the dates of the admissions, discontinuances, "and deaths, the first step towards deducing the law of mortality "was to fix the ages at which the deaths and discontinuances "took place. The ages entered in the registers of admission are, "as usual, the *office ages*, that is to say, the years of age through "which the individuals were respectively passing; for instance, "a person whose age upon admission is stated to be 47, is "understood to be then living in his 47th year, or to have "completed 46 years on his last birthday. Hence, as the "admissions take place equally at all times of the year, it might "be assumed that the office ages at admission are the ages which "the parties respectively attained or completed on the 1st of "January next following; and the usual practice in forming "tables of mortality is to count from this date (or some other "fixed epoch of the year), and to make a subsequent allowance "for the part of the year intervening between the date of "admission and the assumed epoch during which the lives were "exposed to the risk of mortality. But in order to avoid the "necessity of making an allowance for such intervening time, "I have followed a different method, and have assumed that each "person, on the day of his admission, had completed *exactly* the "half-year of his current age; for example, all those who entered "the society at the office age of 47 are assumed to have lived "from birth to admission exactly $46\frac{1}{2}$ years. This assumption "being made, the ages at death and discontinuance, and of the "living on the 5th day of April 1841, when the observation "terminated, were obtained by adding to the age at admission "the number of complete years the individual lived in the society, "reckoned from the day of admission. Thus, suppose a person "who was admitted on the 1st of August 1815, at the office age "of 41, to have died in July 1829, he would be entered in the "table as having completed exactly the age $40\frac{1}{2}$ at admission, "and as having died between $53\frac{1}{2}$ and $54\frac{1}{2}$, inasmuch as he had

"lived 13 complete years in the society, and at the time of his "death was passing through the 14th year of his assurance. In "like manner, if a person admitted on the same day and at the "same age remained alive on the 5th of April 1841, he would "be entered as living between $65\frac{1}{2}$ and $66\frac{1}{2}$, or as passing through "the 26th year of assurance, having been assured 25 complete "years on the previous 1st of August.

"By the adoption of this method of proceeding, the time of "the continuance of each individual in the society begins to be "reckoned from the day of his admission, and the duration of "the lives is determined with the utmost certainty. The single "assumption introduced is, that the admissions take place, one "with another, precisely at the middle of the current year of "age: all the rest is matter of calculation.

"It may be objected that the assumption of the half-year of "the current age being exactly completed on the day of admission, "is not quite accurate, inasmuch as many more persons are likely "to enter an assurance office a few weeks or days before their "birthday than shortly after it, on account of the smaller premium "to be paid. The objection (which, however, applies equally to "the other method), must be allowed to have some weight, for "on making a calculation with respect to the first 100 individuals "admitted into the society after the 5th day of April 1839, it was "found that the average time wanting to complete the current "year of age was 4·15 months, whereas, according to the "assumption, it should have been 6 months. It would therefore "seem that, on the average, the members were in reality nearly, "but not quite, two months older at admission, death, and "discontinuance, than they are represented in the table. I have "not thought it worth while to take any account of this circum-"stance. In applying the results of the table to the office "calculations, the effect of the discrepancy, such as it is, will "be on the side of safety. . . . . .

"It is to be observed that, although all who enter upon any "year of age are exposed to the chances of mortality at the "commencement of the year, the number of persons who entered "upon any given year is not the proper number with which the "deaths that occurred in that year are to be compared in order to "obtain the probability of the failure of life before reaching the "next age; for some of them were passing through the given year "when the observation terminated, and consequently had not "been exposed to risk during the whole year; while others

"quitted the society in the course of the year, and consequently "were exposed to risk during only a part of it. Now, of those "who remained alive in any year of age when the observation "terminated, some must have lived through only a small part "of the year, and others through nearly the whole of it, and "therefore they may be considered as having lived, one with "another, through half the year; or, which comes to the same "thing, it may be assumed that one half of them stood the "chances of mortality for the whole of that year. In like manner, "of those who discontinued in any year of age, some left the "society near the beginning of the year, and others near the "end of it, and therefore they may be taken, one with another, "as having lived in the society half a year; or it may be assumed "that one half of them stood the chances of mortality during the "whole year."

The following example will practically illustrate the method of procedure:

Office Age at Entry, 29—Assumed Age, $28\frac{1}{2}$.

Number of Entrants (1808–41)	103
Less half of Existing (3) and Discontinued (2) between Ages $28\frac{1}{2}$ and $29\frac{1}{2}$	2·5
Exposed to Risk in first Policy Year, among whom 1 died	100·5
Entered on second Policy Year, 103−(3+2+1)	97
Less half of Existing (1) and Discontinued (5) between Ages $29\frac{1}{2}$ and $30\frac{1}{2}$	3
Exposed to Risk in second Policy Year, among whom 2 Deaths took place	94

And so on.

The formula for getting the exposed to risk when the mortality table is formed in the usual way, will therefore be

$$E_{x-\frac{1}{2}} = \Sigma n_x - \Sigma(d_{x-1} + w_{x-1} + e_{x-1}) - \tfrac{1}{2}(w_x + e_x)$$
$$= \Sigma(n_{x-1} - f_{x-1}) + n_x - \tfrac{1}{2}(w_x + e_x)$$

where $f_{x-1} = d_{x-1} + w_{x-1} + e_{x-1}$

Or we may deduce the exposed to risk at one age from that at a previous age by the formula

$$E_{x-\frac{1}{2}} = E_{x-1\frac{1}{2}} + n_x - \tfrac{1}{2}(w_{x-1} + w_x) - \tfrac{1}{2}(e_{x-1} + e_x) - d_{x-1}.$$

Thus, taking a numerical example, the number of entrants at age 29 next birthday is, as above, 103; the numbers withdrawing at ages 28 and 29 next birthday, 14 and 11 respectively; the

number existing at these ages, 9 and 17; and the number dying at age 28 next birthday, 3; and as the number exposed to risk at age $27\frac{1}{2}$ is 334·5, we have

$$\begin{aligned}E_{28\frac{1}{2}} &= E_{27\frac{1}{2}} + n_{29} - \tfrac{1}{2}(w_{28}+w_{29}) - \tfrac{1}{2}(e_{28}+e_{29}) - d_{28}\\ &= 334{\cdot}5 + 103 - \tfrac{1}{2}(14+11) - \tfrac{1}{2}(9+17) - 3\\ &= 334{\cdot}5 + 103 - 12{\cdot}5 - 13 - 3\\ &= 409\end{aligned}$$

and as 2 deaths took place, the unadjusted rate of mortality for age $28\frac{1}{2}$ is $\frac{2}{409} = {\cdot}00489$. In order to obtain "the probabilities "of living from one complete year of age to another, or from x to "$x+1$, ... it is assumed that half the number "exposed to the risk of mortality from $x-\frac{1}{2}$ to $x+\frac{1}{2}$, added to "half the number exposed to risk from $x+\frac{1}{2}$ to $x+1+\frac{1}{2}$, may be "taken for the number exposed to risk from x to $x+1$; and that "half the number who died in the interval between $x-\frac{1}{2}$ and "$x+\frac{1}{2}$, added to half the number who died between $x+\frac{1}{2}$ and "$x+1+\frac{1}{2}$, may be taken for the number who died in the interval "between x and $x+1$." This, however, does not allow for the variation in the numbers entering at different ages.

In 1854 an investigation by Mr. Jellicoe into the mortality experience of the Eagle Insurance Company during the 44 years ending 31 December 1851, was published, in which Mr. Galloway's method of procedure was followed.

The next experience published in which the observations were arranged according to policy years was that of the Scottish Amicable Society, by Mr. Spens, in 1861. A different method, however, from that of Mr. Galloway was adopted for ascertaining the age at entry. "The ages at entry, as for the first year's "experience of a life, were taken as the difference between the "year of birth and the year of entry—a mode which evidently "(on the assumption, at least, of parties being born equally at "any time of a year) states the ages on an average correctly." "In the case of policies cancelled otherwise than by death, when "the broken period was less than half a year, 0 was reckoned; "when half a year, ·5; when more than a half, 1." Each policy, too, was traced to its anniversary in the final policy year of the observations, and the fractions for existing thus avoided. The formula for obtaining the exposed to risk in this case will therefore be

$$\begin{aligned}E_x &= \Sigma n_x - \Sigma(d_{x-1} + w_{x-1} + e_{x-1}) - Cw_x\\ &= \Sigma(n_{x-1} - f_{x-1}) + n_x - Cw_x\end{aligned}$$

where C represents the time which has elapsed, on the average, between the date of withdrawal and the anniversary of the policy, and x is throughout the exact age. The idea of taking the anniversary of each policy in the last calendar year as the termination of the observations instead of a fixed date, is certainly an improvement upon Mr. Galloway's method. It will, as a rule, involve no more labour, and, as already mentioned, avoids fractions in the case of the existing. The improvement is even more apparent in observations where there are no withdrawals, such as the Government Annuitants, it having been adopted by Mr. A. J. Finlaison, the actuary to the National Debt Commissioners, in his recent investigation.

The plan of arranging the observations according to policy years has in recent years been adopted in America.

Comparison of Calendar and Policy Years.

I will now compare these two methods, stating the arguments for and against them. Dealing in the first place with the calendar year of entry, or year "0" as it is called, it will be observed that the rate of mortality for that year is deduced from lives which have been exposed to risk on the average for six months only, and that consequently the light mortality which prevails during the six months after entry is assumed to prevail for a whole year after entry. But even the assumption that the lives will, on the average, be exposed to risk for six months, introduces in many cases a considerable error. It is well known that in individual offices policies are not effected uniformly during the year, but that a larger business is usually transacted at the close of the financial year than at the beginning. Mr. G. Todd says (*J.I.A.*, xxvi, 267) "he "was not sure that if they were now to take the old plan of "regarding the commencement of the risk as being spread evenly "over the 12 months, and taking, therefore, the half-year as being "the amount of risk run, they would not bring out results some-"what wide of the truth. He had found in one case that the "business had only been in force on the average ·24 of a year at "the close of year 0. From the same experience he had found that "the lives assured had, at the date of commencement of the risk, "completed on the average ·81 of their current year of existence, "and if they considered that the lives assured attained their 'age "next birthday' upon the completion of the business year, they "would seldom be more than a small fraction out." The difficulty of dealing with the year "0" seems to me of itself a serious objection to the use of calendar years, more than one of

the assumptions made being quite at variance with the facts. The error Mr. Todd has shown to exist in the experience of an individual office, would not, however, operate to the same extent in the experience of a group of offices. I find, for instance, in the Institute experience that the close of the financial year of only about one half of the offices contributing their experience coincides with the end of a calendar year, the other offices closing their books at various periods throughout the year. In the case of policy years, however, if the practice of dating back policies exists to any extent, allowance should be made for it when comparisons are made with other rates not affected in the same manner.

It is argued by some actuaries that although tables based upon policy years are more suitable for the calculation of premiums than those based upon calendar years, yet they are not so suitable for the valuation of a company and for the calculation of the expected mortality, as these are made at the end of a calendar year, not a policy year. It may be remarked in passing that the financial year of only about 60 per-cent of the insurance offices ends with the close of the calendar year, so that this argument does not hold. But let us examine it and see whether it would hold in the case of a valuation, for instance, assuming all the offices closed their books at the end of the calendar year. In some offices the true age at valuation is taken; in others, the age next birthday. If the experience is made up by policy years and the ages at entry are taken as the difference between the year of birth and the year of entry, then the values of annuities could be calculated for exact ages, and the values of assurances and premiums deduced from them. If these are used in a valuation where the true ages are taken, then they would be consistent with each other, and the reserve value determined with greater accuracy than if they had been deduced from an experience made up by calendar years. This is not, however, in my opinion, the most correct method of determining the net premium which persons of a given age should be charged. I think that no assumptions should be made as to the age at entry, but that the rate of mortality among those who enter an office at a specified age next birthday should be ascertained for each policy year. The true age at entry would, on the average, be about four months less; and if the values of annuities and assurances for ages next birthday were calculated, the true age would in their case also be four months less. When the ages at valuation, however, are taken next birthday, the true ages will, on the average, be six months less; and therefore the values

of the annuities and assurances will, on the average, be for ages two months greater than the true ages. The error, however, is on the safe side, and is not greater than that which exists at present when experiences made up by calendar years are used. The methods here described of forming the premiums and conducting a valuation, are, so far as I am aware, entirely new, and would, in my opinion, give more accurate results than those usually employed. They would also answer perfectly well at whatever period of the calendar year the valuation is made, and would apply with equal facility to the calculation of the expected deaths. I will not, however, stop on the present occasion to discuss the matter further. All that is necessary for me to do is to show that experiences made up by policy years are equal, if not superior, for valuation purposes, to those made up by calendar years. Some actuaries say that as the rate of interest varies and the loading, &c., therefore we need not be so particular about the mortality; but I hardly think that this view will receive general support, especially when more accurate results can be obtained with less labour than by the present methods.

We will now see whether there is any good practical reason why calendar years should be preferred to policy years, as was done by the Institute when they published their experience in 1869. In the introduction it is stated that "in so large a "number of cases to estimate the precise age in parts of a year "was thought to involve more trouble and labour than such "minute accuracy would repay. It was, moreover, of the utmost "consequence to take this opportunity of correcting the defects "of former observations, by obtaining the experience of the offices "at the advanced ages of life. In former years it was not "customary to record the day of birth in the books, and to have "requested the companies to look up old documents long put "away, might have risked the loss of some of the most valuable "fruits of the present labour. But as, in all cases, the office age "on entry is the age next birthday, it was decided that on the "whole the current year of age, or office age, at the date of "assurance, would afford the means of approximating very closely "to the actual age by the single assumption that the assured "attained that age at the end of the year of entry." We have seen, however, that it is not necessary to have the date of birth for policy years, and that, at the very least, the same assumption can be made as in calendar years. Mr. Bailey supplements the above statement by saying (*J.I.A.*, xxvi, 268) that neither the

date of death nor the date of lapse, but only the calendar years of these occurrences were derivable, almost without exception, from the policy records of those offices which contributed. If that is so, then I do not see how any other method than that of calendar years could be adopted. But it must not be forgotten that in the cards contributed by the ten Scotch offices, and which formed the greater portion of the statistics collected by the Institute, the exact dates of birth, entry, and exit were given, although I confess I do not see what use was afterwards made by the Scotch of the more exact material at their command. So far as I can see, the only advantage—if advantage it can be called—which calendar years have over policy years, is that only the calendar years of entry and exit are required, and not the exact dates. On the other hand, the advantages of policy years over calendar years are (1) that the rate of mortality in the first year of assurance can be determined with great accuracy; and that the errors caused by assuming the light mortality experienced during the six months after entry to prevail for a whole year after entry, the under-estimation of the exposed to risk, and the not taking into account those who enter and discontinue in the same year, when calendar years are used, are avoided; (2) that the exact duration of the surrenders and lapses can be ascertained, this being necessary if a large business is done at the end of the year, as surrenders generally take place when a premium falls due, and lapses invariably so; (3) that the fractional ages at entry in America and Germany on account of the ages in these countries being generally taken nearest birthday, and the consequent labour of ascertaining the rate of mortality at integral ages, are thereby avoided; and (4) that they are better suited for the calculation of premiums, valuations, &c.

Having carefully considered all the arguments for and against, I have come to the conclusion that the advantages are, to use the language of an opponent of the method, overwhelmingly in favour of policy years, and that all mortality experiences, whether of assured lives or annuitants, should be made up in that way. Unfortunately this opinion is not shared by British actuaries generally—indeed, so far as I am aware, only one or two have expressed a preference for policy years. Speaking in 1854, the late Mr. Jellicoe said: "Mr. Galloway's arrangement appears to me to be the best which has hitherto been devised." Mr. Sprague says (*J.I.A.*, xv, 340) that Mr. Spens has "adopted a different "mode of estimating the years of insurance from that adopted

"by the committee of the Institute. The latter have based their "calculations on *calendar* years, assuming that on the average six "months elapse between the effecting of each insurance and the "following 31 December, and considering these six months as "the 'year of insurance 0.' Mr. Spens has followed what "appears to me the much preferable course of considering the "12 months which follow the grant of the insurance, as con-"stituting the 'year of insurance, 1'; the next 12 months, the "'year of insurance, 2'; and so on." Mr. Ryan says (*J.I.A.*, xxvi, 258): "The Institute and Mr. Meikle's methods, however, "presuppose an arrangement of the observations to have been "made according to calendar years. The distinguishing feature "of Galloway's method, on the other hand, is that the policies are "followed through complete policy years or years of insurance. "This latter arrangement carries with it several material advan-"tages, especially in that it enables the rate of mortality to be "traced through the true years of insurance. Great confusion "has, I believe, been occasioned by the invaluable statistics "collected by the Institute of Actuaries being so prepared that "the nominal first year of insurance—or year 0 of insurance, "as it has been termed, conveniently but not perspicuously—"embraced a period of six months only. The awkwardness "cannot, however, be avoided when the facts are regarded in "relation to calendar years of experience, and the only way out "of the dilemma, as far as I can see, is to abandon the principle "of calendar years in favour of that of policy years." I would, however, ask my readers to consider the question for themselves, without regard to the opinions of any authority, however eminent.

The best method of making calendar years coincide with policy years is that adopted by Mr. Sprague in his Select Life Tables, where the deaths in year 0 were assumed to be those which occurred in the first six months after entry, and the deaths for the subsequent years were divided into half years. The deaths for the first six months or year 0, and the first half of year 1, were added together, and the rate of mortality for the first policy year deduced; the sum of the deaths for the second half of year 1 and the first half of year 2, enabled the rate of mortality for the second policy year to be deduced; and so on. But this method of procedure will only give it approximately; and when we endeavour to ascertain the rate of discontinuance in the early policy years, we shall find it quite impracticable on account of the great increase in the rate which takes place in year 1.

Experiences therefore made up by calendar years are of no use for the purpose of determining with the greatest accuracy obtainable the rate of mortality among recently-selected lives. They will, however, be of use in indicating the rate at which that mortality increases during the 10 years after insurance, and whether such rate of increase depends upon the rate of discontinuance or not.

Discontinued. I will now state the method adopted to deduce the rate of mortality. In ascertaining the rate of mortality I have followed the usual course of deducting half the discontinued during any year from those who entered on it; but in this way we do not, as has been pointed out by Mr. Sprague (*J.I.A.*, xxii, 102), obtain the correct probability of dying in a year. He there says: "It is quite true that when we have a fluctuating body "of persons, continually increased by entrants and reduced by "secessions, we must make allowance for these in calculating "the effective number of persons at risk during the year; and "when we have reason to suppose that the persons seceding "would, if they could be traced, be found to be subject to the "same rate of mortality as those who remain under observation, "we shall get the correct probability of dying in a year, by "comparing the number of deaths with the effective number at "risk. But, when the persons withdrawing are subject to a "different death rate from those who remain, the ratio of the "dying to the effective number at risk, will not be the probability "of dying in a year, but will be what I have called *the death* "*rate among those who do not withdraw.*" In the same way, when ascertaining the rate of discontinuance I have deducted half those dying during any year from the number entering upon it. If this is not done, erroneous ideas may be formed from a comparison of the rates of mortality and discontinuance. Mr. Sprague, in his paper "On a combined Marriage and Mortality Table" (*J.I.A.*, xxi, 406), says: "These ratios [the marriage rate among "bachelors who do not die, and the force of marriage] are, as we "have seen, independent of the mortality, whereas the probability "of marrying in a year depends partly on the rate of mortality. "It is quite conceivable that there may be two bodies of men, "such that the force of marriage is the same in both, while one "body is subject to a much heavier death rate. In this case, the "probability of marrying in a year will not be the same in the "two bodies; and it follows that, in comparing results deduced "from different observations, the use of the probabilities of "marrying may lead to inaccurate conclusions that would be

"avoided by employing either of the other ratios." We have only to substitute "discontinuance" for "marriage" in order that the above passage may apply to the present case. It would have been easier to have employed the force of mortality and the force of discontinuance in the present investigation; but as the *rate* of mortality is spoken of in the syllabus, it would have been necessary to deduce it, in which case no labour would have been saved.

In America, I find, the rate of discontinuance is generally taken as the ratio of the discontinued to the number exposed to the risk of mortality; for what reason does not appear.

Existing. With regard to the existing, Mr. J. A. Higham says (*J.I.A.*, i, 192): "With the materials at present "available, one can do no more than follow the policies as long "as their history is given to us. But here is the imperfection "which marks all the tables founded on office experience which "we at present possess, they treat all the policies which were "existing at the period of observation as though they had lapsed. "This would be of no consequence if they were of the same "quality as the rest; but if (as I think is clear) they are of the "*worst* quality—worse than the lapsed policies, worse than the "community at large—it is evident that a table of expectations "founded on the *complete* experience of assurance companies "would show values very much below those here given. The "measure of effect which would result from tracing all the policies "to their termination may be judged from the large proportion "which the existing policies bear to the whole number under "observation."

It seems to me, however, that this is incorrect. Suppose that a number of policies on different lives of a given age have been taken out each year in an insurance office: when an investigation is made into the mortality experience of that office, there will, as a rule, be a number of lives existing at the end of each year of insurance. Now, the existing at the end of any given year may or may not be worse than those who have lapsed during the year; but I think it is clear that they exercise no option in this matter, and that they will be, on the average, equal to those who passed on to the next year. If so, then I fail to see how the rate of mortality would differ from that deduced from those who passed on to the next year by keeping the existing longer under observation. To make my meaning clearer, I will take a numerical illustration. We have seen that in the Institute Experience

5,791 healthy males entered at age 30 next birthday, that 319, 252, 230 were existing at the end of year 0, 1, 2, and that the numbers entering on year 1, 2, 3, were 5,393, 4,748, 4,263 respectively. Now, I hold that the 319 existing at the end of year 0 exercise no option in the matter, and that they will, on the average, be equal to the 5,393 who entered on year 1. In the same way the 252 existing at the end of year 1 will, on the average, be equal to the 4,748 who entered on year 2; and so on. Consequently, the rate of mortality, so far as those who entered at age 30 next birthday is concerned, would not be in the least affected if the existing were kept under observation until they had all discontinued or died, supposing the numbers to be sufficiently large to yield average results, and that no material alteration takes place in the mortality of the general population or in the circumstances of the insurance office. When, however, a mortality table is formed in the usual way by adding together all those of the same age, irrespective of the length of time for which they have been assured, then the existing would affect the rate of mortality if the period over which the bulk of the observations extend is short. The rate of mortality for age 30, for instance, will be deduced from the exposed to risk and the deaths for years 0, 1, 2, ... Now, Mr. Sprague has shown that the rate of mortality among assured lives of the same age who have been insured for 0, 1, 2, ... years, attains a maximum and then diminishes,* and the rate will therefore depend on the duration of the policies. In that case it would be necessary to carry the existing forward, dividing them into discontinued and died, in order to obtain a fair indication of what the mortality might be expected to be in the future. This, I understand, is what was done in the Thirty American Offices' Experience, where it is called the method of "final series", on account, I suppose, of the average duration of the policies being very short. No explanation, however, is there given of the reason for its adoption.

III—Methods of Graduation.

The three principal methods of graduation are Woolhouse's, Gompertz's, and the Graphic. A full description of the first mentioned will be found in the *Journal of the Institute,* vol. xv, page 389, and new and improved methods of applying it are given

* See page 164.

by Mr. G. F. Hardy in vol. xxiii, page 351, and by Mr. Ackland in the same volume, page 352. Mr. Peter Gray has described Gompertz's formula and the method of applying it in vol. vii, page 121, and Mr. Sprague has given a full description of the Graphic method and his method of applying it in vol. xxvi, page 77. It is, therefore, quite unnecessary for me to describe these methods, and I will at once address myself to a consideration of their respective merits for the purpose in view, namely, the adjustment of the rate of mortality among recently-selected lives. I am not aware of this point having been fully discussed in the *Journal*, and I wish to explain my reasons for adopting the method I have employed in the present investigation, and to show that there was practically no other course open to me.

Woolhouse's. Mr. Woolhouse's formula may be written

$$\begin{aligned}U_x = {}&\cdot 200u_x + \cdot 192(u_{x-1}+u_{x+1}) + \cdot 168(u_{x-2}+u_{x+2})\\&+\cdot 056(u_{x-3}+u_{x+3}) + \cdot 024(u_{x-4}+u_{x+4})\\&-\cdot 016(u_{x-6}+u_{x+6}) - \cdot 024(u_{x-7}+u_{x+7}).\end{aligned}$$

He says (*J.I.A.*, xv, 394), in explaining the method he devised to graduate the Institute statistics: "As the data for each separate "calculation must extend over an interval of seven years preceding "and following the given age, the formulæ will obviously not "apply to the first seven years of the table, and the numbers for "those years, namely, ages 10 to 16, will therefore be wanting. "To effect a continuous juncture at age 17, I have considered it "most expedient to supply the required numbers by means of "constant third differences.

"At age 10 the radix of the table is $l_{10}=100{,}000$. If Δ_1, "Δ_2, Δ_3, be the differences immediately following age 17, and "$n=7$, we shall have

$$\begin{aligned}l_{10} &= l_{17} - n\Delta_1 + \frac{n(n+1)}{2}\Delta_2 - \frac{n(n+1)(n+2)}{2.3}\Delta_3\\&= l_{17} - 7\Delta_1 + 28\Delta_2 - 84\Delta_3\end{aligned}$$

from which $$\Delta_3 = -\frac{l_{10}-l_{17}}{84} - \frac{\Delta_1}{12} + \frac{\Delta_2}{3}.$$

"When the series of numbers is put down in a retrograde "order, the differences that are of an odd order will change sign. "In such case therefore we shall have to begin with l_{17} and "apply the three orders of differences, after having reversed the "signs of the first and third. The third difference should be "calculated to one, or perhaps two, additional places of figures,

"and then the continued summation of the differences will "sufficiently check the accuracy of the computation.

"The first calculated numbers of the H^{MF} Table beginning "at age 17, with the accompanying differences, are—

	(l)		
Age 17	97,189		
		−469	
„ 18	96,720		−56
		−525	
„ 19	96,195		
	&c.		

"Here we have

$$\Delta_1 = -469, \quad \Delta_2 = -56, \text{ and}$$

$$\Delta_3 = -\frac{l_{10} - l_{17}}{84} - \frac{\Delta_1}{12} + \frac{\Delta_2}{3}$$

$$= -\frac{2811}{84} + \frac{469}{12} - \frac{56}{3} = -13{\cdot}05.$$

"Hence changing the signs of the odd orders, the three com-"mencing differences are $+469{\cdot}00$, $-56{\cdot}00$, $+13{\cdot}05$, and the "retrograde calculation is as follows:

	(l)		$+13{\cdot}05 = \Delta_3$
			−56·00
Age 17	97,189·00	+469·00	
			−42·95
„ 16	97,615·05	426·05	
			−29·90
„ 15	98,011·20	396·15	
			−16·85
„ 14	98,390·50	379·30	
			− 3·80
„ 13	98,766·00	375·50	
			+ 9·25
„ 12	99,150·75	384·75	
			+22·30
„ 11	99,557·80	407·05	
			+35·35
„ 10	100,000·20	+442·40	

"The column of second differences is formed by the repeated "addition of the constant third difference placed above it, and the "other columns are hence obtained by continued addition, or the "inverse operation to that of differencing."

Subsequently he gave (*J.I.A.*, xxi, 46) another method of obtaining the same results which, he says, may in general be preferred and be more readily performed.

It will, therefore, be seen that the original facts are altogether left out of account in determining the rate of mortality between ages 10 and 17, and the effect of this is shown by Mr. King in the discussion which followed the reading of the last-mentioned paper. He says: "I have taken one of my ten 'Analyzed Mortality' "tables at random. I have graduated it by Mr. Woolhouse's "formula, and I wish to call attention to the effects. The "illustration is from the table for age at entry 45.

Age	l_x Ungraduated	(l_x) Graduated by Mr. Woolhouse's Method	$(l_x)-l_x$ Difference
45	77,919	77,919	0
46	77,400	77,177	−223
47	76,568	76,346	−222
48	75,807	75,434	−373
49	74,569	74,448	−121
50	73,448	73,397	− 51
51	72,150	72,287	+137
52	71,068	71,127	+ 59
53	69,846	69,924	+ 78
54	68,525	68,686	+161

"It may, perhaps, be said that the largeness of the corrections " is due to imperfect data, and that I had not sufficient facts. " I think, however, if we look at the quality of the corrections, " that theory will be untenable. We find that they run steadily " from negative up to positive. In fact, we find that the adjust- " ment of Mr. Woolhouse unduly depresses the *l's* immediately " following the radix, and increases those further on. It distributes " unfairly, in my view, the number dying, and tends to destroy " the character of this part of the table. We can easily see the " reason of it. We get from ages 52 to 54 two differences on " which our corrections are built, and we manufacture a third by " bringing in age 45. The result is that we make the early part " of the table depend too much upon ages 52 to 54; and with a " table of this peculiar kind, we do not get a fair adjustment. " Then from age 52 onward we do not for some time find the " formula proper answer, because for age 52, for instance, we " make use of the values of *l* from 45 upwards; and the result is " that the very low mortality which exists in the first ten years, " and which is due to the medical examinations, comes in to " disturb the adjustment when we get a little later down to a " time when the benefit of selection is exhausted. There are two " forces acting on the curve, and the formula of adjustment does " not discriminate between them. Further on, when we get to " 60, I think Mr. Woolhouse's formula gives good results, but " still it does seem to me that the method fails to apply to some " portions of the table." This seems to me to be a fatal objection to the use of Mr. Woolhouse's formula in an investigation like the present; but there are other objections, among the most important of which are that it does not give good results when the observations are few and irregular, and that no greater weight is given to the large number of observations at one age than to the small numbers at others. Mr. Ackland some years later (*J.I.A.*, xxiii,

356) devised an ingenious method of overcoming the defect in Mr. Woolhouse's formula of the early ages, but for a full description of it, I must refer the reader to that paper. It is to a great extent empirical, however, and the results are not altogether satisfactory.

Gompertz's.

The series produced by Gompertz's formula depends to a great extent upon the constants used; and although it is possible by varying these to obtain fair results, yet the results will show only the action of one force on the human constitution, whereas it is assailed by many. With so short a period as 10 years to operate upon, I should think that the labour would be out of all proportion to the results. Besides, an awkward break occurs wherever new constants are introduced, and this method can, therefore, hardly be considered satisfactory. On one occasion only am I aware of its having been used when selection is at work, namely, by Mr. A. J. Finlaison in the "Government Annuitants' (1883) Experience." To obtain the adjusted rate of mortality, he used Mr. Gompertz's formula for the four years following the date of purchase of annuities, and Gompertz's and Woolhouse's after the lapse of four years, the latter formula only being used when the facts were sufficiently numerous.

Graphic.

The graphic method, on the other hand, can be applied with success where the rate of mortality is rapidly changing under the influence of selection, whether the observations are small or large, and the greatest weight can be given to those ages where they are numerous. Besides, with very little labour the observations can be made to tell their own tale, and it will then be readily seen whether it is worth while to expend more labour upon them. It is true that it requires skill in its application; but as I have had considerable experience with it, I have not hesitated to employ it on the present occasion. It will be convenient if I give a short description of how it is applied when selection is at work, as I am not aware this has been previously done in the *Journal*, in order that what follows may be the more readily understood. The facts for ages $x-2$, $x-1$, x, $x+1$, $x+2$, where x is a multiple of 5, are added together for each year of insurance, in order that as steady a progression as possible may be obtained, and the unadjusted rate of mortality calculated. When this has been done, it will be found that there are irregularities here and there; and that it will consequently be necessary to group together two or three or even more years of

insurance where the facts are few, to obtain a regular progression. The rate of mortality thus obtained is placed opposite the central age of the group; but it is obvious that this will only be strictly correct when the average age of the exposed to risk coincides with it, which will hardly ever be the case. The error will, however, as a rule be small. The rates of mortality thus brought out are plotted down on cross-ruled paper as ordinates to a curve and joined by straight lines. This gives a very fair indication of the law, and shows where the rates have a tendency to run into one another and form a common curve. Curves are then drawn with the hand, cutting off the angularities; and when the ordinates are read off, the adjusted rate of mortality is obtained. The expected deaths are then calculated and compared with the actual, and brought into as close an agreement as possible. When this method has been used, I have given the exposed to risk and the expected and actual deaths, so that the reader may judge for himself of the closeness of the adjustment. The tests of a good adjustment have been ably given by Professor McCay in his paper "On the Adjustment of Mortality Tables" (*J.I.A.*, xxii, 24). He says: "If the rates of mortality "be multiplied by the exposures, and the number of deaths which "they would give for all ages be added together from the earliest "age to the oldest, these sums, compared with the corresponding "sums of the observed deaths, will be a severe test of the accuracy "of the adjustment. If the rates were too high or too low at any "part of the table, the errors would be accumulated and continued "until they were balanced by opposite errors; and if they were "not balanced, they would show an excess or deficiency in the "total rates. If they were balanced soon, the sum of the errors "would be small, unless the errors themselves were large.

"The excellence of the adjustment would therefore be shown:

(1) by the general regularity in the rates,
(2) by the near agreement in the whole number of deaths,
(3) by the equality of the positive and negative deviations,
(4) by the smallness of the accumulated deviations,
(5) by the frequency of the changes in their signs from positive to negative."

IV.—DIRECTIONS FOR EXAMINING MORTALITY EXPERIENCES.

It may be useful to state briefly here the principal points to which I have directed my attention when examining an experience for the purpose of deducing the rate of mortality and making

comparisons. They may be of service to those engaged in compiling an experience, as some of them are apt to be overlooked in the mass of information to be dealt with.

They are as follows:

(1) Period over which the observations extend,

(2) Material and extent of the observations,

(3) Whether the lives are all males, or males and females combined,

(4) Whether the lives are all healthy, or whether those rated up on account of health or occupation are included,

(5) Country and class to which they belong, with their occupations,

(6) Class and amount of the insurances,

(7) Method in which the observations are treated—how the age at entry is arrived at, whether calendar years or policy years have been used, whether duplicate policies have been eliminated, &c.

That it is desirable, if not essential, that information should be given on all these points, the following considerations will show.

Period over which the Observations extend.

The mortality which has been experienced in the past is only a guide as to the mortality which may be expected to prevail in the future, and will not in all probability be exactly reproduced. It has been shown that the progress of science and the enactment of new sanitary laws, have exercised an influence upon the health of the people; and although the duration of life may not have altered to any great extent during the past century, still the mortality has altered at different periods of life, while the conditions under which the business of life insurance is conducted have undergone a considerable change, so that an experience which extends over the last fifty years will be a surer index of the mortality likely to prevail during the next fifty years, than an experience which extends over the first fifty years of the present century.

Material and Size of the Observations.

It has been shown that the rate of mortality varies according to the material used,—lives, policies, or amounts; and it is important, therefore, to know which has been employed in the investigation. The observations should also be sufficiently large to yield average results. An experience may yield very fair results when lives are used, but this might not be the case if amounts were used.

Whether the Lives are all Males, or Males and Females combined.

As is well known, the rate of mortality among females differs from that among males. Although female annuitants are better than males, yet it is found that for certain periods of life female insured lives are worse than males; and it is therefore necessary to know whether the experience contains male lives only, or males and females combined.

Whether the Lives are all healthy, or whether those rated up on account of Health or Occupation are included.

It is obvious that if unhealthy lives are included in any observation, the rate of mortality will be higher than if they were excluded, unless the rated-up ages are used and the additions made have exactly sufficed to cover the extra risk incurred. The inclusion of a comparatively small proportion of persons who have been rated up on account of being engaged in hazardous occupations, for instance, may have such an effect upon the observations as to render them at some ages almost worthless as an exponent of the law of mortality.

Country and Class to which they belong, with their Occupations.

It is well known that the mortality in different countries varies, and that in different classes it varies perhaps to a still greater extent. For instance, the mortality among the clergy is much lighter than that of the general population, while the mortality among publicans is heavier. We should, therefore, expect to find the rate of mortality in an office which looks to the clergy for support, much lighter than that in an office to which publicans are attracted by the offer of more favourable terms than is customary.

Class and Amount of the Insurances.

It is also necessary to know the proportions of the various classes of insurance. It is found that in some offices the rate of mortality among short-term assurances is much greater than among whole-life assurances, while the latter is believed to be greater than that among endowment assurances. It is believed, too, that the mortality among assurances effected with profits differs from that among those effected without profits. It is found also in some cases that the mortality differs according to the sum insured.

Method in which the Observations are treated.

The importance of this has been already shown, and it is unnecessary to refer to it here.

In a number of the experiences, however, no information is given on several of the above mentioned points. For instance, in the Thirty American Offices' Experience published so recently as 1881, it is not stated what age has been used when tabulating the

particulars relating to rated-up lives, or indeed whether such lives were included in the published observations, although this is very important; and no information is given as to the class of policy, the occupation of the insured, &c. In the older experiences the last-mentioned particulars are, perhaps, not important, as almost all the policies would doubtless be whole-term ones on the lives of healthy males drawn from the middle classes; but, in view of the changed conditions of life insurance business as it is now conducted, it would be desirable to have information on these points also in any future experience.

V.—Description of Mortality Experiences used.

I will now give a description of the Equitable, Amicable, Scottish Amicable, Institute of Actuaries, New York Mutual, Thirty American Offices, Gotha, and Connecticut Mutual mortality experiences, these being the ones I have examined. In order, however, that this essay may be kept within reasonable limits, it will be necessary for me to confine myself strictly to particulars which are pertinent to the present investigation.

I should mention that I have dealt with healthy male lives only, leaving out of account female and rated-up lives.

Mortality Experience of the Equitable Society from its commencement in September 1762 to 1 January 1829, by Arthur Morgan, actuary to the society, published in 1834.

This is really the first investigation into the mortality among assured lives based upon sound principles, published in this country. 21,398 healthy persons, who, I presume, would be almost all males*, entered the society during the period under review, a period exceeding 66 years. Of these 6,930 were existing at the close of the observations, 9,324 had discontinued, and 5,144 had died. The years of life were 252,708, and the average duration of the policies was therefore nearly 12 years. Full particulars are given for each age at entry and each year of insurance in a table from which the following is an extract:

* I find this assumption is correct. In Walford's *Insurance Cyclopædia* it is said that Mr. A. Morgan stated that the number of female lives insured in the Equitable from its commencement "has been so small, that the probability of life "given in Table A of the published experience may be confidently taken to "represent the value of lives of *males* only."

"TABLE A."

Age on Admission	Age 32				Age 33				Age 34			
	Attained the above Age	Living 1 Jan. 1829, at the above Age	Discontinued their Assurances	Died	Attained the above Age	Living 1 Jan. 1829, at the above Age	Discontinued their Assurances	Died	Attained the above Age	Living 1 Jan. 1829, at the above Age	Discontinued their Assurances	Died
...	...	...	...	...	...	...	...	...	...	...	...	...
32	780	...	...	1	779	7	45	5	722	12	44	5
33	...	...	...	...	726	...	...	2	724	2	32	5
34	...	...	...	...	...	...	...	...	750	...	1	...
...	...	...	...	...	...	...	...	...	...	...	...	...

Mr. Morgan, in a foot note on page 4 of the experience, says: "185 new members were admitted into the society in the course "of the whole year 1829, at various ages; these are not included "in the Table A, although the mortality of that year is included "in that table. The reason for this omission will be apparent "from a consideration of the principle upon which tables of this "kind are formed, and of the information they are intended to "convey, namely, the number of deaths which have happened, "or may be expected to happen, out of a certain number of "individuals at any given age before they shall attain to the next "year of age; in other words, how many persons out of a given "number existing at any age will survive to the next succeeding "year of age. In conformity with this principle, the duration of "the lives of those who are or have been members of the society, "is reckoned according to the number of entire years of age which "they have completed since the time of their admission. A "person, therefore, making assurance in the course of the year "1829, in the 31st year of his age, cannot be reckoned as having "lived a year in the society on the 31st of December 1829; and is "not to be added until 1830, when he must be classed under the "age of 31, as he would complete his 32nd year in 1830, and "have entered his 33rd year on the 1st of January 1831, should "his life have so long endured." Considerable difficulty has been experienced by previous writers in understanding this statement, and for a summary of their views and conclusions I may refer the reader to a paper by Mr. Spens "On the Mortality Experience of the Scottish Amicable Life Assurance Society", printed in the tenth volume of the *Journal*, page 66. It is there

stated that Mr. Edmonds, in a paper in the *Lancet*, 1842, says: "He [Mr. Morgan] assumes that all who have been admitted "during any year of the observation, in any specific year of age, "will have completed that year of age at the end of such year of "the observation. Having made this assumption, he disregards "any mortality occurring in the fractional part of the first year "of admission, and commences the observation from the beginning "of the ensuing year, when the age is expressible without fractions "of a year. For example, if a member had been admitted in the "45th year of age in any part of the year 1821, he will have been "regarded as having completed his 45th year, or as being exactly "45 years old, on the 31st of December 1821; and, for the "purposes of the observation, no account will have been taken of "this member before the 1st January 1822. Although, as I "have learned from enquiries, this is the mode of proceeding "adopted by Mr. Morgan, yet there is no statement to this effect "published by him." I was at first inclined to agree with Mr. Edmonds—indeed, I had arrived at the same conclusion before I had seen his explanation—that Mr. Morgan had simply excluded from observation year "0"; but on examining the figures themselves, I am doubtful whether this is actually the case. If we look at the above table we see that at ages 32 and 33 on admission, where the numbers are 780 and 726 respectively, none discontinued their assurances on this assumption during the first complete calendar year after entry, that is to say, that all of them paid the second premium, while in the next calendar year 45 and 32 respectively discontinued: also that for age on admission 34, out of 750 entrants only one discontinued in the first complete calendar year after entry. I can hardly credit this. It seems to me more likely that the figures relate to year "0", and that the deaths ought to be doubled in deducing the rate of mortality for the first year; and this would account for the great discrepancy noticed by Mr. Spens between the rate of mortality for the first and second calendar years after entry. On this assumption, however, the rate of mortality for the year "0" would be very high; and as the point is not free from doubt, I have preferred to take the figures as they stand. Mr. Spens, in his paper above mentioned, says: "Apparently the mortality for the "year 1829, noted separately on page 29 of the Equitable "experience, is not included in the mortality in Table A, as "it would seem it ought to be, and as it is stated by Mr. Morgan "to be. . . . ." What his reasons are for saying so are

not given, but on enquiry I have ascertained that the deaths for that year among old insurers were included; and this may account for the slip made by Mr. Morgan in the introduction: "The first of the following tables (Table A) is so formed as to "exhibit in one view the duration (so far as the same can be "traced) of *all* the lives which have been assured in the Equitable "Society from the time of its commencement in September 1762 "to the 1st of January 1829, a period exceeding 67 (?) years."

As one of the reasons stated by the directors to the society for undertaking this investigation was, that no allowance had been made for the plurality of policies effected on one life in the tables already published, professing to be founded upon the Equitable experience, I assume that duplicate policies were eliminated on this occasion.

It will be observed from the manner in which the above table is made up, that the existing are to be deducted at the beginning of the year, instead of at the end as is usual. For instance, at age 32 there came under observation 780, among whom 1 died, and the number existing at the end of the year, 7, is placed under age 33. Consequently, if we wish to get the number exposed to risk between ages 33 and 34, we should deduct the 7 existing from the 779 who attained the age of 33, since they are included in that number, and half the discontinued, 22·5, which gives 749·5 as the number exposed to risk, among whom 5 deaths took place; and so on. In the introduction to the Seventeen Offices' Experience, page ix, it is stated: "When the number existing at any "given age had been exposed to the risk of mortality for the "whole of the year, as is the case in the tables of the Equitable "Society, only one-half of the discontinued was deducted from "the number that attained the given age; and when the number "existing at the termination of the experience had not been "exposed to the risk of mortality for any part of the year, as is "the case in some of the returns, the whole of the number "existing and half the discontinued were deducted from the "number which attained the age, in order to ascertain the number "exposed to the risk of mortality during the year, out of which "the deaths were reckoned to have taken place." In Tables H(1), H(2), H(3), and H(4) of that experience which relate to the Equitable, we have the means of checking the figures, and it seems to me that the existing have been kept under observation for a year longer than they ought to have been. For instance, the 7 in the example given above as attaining 33 are not deducted

until they have attained age 34, that is to say, the number exposed to risk between ages 33 and 34 is taken as $779-22\cdot5=756\cdot5$. I think there can be little doubt that they ought to have been deducted at the beginning instead of at the end, and I have done so in deducing the rate of mortality; but however this may be, it will not affect the rate in the first 10 years to any great extent. Mr. Spens, when dealing with the same facts in Table No. VII of his paper above mentioned, has followed the same course as was adopted in the Seventeen Offices' Experience. But even then, my figures and his for the exposed to risk do not always agree. For instance, in the fourth year of insurance he gives the exposed to risk as 17,129·5, whereas, according to the principle on which he has gone, it should be 17,160. In the same table he gives the rate of mortality for the second year as 1·181 per-cent, and on page 66 he says it is 1·167 per-cent, while according to my interpretation of the figures it should be 1·191 per-cent.

Mr. Spens in 1862 said it would be a boon if an authoritative statement were made as to the manner in which this experience has been made up, but I am not aware of this suggestion having been complied with.

Tables of Mortality deduced from the Experience of the Amicable Society for the 33 years ending 5 April 1841, by Thomas Galloway, Registrar of the Society, published in 1841.

This was the next experience published, and, as already mentioned, it is arranged according to policy years. It is not nearly so large as that of the Equitable, the period over which the observations extended being only about one-half. 3,530 entered the society, of whom 2,227 were existing at the close of the observations, 505 had discontinued, and 798 had died. They were almost all males, and all of them healthy or select at entry, and assured for the whole of life. The years of life were 38,370, and the average duration of the policies 10·87 years, or nearly the same as in the Equitable. Duplicate policies were eliminated, so that it is a record of lives. Full particulars of the lives are given for each age at entry and each policy year. They were assumed to be on the average six months less than age next birthday at entry, although it was found from an examination of part of the experience that they were only 4·15 months less. As a fixed date was taken for the termination of the observations, namely, 5 April 1841, it was necessary in ascertaining the exposed

to risk for ages $x-\frac{1}{2}$, $x+\frac{1}{2}$, &c., to deduct half the existing as well as half the discontinued from the number who entered on the year. In order to obtain the rate of mortality for age x, half the exposed to risk at age $x-\frac{1}{2}$ was added to half the exposed to risk at age $x+\frac{1}{2}$, and the deaths treated in the same way.

Although among the earliest of published mortality experiences, the description of the methods employed is easily followed, each step being clearly shown, which cannot be said of some of the later experiences.

Table of the Mortality Experience of the Scottish Amicable Life Assurance Society, from 1826 to 1860, by William Spens, Manager of the Society, published in 1861.

This experience is divided into four sections:

(1) Non-hazardous—Males.
(2) Non-hazardous—Females.
(3) Hazardous (both sexes)—exclusive of West Indies.
(4) Hazardous (both sexes)—West Indies.

As already mentioned, I shall deal with the first only, which relates to 8,596 persons, of whom 5,414 were existing, 2,550 had discontinued, and 632 had died. The years of life were 48,032, and the average duration of the policies 5·59 years. It is a record of lives arranged according to policy years, each policy being traced to its anniversary in the final policy year of the observations, but it does not embrace the lives on which annuities or endowments were granted. The ages at entry "were taken as the difference between "the year of birth and the year of entry—a mode which evidently "(on the assumption, at least, of parties being born equally at any "time of a year) states the ages on an average correctly." "In the "case of policies cancelled otherwise than by death, when the broken "period was less than half a year, 0 was reckoned; when half a "year, ·5; when more than a half, 1." Only the exposed to risk and the deaths are given for each age at entry and each year of insurance; but in the supplementary observations made by Mr. Spens in presenting this experience to the Institute, he gave (*J.I.A.*, x, 204) a table showing the percentage of discontinuance in the society for each year of insurance.

Mortality Experience of Life Assurance Companies collected by the Institute of Actuaries, published in 1869.

The data for this experience was furnished to the Institute of Actuaries on cards by 20 British offices, and arranged under the

superintendence of a committee of that body. The cards were divided into four sections:

(1) Healthy Lives—Male.
(2) Healthy Lives—Female.
(3) Diseased Lives—Male and female.
(4) Lives exposed to extra risks from climate, occupation, &c.

There were 130,243 entrants in the first section, of whom 74,698 were existing, 35,024 had discontinued, and 20,521 had died. There were 1,190,140 years of life, and the average duration of the policies was 9·13 years. Duplicate policies were eliminated, and the experience was arranged according to calendar years. Full particulars of the lives are given for each age at entry and each year of insurance. It is stated that, for all practical purposes, the benefit of selection may perhaps be said to be lost after the fifth year of insurance.

Mr. King has given (*J.I.A.*, xix, 383) the following table, showing an analysis of the business, when last they valued, of 19 out of the 20 offices which contributed their experiences:

Description of Assurances	Amounts Assured, less Re-assurances	Percentage to Total Business
Whole Life, Uniform Premiums .	£97,377,572	90·33
Do., other descriptions . .	5,410,688	5·02
Joint Lives	682,297	·63
Endowment Assurances . . .	1,662,165	1·54
Short Term	1,113,001	1·03
Contingent Survivorship . . .	756,999	·70
Last Survivor	701,173	·65
Endowments	108,175	·10
	£107,812,070	100·00

Mortality Experience of the Mutual Life Insurance Company of New York from 1843 to 1873, by William H. C. Bartlett, LL.D., published in 1876.

This is not the first occasion on which the mortality experience of this company has been published, and the volume under review at present is replete with a mass of interesting information, upon the compilation of which much labour must have been bestowed. 101,967 healthy persons entered during the period (about 3 per-cent were females), of whom 68,688 were existing, 27,764 had discontinued, and 5,515 had died. It is stated that some of the lives had been exposed to extra risk of climate and warfare, and that duplicate policies were eliminated. There were 575,355 years of life, and the average duration of each policy was 5·64 years. The age at entry is, according to the American

custom, the age nearest birthday. "In the subsequent work the method adopted by the British actuaries was carefully pursued"; but in the table of mortality deduced from the observations no adjustment appears to have been made for the effect this would have on the ages nearest birthday, namely, reducing them by half a year. Age 30, for instance, nearest birthday, would thus become $29\frac{1}{2}$, but the former age seems to have been retained. The exposed to risk and died are given for each age at entry and each year of insurance, and a table of discontinued is given for quinquennial groups of ages at entry, and quinquennial groups of insurance years; also for each year of insurance, all ages combined, the ratio being $\frac{\text{Retirants}}{\text{Exposed to risk of death}}$.

According to the New York Insurance Report relating to the year 1872, the policies in force at the end of that year in this company, including additions, classified, were:

	No.	Amount	Percentage of No. to Total No.
Whole-Life Policies	58,603	£41,720,214	75·00
Endowment Policies	19,391	11,156,265	24·81
Joint Lives and Survivorships	152	42,257	·19
	78,146	£52,918,736	100·00

As the amount of insurance varies in the different classes—endowment assurances, for instance, being generally for a smaller amount than whole-life—the effect of the various classes on the mortality will be best ascertained by taking the number of policies; and a column has been added showing the percentage of the policies in each class to the total number. It will be seen that the proportion of endowment assurances in this experience is very much higher than in the experience of the Twenty British Offices, and that of whole-life policies very much lower; also that the special classes form a very small proportion of the whole.

System and Tables of Life Insurance, or Thirty American Offices' Experience, by Levi W. Meech, published in 1881.

This is the largest experience which has ever been published. The period over which the observations extend may be gathered from the fact that it is stated that "the first 10 or 15 years' "duration comprise the greatest part of the business; and the "whole is practically limited to 30 years, although a few cases "are of older date." The experience was divided into two

sections—males and females. In the former section there were 982,734 entrants, of whom 527,157 were existing, 411,092 had discontinued, and 44,485 had died. The years of life were 4,304,843·5, and the average duration of each policy was 4·38 years. I presume that it is a record of policies and not of lives, as neither the name nor the date of birth was given on the card; and in the instructions offices are not warned to eliminate duplicate policies. Even if they had, however, this would not have prevented them occurring, as one person is often insured in several offices. Tables of the existing, discontinued, died, and exposed to risk, for lives and amounts, are given for each age at entry and each year of insurance. The method of tabulating the experience is that adopted by the Institute of Actuaries. The ages at entry are, according to the American custom, those nearest birthday; and, as already stated, this has the effect of reducing age at entry 30, for instance, to 29½. The rate of mortality for integral ages was found by taking the mean of the data at two adjacent ages. This, however, does not allow for the variation in the numbers entering at the different ages. When for any special reason the age had been rated higher or lower than the true age, the latter was also given; but, as already mentioned, it is not stated which was used when tabulating the cards. Mr. Ryan, in a recent number of the *Journal* (*J.I.A.*, xxviii, 221), says that "the rated-up ages were apparently adopted in all cases." What his reason is for saying so, however, he does not state. From the fact that instructions were given to "indicate the kind of policy by L for life, E for endowment, and T for term policies", I infer that the experience relates to these classes only. It is stated that "in relation to *endowment insurances* composed of two parts, that "is, of a temporary insurance, and a pure endowment for the "same amount, since the maturity of the endowments was not "specified on our cards, their effect on the whole experience is "that of temporary insurance only." It will be seen from the figures given above, that more than 50 per-cent of the entrants were existing at the close of the observations; and as the duration of the policies was short, in forming the mortality table the existing at any given age were, as already mentioned, carried forward and proportioned into discontinuances and deaths each year, until they had all discontinued or died. This they called the method of "final series." It is stated that the effect of selection is lost after about 2½ years.

Mittheilungen aus der Geschäfts und Sterblichkeits-Statistik der Lebensversicherungsbank für Deutschland zu Gotha, für die fünfzig jahre von 1829 bis 1878, herausgegeben von Dr. A. Emminghaus. (Weimar, 1880.) (Statistics relating to the Business and to the Mortality Experience of the Life Insurance Office of Germany at Gotha, for the 50 years from 1829 to 1878, by Dr. A. Emminghaus.)

This is by far the most valuable experience ever published. Not only is the Gotha Life Office one of the largest European companies transacting ordinary life assurance business, but the information contained in the volume far surpasses that given by any other office or group of offices—indeed, it is replete with information of the most interesting kind upon almost every conceivable point. Insurances are only granted by this office on the lives of healthy persons who live within its district. The experience is divided into males and females, the number of entrants in the former being 76,986, of whom 47,596 were existing, 9,391 had discontinued, and 19,999 had died. There were 964,036·5 years of life, and the average duration of the policies is 12·39 years. Full particulars of the existing, discontinued, died, and exposed to risk, are given for each age at entry and each year of insurance. The experience is arranged according to policy years, each policy being traced to its anniversary in the final policy year of the observations. I am unable to find it anywhere stated that duplicate policies were eliminated; but as "persons" are spoken of, whereas the word "policies" is used in referring to the Seventeen English Offices' experience, I assume that they were. Tables are given, showing the occupation of the insured, and the amount and class of their policies. As might be anticipated in a work of such magnitude, several mistakes have occurred: for instance, Table 10, on page 83, which shows the percentage of discontinuances in quinquennial insurance years for various groups of ages at entry, relates to males and females—not males only, as stated in the heading. On page 49, where the Gotha experience is compared with the healthy lives of the Twenty English Offices, the numbers given as the existing and the years of life, include the diseased and those rated up for climate and occupation.

Mortality Experience of the Connecticut Mutual Life Insurance Company, of Hartford, Conn., from 1846–78, published in 1884.

This experience is arranged according to policy years, the observations being carried to the anniversaries of the policies in

the last calendar year, and the age at entry is at nearest birthday. The male lives are divided into three sections:

	Years of Life	Died
(1) Whole-life and Endowment Assurances .	711,077	8,009
(2) Paid-up Policies issued in lieu of Policies surrendered	57,512	618
(3) Short-term Policies	11,121	188
	782,710	8,815

It is stated that "the ratio of actual to probable deaths among "male lives insured under term policies . . . . is 1·715, "or more than twice as great as among male lives insured under "other forms of policies, or as a whole. This is due in great part "to the fact that these policies were largely taken to cover special "risks, especially on the lives of persons going to California to "engage in gold-mining in 1849 and the years succeeding. The "excessive mortality at the ages under 25, and especially under "20, is probably due to the immaturity of the lives and their "consequent inability to endure the unusual strain put upon "them. The same cause may also have operated to increase "somewhat the mortality under premium-paying life and endow-"ment policies, especially at the younger ages." The exposed to risk and the died for each age at entry and each policy year, are only given for the three sections combined, the number of entrants being 97,790, the deaths 8,746, and the years of life 780,353. These figures differ from the totals given above, on account, no doubt, of a number of the lives being insured in more than one class, and their consequent elimination when combined. Tables of the discontinued are given for the whole-term policies for each year of insurance, and for decennial ages at entry; also for endowment assurance policies. Tables are also given showing the discontinuances at different periods in the company's history, which show that they are on the increase; the percentage of whole-term policies, for instance, discontinued in the period 1846–64 being 4·10, while in 1870–77 it was 12·05. Among other tables there is one showing the occupations of the assured.

In order that the foregoing experiences may be readily compared, they are summarized in the following table. In arriving at the years of life, I have taken the years entered upon and deducted half the deaths and withdrawals, and, where necessary, half the existing; also in the case of calendar years, half the entrants. The average duration of the policies has been ascertained

Summary of the Mortality Experiences.

Name	Date of Publication	Duration in Years	Material	Entrants	Existing	Discontinued	Died	Percentage of Existing	Percentage of Discontinued	Percentage of Died	Years of Life	Average Duration of Policies	Males or Females	Healthy or Rated-up	Class of Assurance	Calendar or Policy Years
Equitable . . .	1834	66⅓	Lives	21,398	6,930	9,324	5,144	32·4	43·6	24·0	252,708	11·81	Almost all Males	Healthy	...	Calendar
Amicable . . .	1841	33	Lives	3,530	2,227	505	798	63·1	14·3	22·6	38,370	10·87	Almost all Males	Healthy	Life	Policy
Scottish Amicable .	1861	34	Lives	8,596	5,414	2,550	632	63·0	29·7	7·3	48,032	5·59	Males	Healthy	...	Policy
Institute of Actuaries.	1869	142	Lives	130,243	74,698	35,024	20,521	57·4	26·9	15·7	1,190,140	9·13	Males	Healthy	Chiefly Life	Calendar
New York Mutual .	1876	30 11/12	Lives	101,967	68,688	27,764	5,515	67·4	27·2	5·4	575,355	5·64	Males and Females	Healthy	Chiefly Life and Endowment Assurance	Calendar
Thirty American .	1881	30	Policies	982,734	527,157	411,092	44,485	53·7	41·8	4·5	4,304,813·5	4·38	Males	Probably both	Life, Endowment Assurance, and Short Term	Calendar
Gotha . . .	1881	50	Lives	76,986	47,596	9,391	19,999	61·8	12·2	26·0	964,036·5	12·39	Males	Healthy	...	Policy
Connecticut Mutual .	1884	32	Lives	97,790	...	...	8,746	...	...	8·9	780,353	7·98	Males	Both	Life, Endowment Assurance, and Short Term	Policy

by dividing the result thus obtained by the entrants. It sometimes differs slightly from that given by other writers on account of their having treated the figures differently.

Perhaps the most notable features in the above table are the large percentage of the discontinued in the Equitable and the Thirty American, and the small percentage in the Amicable and the Gotha. In the case of the Thirty American, however, it must be remembered that probably endowment assurances formed a not inconsiderable portion of the experience, and that "their effect on the whole experience is that of temporary insurance only." It is the only experience in which policies have been used instead of lives. All the experiences or portions of experiences here tabulated may be said to relate to males, and, with two exceptions, to healthy males. These two exceptions are the Thirty American and Connecticut Mutual, which, no doubt, contain lives which have been charged an extra on account of exposure to extra risk, &c., although the number is probably small. The observations in one half of the experiences have been arranged according to calendar years, and in the other half according to policy years; but it is to be regretted that the three largest are contained in the former.

VI.—Numbers at Risk and Deaths in the Mortality Experiences.

I will now give the numbers at risk, and the deaths, in the mortality experiences, deduced by me in the manner previously described, for quinary groups of ages at entry, with the unadjusted rate of mortality; also the grouping adopted to indicate the rate at which the mortality increases during the 10 years after insurance, the results thus obtained being shown in Diagram No. 1 appended to this essay. I will first of all deal with those experiences which are arranged according to policy years.

Amicable. The number of entrants in this experience, 3,530, is small; and when they are divided into quinquennial groups of ages at entry, the rate of mortality is very irregular. Mr. Galloway indeed refers to this himself. He says: "It is "true that the materials from which the present table is "constructed, are much too scanty to allow of its indications "being safely adopted for the solution of such questions [such, "for example, as the effects of selection at the different ages]; "but as the present investigation is the first of a series "directed by the society's charter to be undertaken every "tenth year, and as a much larger number of facts will

"necessarily be accumulated at the future decennial periods, it "appeared to be desirable, in beginning the series, to arrange "the data in a form which affords the greatest facilities for "deducing from them all the information they are capable of "yielding, supposing them sufficiently numerous." I therefore decided to abandon it. It is true that it would have been possible, by means of the graphic method, to have obtained a fairly satisfactory law, but this would have been more a matter of judgment than of allowing the facts to indicate plainly the law they follow; and I take this opportunity of stating that I have carefully abstained from making any such attempt in the present investigation, preferring to base any conclusions at which I have arrived on solid grounds.

Scottish Amicable. The exposed to risk and the deaths for the quinary groups of ages at entry 23–27, 28–32, ... 48–52, which will for convenience be represented by the central ages 25, 30, ... 50, are given in the following table for each year of insurance up to the tenth, the numbers beyond these ages being too small to admit of their being dealt with:

Scottish Amicable Experience—Males.

Year of Insurance	Number at Risk	Deaths	Rate of Mortality	Years of Insurance	Number at Risk	Deaths	Rate of Mortality
Age at Entry 25							
1	1,231·5	4	·0033	1	1,231·5	4	·0033
2	1,007	6	·0060	2	1,007	6	·0060
3	813·5	11	·0135				
4	651	3	·0046	3–5	1,967·5	20	·0102
5	503	6	·0119				
6	408	3	·0074				
7	328·5	1	·0030				
8	258	7	·0271				
9	183	3	·0164	6–11	1,411·5	15	·0106
10	145	1	·0069				
Age at Entry 30							
1	1,777	7	·0039	1	1,777	7	·0039
2	1,467	11	·0075				
3	1,223	11	·0090	2–4	3,718·5	26	·0070
4	1,028·5	4	·0039				
5	834·5	9	·0108	5	834·5	9	·0108
6	676	9	·0133	6, 7	1,226·5	14	·0114
7	550·5	5	·0091				
8	424	2	·0047				
9	322	5	·0155	8–11	1,164·5	14	·0120
10	236	4	·0170				

Scottish Amicable Experience—Males—(continued).

Year of Insurance	Number at Risk	Deaths	Rate of Mortality	Years of Insurance	Number at Risk	Deaths	Rate of Mortality
			Age at Entry 35				
1	1,749·5	9	·0051	1	1,749·5	9	·0051
2	1,463·5	8	·0055	2	1,463·5	8	·0055
3	1,262·5	5	·0040	3, 4	2,318·5	19	·0082
4	1,056	14	·0133				
5	851	11	·0129	5	851	11	·0129
6	698·5	12	·0172	6, 7	1,266·5	18	·0142
7	568	6	·0106				
8	444·5	9	·0203				
9	336	4	·0119				
10	248	6	·0242	8–15	1,747	27	·0155
			Age at Entry 40				
1	1,323	9	·0068	1	1,323	9	·0068
2	1,107	7	·0063				
3	963·5	6	·0062	2–4	2,889·5	24	·0083
4	819	11	·0134				
5	671	8	·0119	5, 6	1,232·5	17	·0138
6	561·5	9	·0160				
7	476	9	·0189				
8	381	11	·0289				
9	310	2	·0065	7–11	1,586	25	·0157
10	240	2	·0083				
			Age at Entry 45				
1	903·5	6	·0066	1	903·5	6	·0066
2	764	10	·0131				
3	651	8	·0123	2–5	2,441	29	·0119
4	556·5	10	·0180				
5	469·5	1	·0021				
6	408	4	·0098	6, 7	755	12	·0159
7	347	8	·0231				
8	277	5	·0181	8	277	5	·0181
9	216	7	·0324				
10	152	5	·0329				
			Age at Entry 50				
1	541·5	5	·0092	1	541·5	5	·0092
2	461·5	5	·0108	2	461·5	5	·0108
3	400·5	6	·0150				
4	343·5	2	·0058	3–5	1,017	13	·0128
5	273	5	·0183				
6	227	3	·0132	6, 7	420	9	·0214
7	193	6	·0311				
8	153	5	·0327				
9	120	2	·0167				
10	82	2	·0244	8–12	474	14	·0295

Gotha. The following table gives the exposed to risk and the deaths in this experience for ages at entry 23–27, 28–32, ... 58–62, for each year of insurance up to the tenth, denoting the groups, as before, by the central age:

Gotha Experience—Males.

Year of Insurance	Number at Risk	Deaths	Rate of Mortality	Years of Insurance	Number at Risk	Deaths	Rate of Mortality
Age at Entry 25							
1	6,487	20	·0031	1	6,487	20	·0031
2	5,725·5	31	·0054	2	5,725·5	31	·0054
3	5,094	34	·0067	3	5,094	34	·0067
4	4,578	39	·0085	4, 5	8,672	64	·0074
5	4,094	25	·0061				
6	3,725	25	·0067	6–9	13,162	105	·0080
7	3,406·5	28	·0082				
8	3,137	35	·0112				
9	2,893·5	17	·0059				
10	2,620·5	18	·0069	10, 11	4,997	44	·0088
Age at Entry 30							
1	16,335·5	65	·0040	1	16,335·5	65	·0040
2	14,933·5	91	·0061	2	14,933·5	91	·0061
3	13,687	91	·0066	3	13,687	91	·0066
4	12,640·5	92	·0073	4	12,640·5	92	·0073
5	11,653·5	91	·0078	5, 6	22,361	174	·0078
6	10,707·5	83	·0078				
7	9,823	90	·0092	7–9	27,478	250	·0091
8	9,095·5	71	·0078				
9	8,559·5	89	·0104				
10	7,853	72	·0092	10, 11	15,075	157	·0104
Age at Entry 35							
1	17,945	102	·0057	1	17,945	102	·0057
2	16,653	112	·0067	2	16,653	112	·0067
3	15,489	103	·0067	3, 4	29,963·5	214	·0071
4	14,474·5	111	·0077				
5	13,518·5	104	·0077	5	13,518·5	104	·0077
6	12,624·5	109	·0086	6	12,624·5	109	·0086
7	11,718	116	·0099	7	11,718	116	·0099
8	10,924·5	91	·0083	8–10	30,717	317	·0103
9	10,294·5	120	·0117				
10	9,498	106	·0112				
Age at Entry 40							
1	14,462	96	·0066	1, 2	28,025	187	·0067
2	13,563	91	·0067				
3	12,738·5	111	·0087	3	12,738·5	111	·0087
4	12,022·5	119	·0099	4	12,022·5	119	·0099
5	11,297	137	·0121	5–7	31,748	377	·0119
6	10,581·5	122	·0115				
7	9,869·5	118	·0120				
8	9,254	113	·0122	8	9,254	113	·0122
9	8,749	118	·0135	9, 10	16,832	231	·0137
10	8,083	113	·0140				

Gotha Experience—Males—(continued).

Year of Insurance	Number at Risk	Deaths	Rate of Mortality	Years of Insurance	Number at Risk	Deaths	Rate of Mortality
			Age at Entry 45				
1	8,966	60	·0067	1	8,966	60	·0067
2	8,396·5	66	·0079	2	8,396·5	66	·0079
3	7,905	99	·0125				
4	7,450·5	88	·0118	3–5	22,389·5	264	·0118
5	7,034	77	·0110				
6	6,651·5	100	·0150	6	6,651·5	100	·0150
7	6,220	113	·0182	7, 8	12,029·5	214	·0178
8	5,809·5	101	·0174				
9	5,476	108	·0197	9, 10	10,513·5	199	·0189
10	5,037·5	91	·0181				
			Age at Entry 50				
1	5,952·5	57	·0096	1	5,952·5	57	·0096
2	5,607	84	·0150	2	5,607	84	·0150
3	5,275·5	92	·0174	3, 4	10,251·5	179	·0175
4	4,976	87	·0175				
5	4,683	86	·0184	5	4,683	86	·0184
6	4,409	91	·0206	6	4,409	91	·0206
7	4,114·5	87	·0211	7	4,114·5	87	·0211
8	3,816	102	·0267	8	3,816	102	·0267
9	3,563	104	·0292	9	3,563	104	·0292
10	3,260·5	100	·0307	10–12	8,982·5	274	·0305
			Age at Entry 55				
1	3,119·5	40	·0128	1	3,119·5	40	·0128
2	2,921·5	65	·0223	2, 3	5,663	128	·0226
3	2,741·5	63	·0230				
4	2,590	71	·0274				
5	2,438	65	·0267	4–6	7,303·5	197	·0270
6	2,275·5	61	·0268				
7	2,101·5	67	·0319	7	2,101·5	67	·0319
8	1,958	65	·0332	8	1,958	65	·0332
9	1,818	84	·0462	9, 10	3,461	153	·0442
10	1,643	69	·0420				
			Age at Entry 60				
1	1,338·5	32	·0239	1, 2	2,583	58	·0225
2	1,214·5	26	·0209				
3	1,171·5	40	·0341	3, 4	2,266	71	·0313
4	1,094·5	31	·0283				
5	1,029·5	35	·0340	5	1,029·5	35	·0340
6	952	54	·0567				
7	866	39	·0450	6–8	2,621	128	·0488
8	803	35	·0436				
9	730	35	·0480	9, 10	1,395	76	·0545
10	665	41	·0617				

The observations in this experience are numerous, and consequently it has not been necessary to group many years of insurance together. The rates of mortality brought out are represented by the continuous lines in Diagram No. 1. The dotted, crossed, and broken lines there shown will be explained afterwards.

Connecticut Mutual.

The figures were extracted for this experience also; but it was evident, from the unusual course which the rates of mortality followed, that some cause was operating to disturb them. This is no doubt due to the fact that the term policies, among which the mortality was, from special causes, very heavy (see page 123), are included in the observations; and I therefore decided to discard this experience, as any results founded upon it would have been untrustworthy—at least, in the early years of insurance.

Equitable, Institute, New York Mutual, and Thirty American.

As already mentioned, the experiences arranged according to calendar years will be of service in showing the rate at which the mortality increases during the 10 years after insurance, and whether such rate of increase depends upon the rate of discontinuance or not; and it will be convenient to give here also the figures relating to the numbers at risk and the deaths for the Equitable, Institute, New York Mutual, and Thirty American experiences. As before, the age at entry in each case is the central age of a quinary group; but on account of the age at entry in the two American experiences being age nearest birthday, a different method of treating the figures, which will be explained later on, was adopted.

Equitable Experience.

Year of Insurance	Number at Risk	Deaths	Rate of Mortality	Years of Insurance	Number at Risk	Deaths	Rate of Mortality
			Age at Entry 25				
1	2,957·5	9	·0030	1	2,957·5	9	·0030
2	2,804·5	28	·0100	2, 3	5,298·5	48	·0091
3	2,494	20	·0080				
4	2,228	22	·0099	4	2,228	22	·0099
5	2,026	19	·0094				
6	1,843·5	14	·0076				
7	1,690	18	·0107				
8	1,553	17	·0110	5–11	10,995	113	·0103
9	1,410·5	14	·0099				
10	1,279·5	15	·0117				

Equitable Experience—(continued).

Year of Insurance	Number at Risk	Deaths	Rate of Mortality	Years of Insurance	Number at Risk	Deaths	Rate of Mortality
			Age at Entry 30				
1	3,839·5	10	·0026	1	3,839·5	10	·0026
2	3,697·5	22	·0060	2	3,697·5	22	·0060
3	3,404·5	24	·0071	3	3,404·5	24	·0071
4	3,102	27	·0087	4	3,102	27	·0087
5	2,858	26	·0091	5	2,858	26	·0091
6	2,654·5	28	·0106	6	2,654·5	28	·0106
7	2,469	28	·0113				
8	2,284	25	·0110	7–10	8,780	94	·0107
9	2,090	22	·0105				
10	1,937	19	·0098				
			Age at Entry 35				
1	3,680·5	11	·0030	1	3,680·5	11	·0030
2	3,577	42	·0117				
3	3,330	49	·0147				
4	3,052	33	·0108	2–7	17,819	209	·0117
5	2,811	31	·0110				
6	2,610	27	·0103				
7	2,439	27	·0111				
8	2,275	26	·0114				
9	2,090·5	21	·0101				
10	1,943·5	23	·0118	8–12	9,832	116	·0118
			Age at Entry 40				
1	3,042·5	12	·0039	1	3,042·5	12	·0039
2	2,950·5	31	·0105				
3	2,748	29	·0106	2–5	10,607	112	·0106
4	2,540·5	27	·0106				
5	2,368	25	·0106				
6	2,232·5	27	·0121				
7	2,121	23	·0108	6–8	6,332·5	73	·0115
8	1,979	23	·0116				
9	1,816·5	25	·0138	9	1,816·5	25	·0138
10	1,686	28	·0166	10–12	4,775	78	·0163
			Age at Entry 45				
1	2,355	13	·0055	1	2,355	13	·0055
2	2,279·5	28	·0123				
3	2,114·5	27	·0128	2–4	6,334·5	79	·0125
4	1,940·5	24	·0124				
5	1,810	27	·0149	5	1,810	27	·0149
6	1,697	29	·0171	6	1,697	29	·0171
7	1,587·5	29	·0183				
8	1,460·5	26	·0178	7–10	5,593	105	·0188
9	1,330	28	·0211				
10	1,215	22	·0181				

Equitable Experience—(continued).

Year of Insurance	Number at Risk	Deaths	Rate of Mortality	Years of Insurance	Number at Risk	Deaths	Rate of Mortality
			Age at Entry 50				
1	1,635·5	19	·0116	1	1,635·5	19	·0116
2	1,573	28	·0178	2	1,573	28	·0178
3	1,455	33	·0227				
4	1,330	32	·0241	3–5	4,002	89	·0222
5	1,217	24	·0197				
6	1,133	23	·0203				
7	1,066	29	·0272				
8	984·5	23	·0234	6–10	4,916	117	·0238
9	901	20	·0222				
10	831·5	22	·0265				

Institute Experience—Males.

Year of Insurance	Number at Risk	Deaths	Rate of Mortality	Years of Insurance	Number at Risk	Deaths	Rate of Mortality
			Age at Entry 25				
0	11,142	27	·0024	0	11,142	27	·0024
1	20,037	124	·0062	1	20,037	124	·0062
2	17,368	131	·0075	2	17,368	131	·0075
3	15,260·5	121	·0079	3	15,260·5	121	·0079
4	13,173·5	123	·0091	4, 5	25,522	228	·0089
5	12,048·5	105	·0087				
6	10,850	101	·0093	6	10,850	101	·0093
7	9,778	88	·0090				
8	8,814	82	·0093	7–9	26,547·5	255	·0096
9	7,955·5	85	·0107				
10	7,061	83	·0117	10, 11	13,360·5	137	·0103
			Age at Entry 30				
0	13,232	59	·0045	0	13,232	59	·0045
1	24,222·5	121	·0050	1	24,222·5	121	·0050
2	21,543·5	158	·0073	2	21,543·5	158	·0073
3	19,333·5	169	·0087	3	19,333·5	169	·0087
4	17,462	183	·0105	4, 5	33,353	337	·0101
5	15,891	154	·0097				
6	14,595·5	154	·0106	6, 7	27,920	284	·0102
7	13,324·5	130	·0098				
8	12,085	134	·0111	8, 9	23,066	247	·0107
9	10,981	113	·0103				
10	9,942·5	125	·0126	10, 11	18,844	221	·0117
			Age at Entry 35				
0	11,585·5	53	·0046	0	11,585·5	53	·0046
1	21,329	141	·0066	1	21,329	141	·0066
2	19,143	164	·0086	2	19,143	164	·0086
3	17,465	170	·0097	3	17,465	170	·0097
4	15,879	176	·0111				
5	14,518	152	·0104	4–6	43,835·5	466	·0106
6	13,408·5	138	·0103				
7	12,323·5	133	·0108				
8	11,231·5	122	·0109	7–9	33,784·5	362	·0107
9	10,229·5	107	·0105				
10	9,305	115	·0124	10	9,305	115	·0124

Institute Experience—Males—(continued).

Year of Insurance	Number at Risk	Deaths	Rate of Mortality	Years of Insurance	Number at Risk	Deaths	Rate of Mortality
			Age at Entry 40				
0	8,695·5	30	·0035	0	8,695·5	30	·0035
1	16,144·5	115	·0071	1	16,144·5	115	·0071
2	14,522·5	132	·0091	2	14,522·5	132	·0091
3	13,172	141	·0107	3	13,172	141	·0107
4	12,051·5	155	·0129	4	12,051·5	155	·0129
5	11,063·5	129	·0117	5, 6	21,265·5	281	·0132
6	10,202	152	·0149				
7	9,329	128	·0137	7	9,329	128	·0137
8	8,532·5	120	·0141	8	8,532·5	120	·0141
9	7,804	131	·0168	9–12	27,068	465	·0172
10	7,097·5	122	·0172				
			Age at Entry 45				
0	5,974	34	·0057	0	5,974	34	·0057
1	11,101·5	103	·0093	1	11,101·5	103	·0093
2	9,978	119	·0119	2	9,978	119	·0119
3	9,062	129	·0142	3	9,062	129	·0142
4	8,264	120	·0145	4	8,264	120	·0145
5	7,582	115	·0152	5	7,582	115	·0152
6	6,989·5	100	·0143	6, 7	13,396	223	·0166
7	6,406·5	123	·0192				
8	5,827·5	117	·0201	8–10	16,020	319	·0199
9	5,342	104	·0195				
10	4,850·5	98	·0202				
			Age at Entry 50				
0	3,937	26	·0066	0	3,937	26	·0066
1	7,301	67	·0092	1	7,301	67	·0092
2	6,598	91	·0138	2	6,598	91	·0138
3	6,015	104	·0173	3	6,015	104	·0173
4	5,488	98	·0179	4	5,488	98	·0179
5	5,033	109	·0217	5	5,033	109	·0217
6	4,614·5	111	·0241	6–8	12,652	285	·0225
7	4,211	88	·0209				
8	3,826·5	86	·0225				
9	3,506·5	96	·0274	9	3,506·5	96	·0274
10	3,168·5	101	·0319	10, 11	6,028	173	·0287
			Age at Entry 55				
0	2,220·5	22	·0099	0	2,220·5	22	·0099
1	4,154·5	71	·0171	1	4,154·5	71	·0171
2	3,718	69	·0186	2	3,718	69	·0186
3	3,417·5	67	·0196	3	3,417·5	67	·0196
4	3,141	84	·0267	4	3,141	84	·0267
5	2,896	86	·0297	5	2,896	86	·0297
6	2,648·5	93	·0351	6, 7	5,025	172	·0342
7	2,376·5	79	·0332				
8	2,147·5	82	·0382	8	2,147·5	82	·0382
9	1,941·5	69	·0355	9, 10	3,686	153	·0415
10	1,744·5	84	·0482				

Institute Experience—Males—(continued).

Year of Insurance	Number at Risk	Deaths	Rate of Mortality	Years of Insurance	Number at Risk	Deaths	Rate of Mortality
			Age at Entry 60				
0	1,246	14	·0112	0	1,246	14	·0112
1	2,334·5	52	·0223	1	2,334·5	52	·0223
2	2,086	62	·0297	2	2,086	62	·0297
3	1,892·5	81	·0428				
4	1,703·5	64	·0376	3–5	5,142·5	192	·0373
5	1,546·5	47	·0304				
6	1,417	61	·0430	6	1,417	61	·0430
7	1,265·5	73	·0577	7	1,265·5	73	·0577
8	1,115·5	71	·0636	8	1,115·5	71	·0636
9	982	65	·0662	9	982	65	·0662
10	869·5	57	·0656	10, 11	1,626·5	110	·0676

New York Mutual Experience.

Year of Insurance	Number at Risk	Deaths	Rate of Mortality	Years of Insurance	Number at Risk	Deaths	Rate of Mortality
			Age at Entry 25				
0	8,840·25	34	·0038	0	8,840·25	34	·0038
1	15,124·5	91·5	·0060	1	15,124·5	91·5	·0060
2	12,547·5	80·5	·0064	2	12,547·5	80·5	·0064
3	10,854·25	75	·0069	3	10,854·25	75	·0069
4	9,353·5	76·5	·0082	4, 5	17,148·75	140·5	·0082
5	7,795·25	64	·0082				
6	6,097·5	46·5	·0076				
7	4,455·5	45	·0101	6–9	16,181·75	136·5	·0084
8	3,167·75	22·5	·0071				
9	2,461	22·5	·0091				
10	2,055·25	13·5	·0066	10–12	5,383·75	48	·0089
			Age at Entry 30				
0	10,792·25	34	·0032	0	10,792·25	34	·0032
1	18,864·5	102	·0054	1	18,864·5	102	·0054
2	16,194·25	101	·0062	2, 3	30,538·75	188	·0062
3	14,344·5	87	·0061				
4	12,681·5	99·5	·0078	4	12,681·5	99·5	·0078
5	10,776·25	99·5	·0092				
6	8,656	86·5	·0100	5–8	30,604·25	273·5	·0089
7	6,448	57·5	·0089				
8	4,724	30	·0064				
9	3,714·5	31·5	·0085				
10	3,071	23·5	·0077	9–11	9,415·5	84·5	·0090
			Age at Entry 35				
0	9,965·25	49·5	·0050	0	9,965·25	49·5	·0050
1	17,633·75	103·5	·0059	1	17,633·75	103·5	·0059
2	15,380	90·5	·0059	2, 3	29,071·5	212	·0073
3	13,691·5	121·5	·0089				
4	12,145·75	95·5	·0079	4	12,145·75	95·5	·0079
5	10,419	91·5	·0088				
6	8,450·75	78·5	·0093	5–8	29,540·25	254	·0086
7	6,268	47·5	·0076				
8	4,372·5	36·5	·0083				
9	3,403·5	31	·0091	9	3,403·5	31	·0091
10	2,845·5	29	·0102	10, 11	5,307	59	·0111

New York Mutual Experience—(continued).

Year of Insurance	Number at Risk	Deaths	Rate of Mortality	Years of Insurance	Number at Risk	Deaths	Rate of Mortality
			Age at Entry 40				
0	7,543·25	31·5	·0042	0	7,543·25	31·5	·0042
1	13,407	89	·0066	1, 2	25,225·75	169·5	·0067
2	11,818·75	80·5	·0068				
3	10,584·75	91	·0086	3	10,584·75	91	·0086
4	9,425·5	86	·0091	4	9,425·5	86	·0091
5	8,147·25	80·5	·0099	5	8,147·25	80·5	·0099
6	6,503	65·5	·0101	6	6,503	65·5	·0101
7	4,719·25	48·5	·0103	7	4,719·25	48·5	·0103
8	3,215·5	37	·0115	8	3,215·5	37	·0115
9	2,460·25	34·5	·0140	9, 10	4,457	59	·0132
10	1,996·75	24·5	·0123				
			Age at Entry 45				
0	4,771·75	37	·0078	0, 1	13,252·75	92	·0069
1	8,481	55	·0065				
2	7,503·25	74·5	·0099				
3	6,738	66	·0098	2–5	25,348·75	250·5	·0099
4	5,973	62	·0104				
5	5,134·5	48	·0093				
6	4,107·75	47·5	·0116	6	4,107·75	47·5	·0116
7	2,914·5	41	·0141	7	2,914·5	41	·0141
8	1,960·75	30	·0153	8	1,960·75	30	·0153
9	1,432·5	24	·0168	9	1,432.5	24	·0168
10	1,131·75	26·5	·0234	10–12	2,901·25	58	·0200
			Age at Entry 50				
0	2,770·5	25	·0090	0	2,770·5	25	·0090
1	4,898·25	50	·0102	1	4,898·25	50	·0102
2	4,326·75	48·5	·0112	2	4,326·75	48·5	·0112
3	3,873	48	·0124	3, 4	7,290	89	·0122
4	3,417	41	·0120				
5	2,905·25	42·5	·0146	5	2,905·25	42·5	·0146
6	2,240·75	39	·0174	6	2,240·75	39	·0174
7	1,572	28·5	·0181				
8	1,007	18	·0179	7–10	3,855	77·5	·0201
9	710	13	·0183				
10	566	18	·0318				

Thirty American Offices' Experience—Males.

Year of Insurance	Number at Risk	Deaths	Rate of Mortality	Years of Insurance	Number at Risk	Deaths	Rate of Mortality
			Age at Entry 25				
0	77,250·5	380·5	·0049	0	77,250·5	380·5	·0049
1	127,422·75	793·5	·0062	1	127,422·75	793·5	·0062
2	95,459·5	671·5	·0070	2	95,459·5	671·5	·0070
3	75,384	588	·0078				
4	60,970	518·5	·0085	3–6	222,600·5	1,804·5	·0081
5	48,439·5	397·5	·0082				
6	37,807	300·5	·0079				
7	28,385·25	236·5	·0083				
8	20,642·75	161·5	·0078	7–10	74,501·5	609	·0082
9	14,758	125·5	·0085				
10	10,715·5	85·5	·0080				
			Age at Entry 30				
0	96,139	459·5	·0048	0	96,139	459·5	·0048
1	162,650	988·5	·0061	1	162,650	988·5	·0061
2	126,959·5	872·5	·0069	2	126,959·5	872·5	·0069
3	103,364·25	775	·0075	3	103,364·25	775	·0075
4	85,920·5	695·5	·0081	4	85,920·5	695·5	·0081
5	70,132·75	590·5	·0084	5	70,132·75	590·5	·0084
6	56,551	525	·0093				
7	44,094·25	410·5	·0093				
8	33,242	272·5	·0082	6–10	175,787·5	1,573	·0089
9	24,355·75	211	·0087				
10	17,544·5	154	·0088				
			Age at Entry 35				
0	91,088·25	469	·0051	0	91,088·25	469	·0051
1	155,885·25	1,075·5	·0069	1	155,885·25	1,075·5	·0069
2	123,922	962	·0078	2	123,922	962	·0078
3	103,522	907·5	·0088	3	103,522	907·5	·0088
4	86,687	841·5	·0097	4	86,687	841·5	·0097
5	71,348·75	715·5	·0100				
6	57,800	569	·0098	5–7	174,314·5	1,739	·0100
7	45,165·75	454·5	·0101				
8	33,612	349·5	·0104				
9	24,357·75	234	·0096	8–10	75,346	786	·0104
10	17,376·25	202·5	·0117				
			Age at Entry 40				
0	71,675·5	436	·0061	0	71,675·5	436	·0061
1	123,503·75	1,021·5	·0083	1	123,503·75	1,021·5	·0083
2	99,406·5	929	·0093	2	99,406·5	929	·0093
3	83,321·5	835	·0100	3	83,321·5	835	·0100
4	69,925·75	761	·0109	4	69,925·75	761	·0109
5	57,665·75	665·5	·0115				
6	46,608·25	512	·0110	5–8	166,697·75	1,883·5	·0113
7	36,001·75	398	·0111				
8	26,122	308	·0117				
9	18,656·75	238	·0128	9, 10	31,779·75	395	·0124
10	13,123	157	·0120				

Thirty American Offices' Experience—Males—(continued).

Year of Insurance	Number at Risk	Deaths	Rate of Mortality	Years of Insurance	Number at Risk	Deaths	Rate of Mortality
Age at Entry 45							
0	48,470	351	·0072	0	48,470	351	·0072
1	83,910·5	841·5	·0100	1	83,910·5	841·5	·0100
2	68,018	772·5	·0114	2	68,018	772·5	·0114
3	57,035·5	665	·0117	3	57,035·5	665	·0117
4	47.783	623	·0130	4, 5	87,004·25	1,110·5	·0128
5	39,221·25	487·5	·0124				
6	31,516·75	454·5	·0144	6	31,516·75	454·5	·0144
7	24,015	364	·0152	7	24,015	364	·0152
8	17,493·75	280	·0160	8	17,493·75	280	·0160
9	12,273·25	213	·0174	9, 10	20,631·75	351	·0170
10	8,358·5	138	·0165				
Age at Entry 50							
0	28,679·25	292	·0102	0	28,679·25	292	·0102
1	49,659·25	643	·0130	1	49,659·25	643	·0130
2	40,373·5	590	·0146	2	40,373·5	590	·0146
3	33,728·5	534·5	·0158	3	33,728·5	534·5	·0158
4	28,119·5	458·5	·0163	4	28,119·5	458·5	·0163
5	22,787·5	413	·0181	5	22,787·5	413	·0181
6	17,857	337·5	·0189	6	17,857	337·5	·0189
7	13,331·25	293	·0220	7	13,331·25	293	·0220
8	9,326·5	212·5	·0228	8, 9	15,615·75	355	·0227
9	6,289·25	142·5	·0227				
10	4,324·25	122	·0282	10–12	9,730·5	268·5	·0276
Age at Entry 55							
0	13,646·25	189	·0139	0	13,616·25	189	·0139
1	23,467·5	375	·0160	1	23,467·5	375	·0160
2	19,103	359·5	·0188	2	19,103	359·5	·0188
3	15,921·25	346	·0217	3	15,921·25	346	·0217
4	13,257·75	333·5	·0252	4–6	32,375	830	·0256
5	10,788·25	287	·0266				
6	8,329	209·5	·0252				
7	6,139·75	183	·0298	7, 8	10,427·25	299·5	·0287
8	4,287·5	116·5	·0272				
9	2,852·5	87	·0305	9–12	7,039·75	229·5	·0326
10	1,869·5	52	·0278				
Age at Entry 60							
0	5,226	104·5	·0200	0	5,226	104·5	·0200
1	9,070·75	230·5	·0254	1	9,070·75	230·5	·0254
2	7,452·75	200	·0268	2	7,452·75	200	·0268
3	6,290	191·5	·0304	3	6,290	191·5	·0304
4	5,208·5	169·5	·0325	4	5,208·5	169·5	·0325
5	4,141·25	139·5	·0337	5	4,141·25	139·5	·0337
6	3,153·25	109	·0346	6	3,153·25	109	·0346
7	2,244·25	109	·0486	7, 8	3,716	166·5	·0448
8	1,471·75	57·5	·0391				
9	929·5	44	·0473	9	929·5	44	·0473
10	623·75	31·5	·0505	10	623·75	31·5	·0505

It has been necessary in some cases to go beyond the tenth year of insurance when grouping, in order to get a fairer indication of the rate of mortality.

The rates for these experiences after grouping are also represented in Diagram No. 1. On examining them along with those for the other two experiences, one cannot help noticing the rapid rise in the rate of mortality among recently-selected lives in the Equitable and the Scottish Amicable compared with that in the New York Mutual, the Thirty American, and the Gotha, while the Institute rate comes between the two. The rate for year "0" in the Thirty American Offices is much steadier than in the New York Mutual, which confirms the remark I previously made, that if there is a group of offices there is not the same chance of error in estimating the exposed to risk for that year.

Average Age at Entry. To compare these mortality experiences, the numbers entering at the various ages might have been raised to a common radix, and the central age representing a quinary group would then have become the real age. This, however, would have entailed a great deal of labour; and is open to the objection that the same weight is assigned to the entrants at each age whatever may be their relative numbers. Now, the ages at which insurances are effected follow a law as sure as that of the mortality, the numbers entering attaining a maximum and then diminishing; and it occurred to me that advantage might be taken of this. The average age at entry will therefore be greater than that of the central age of the group at the earlier part of the table, and afterwards less. The following table shows the average age at entry of the various groups in the mortality experiences. In the case of the Scottish Amicable experience the entrants at individual ages are not given, and the numbers at risk have therefore been used to calculate the average age at entry; but this will not, I think, materially affect the results.

Table 3.

Central Age of Group	Average Age of Group					
	Scottish Amicable	Gotha	Equitable	Institute	New York Mutual	Thirty American
25	25·29	25·57	25·16	25·19	25·21	25·20
30	29·98	30·16	30·03	29·98	30·00	30·00
35	34·95	34·95	35·00	34·90	34·90	34·92
40	39·88	39·84	39·89	39·84	39·83	39·84
45	44·73	44·80	44·89	44·83	44·77	44·80
50	49·73	49·79	49·81	49·80	49·70	49·70
55	54·72	54·75	54·83	54·77	54·63	54·62
60	59·52	59·32	59·73	59·66	59·41	59·41

It will be seen that the average ages at entry differ very little from each other, taking into account the circumstance that the experiences relate to various countries. Excluding the first and last groups, where the number of entrants is limited, the greatest difference between the average age in any two of the experiences is ·21 of a year, while in many cases the average age is identical, or nearly so. The agreement between the New York Mutual and Thirty American Offices is very close: in three cases the average age is the same, while in other three the difference is only ·01 of a year, or less than four days. Having regard to all the circumstances, I think we may compare the experiences as they stand, and that the groups may properly, as well as conveniently, be represented by the central ages, and I shall, therefore, adopt this course in what follows. The deviations from the real ages might have affected the results to some extent had it been necessary to deal with very old ages, but the oldest age to which we shall afterwards find we can carry our investigation is 69.

As already mentioned, the numbers at risk and the deaths in the New York Mutual and the Thirty American Offices, had to be grouped in a different manner from that of the other experiences, on account of the age at entry being, according to the American custom, nearest birthday. As both these experiences were arranged according to calendar years, age at entry 30, for example, was in this way reduced to $29\frac{1}{2}$; and the quinary groups would have been $22\frac{1}{2}$ to $26\frac{1}{2}$, $27\frac{1}{2}$–$31\frac{1}{2}$, ... instead of 23–27, 28–32, ... This arrangement, however, would have made the average age at entry too small; and accordingly one-half of those entering at the first age in the group was thrown off, and one-half of those entering at the first age in the next group added on. For instance, to get the number at risk and the deaths in the first year of insurance in the New York Mutual for the group which is represented by average age at entry 25, we proceed as follows:

Age $22\frac{1}{2}$,	one-half of number at risk	620·25,	one-half of deaths	1
„ $23\frac{1}{2}$	number at risk	1,470	deaths	5
„ $24\frac{1}{2}$	„ „	1,856	„	8
„ $25\frac{1}{2}$	„ „	1,847	„	7
„ $26\frac{1}{2}$	„ „	1,990	„	7
„ $27\frac{1}{2}$,	one-half of number at risk	1,057	one-half of deaths	6
		8,840·25		34

Taking the ages at entry nearest birthday will probably have an effect similar to taking them next birthday; that is to say, that just

as those entering next birthday are found to be on the average more than midway between last and next birthdays, so those entering at nearest birthday will be found to be on the average somewhat over that age, but probably not to the same extent.

VII.—RATES OF MORTALITY IN VARIOUS COUNTRIES.

Here the question presented itself for consideration whether these mortality experiences, which relate to England, Scotland, America, and Germany, could properly be compared with each other; or whether we might not err in attributing to other causes, such as a difference in the class of policy or the rate of discontinuance, what was really due to a difference in the mortality of the general population in the respective countries, either at particular periods or throughout life. I therefore obtained the mortality tables applicable to the general population in each country, and will give a very brief description of them.

England. A life table, based upon the mortality in 1871–80, has been given by Dr. Ogle in the supplement to the *Forty-fifth Annual Report of the Registrar-General*, by means of which I have calculated the rate of mortality for males.

Scotland. The latest life table was formed by Dr. Robertson from the Census of 1871 and the registered deaths for that year. The probability of living a year is given, from which the rate of mortality has been deduced.

America. The life table for this country was formed by Mr. Levi Meech, and was based on four enumerations of the general Census of the United States from 1830 to 1860.

Germany. This life table is based upon the mortality of the population of the Empire in the 10 years 1871–2 to 1880–1.

The following table gives the rate of mortality for males at each age after age 20 in the respective countries:

TABLE 4.

Age	England	Scotland	America	Germany	Age	England	Scotland	America	Germany
20	·00627	·00851	·00915	·00750	62	·04003	·03931	·03137	·04409
21	·00655	·00877	·00939	·00805	63	·04270	·04225	·03355	·04748
22	·00684	·00903	·00945	·00853	64	·04564	·04549	·03583	·05118
23	·00713	·00930	·00949	·00852	65	·04886	·04877	·03840	·05520
24	·00743	·00957	·00957	·00847	66	·05240	·05239	·04120	·05956
25	·00773	·00986	·00963	·00848	67	·05625	·05627	·04418	·06429
26	·00804	·01016	·00971	·00855	68	·06044	·06042	·04753	·06942
27	·00836	·01046	·00979	·00868	69	·06498	·06488	·05116	·07500
28	·00869	·01077	·00988	·00885	70	·06988	·06965	·05509	·08108
29	·00903	·01108	·00996	·00905	71	·07538	·07477	·05938	·08770
30	·00939	·01141	·01008	·00928	72	·08128	·08024	·06414	·09489
31	·00972	·01176	·01020	·00954	73	·08763	·08609	·06919	·10267
32	·01008	·01211	·01032	·00984	74	·09442	·09140	·07484	·11105
33	·01046	·01246	·01047	·01019	75	·10163	·09904	·08084	·12004
34	·01085	·01284	·01061	·01058	76	·10936	·10619	·08748	·12965
35	·01128	·01321	·01080	·01101	77	·11749	·11382	·09469	·13989
36	·01172	·01361	·01097	·01148	78	·12613	·12197	·10251	·15077
37	·01222	·01401	·01118	·01199	79	·13522	·13066	·11094	·16230
38	·01274	·01443	·01140	·01253	80	·14480	·13992	·12019	·17448
39	·01328	·01486	·01164	·01308	81	·15587	·14979	·13017	·18731
40	·01389	·01530	·01189	·01363	82	·16749	·16029	·14094	·20074
41	·01435	·01576	·01221	·01418	83	·17976	·17146	·15256	·21467
42	·01485	·01622	·01251	·01475	84	·19267	·18251	·16516	·22900
43	·01540	·01670	·01286	·01537	85	·20624	·19594	·17886	·24363
44	·01598	·01719	·01325	·01605	86	·22025	·20931	·19354	·25846
45	·01660	·01770	·01368	·01680	87	·23502	·22349	·20900	·27344
46	·01726	·01823	·01412	·01761	88	·25041	·23849	·22616	·28852
47	·01798	·01876	·01463	·01848	89	·26643	·25436	·24451	·30370
48	·01872	·01932	·01518	·01941	90	·28285	·27112	·26359	·31902
49	·01953	·01989	·01579	·02040	91	·29976	·28880	·28412	·33457
50	·02039	·02048	·01643	·02145	92	·31702	·30743	·30576	·35047
51	·02190	·02113	·01717	·02256	93	·33503	·32703	·32988	·36689
52	·02300	·02171	·01796	·02374	94	·35284	·34760	·35309	·38404
53	·02415	·02236	·01883	·02501	95	·37278	·36924	·38247	·40217
54	·02536	·02301	·01977	·02639	96	·39084	·39186	·40645	·42158
55	·02667	·02370	·02080	·02790	97	·41150	·41551	·43478	·44259
56	·02809	·02548	·02196	·02956	98	·43233	...	·46154	·46560
57	·02965	·02739	·02317	·03139	99	·45695	...	·50000	·49102
58	·03139	·02945	·02455	·03342	100	1·00000	...	·50000	·51930
59	·03332	·03166	·02605	·03568	101	...	...	·57143	...
60	·03545	·03403	·02767	·03820	102	...	...	·66666	...
61	·03764	·03658	·02944	·04100					

To enable the reader more readily to compare these rates, each series is represented in the appended Diagram No. 2 by a curve in which the age is the abscissa, and the probability of dying in a year is the ordinate. It would be out of place here to analyze the rates minutely, but several of the more salient features may be noted. It will be observed that the rates of mortality at ages 33 and 34 in three of the countries, England, America, and Germany, are nearly the same, and that the rate of mortality in the first and last of these three countries is very similar up to

nearly age 50, while in America it is very much lower. The rate of mortality in Scotland, again, is higher than that in England up to the same age, and afterwards becomes less; but after age 60 they agree very well. We must not, however, lay too much stress upon these differences. A great deal depends upon the estimate which is made of the personal errors in the returns, and upon the manner in which the facts are adjusted. Had the same method of graduation, for instance, been used for Scotland as for England, and the basis been as broad, I think they would have agreed more closely than they do at the outset. I may mention, however, that in the introduction to the Institute Experience it is stated that the experience of one large office showed a considerable difference in the mortality of England and Scotland. Perhaps that office issued policies on the lives of special classes, such as those engaged in the liquor trade, which might account to some extent for the difference.

VIII.—Rate of Mortality among recently-selected Lives.

We have only two experiences arranged according to policy years by means of which we can determine accurately the rate of mortality in the first year of insurance, namely, the Scottish Amicable and the Gotha; but they so entirely corroborate each other, and the numbers at risk in the latter are so large, that I have no hesitation in employing them for that purpose. The unadjusted rates of mortality for the first year of insurance for quinquennial ages at entry are given in the following table, along with those of the Equitable, Institute, New York Mutual, and Thirty American Offices, for comparison. In one or two cases the mean of the first and second years has been taken, where the rate for the first happened to be greater than that for the second.

Table 5.—*First Year of Insurance—Unadjusted Rate of Mortality.*

Age	Scottish Amicable	Gotha	Equitable	Institute	New York Mutual	Thirty American
25	·0033	·0031	·0030	·0024	·0038	·0049
30	·0039	·0040	·0026	·0045	·0032	·0048
35	·0051	·0057	·0030	·0046	·0050	·0051
40	·0068	·0066	·0039	·0035	·0042	·0061
45	·0066	·0067	·0055	·0057	·0069	·0072
50	·0092	·0096	·0116	·0066	·0090	·0102
55	...	·0128	...	·0099	...	·0139
60	...	·0225	...	·0112	...	·0200

The agreement between the Scottish Amicable and the Gotha is very close; but perhaps not closer than might have been expected, when we remember that these experiences are arranged according to policy years, that they both relate to healthy lives, and that the rate of mortality in the two countries is very similar, most of the insurances in Scotch offices being drawn from England. On the other hand, in the experiences arranged according to calendar years, the rates are very irregular; and any conclusions based upon them as to the rate of mortality in the first year would be untrustworthy. The rate of mortality in the Equitable is very low compared with the others; but if my view (see page 115) is correct, then it should be doubled. We shall find afterwards that the rate of mortality for the next year of insurance is very much higher in the Equitable than in the Institute or the New York Mutual. On the other hand, the rate of mortality in the Thirty American Offices at ages 25 and 30 is high, and probably the normal ages in the case of rated-up lives have been used; or if they have not, the addition has not sufficed to cover the risk. However this may be, I think it will be better not to base any conclusions on the results deduced from this experience until an authoritative statement is made as to whether the rated-up lives or those exposed to extra risk have been included, and if so, how they have been treated. I thought that all the lives would have been healthy, and that those exposed to extra risk would have been excluded; but I found that the Connecticut Mutual was one of the thirty offices that contributed to the experience, and I am doubtful whether this has been done.

I have taken the Gotha figures almost as they stand above as the rate of mortality for the first year of insurance, and interpolated for the other ages by means of the formula stated and explained by Mr. Sprague (*J.I.A.*, xxii, 270); and the values thus obtained are given in the following table:

Table 6.—*First Year of Insurance—Adjusted Rate of Mortality.*

Age	Rate of Mortality	Age	Rate of Mortality	Age	Rate of Mortality
25	·0031	37	·0062	49	·0089
26	·0031	38	·0064	50	·0095
27	·0032	39	·0065	51	·0101
28	·0034	40	·0066	52	·0107
29	·0037	41	·0066	53	·0113
30	·0040	42	·0066	54	·0120
31	·0044	43	·0066	55	·0128
32	·0047	44	·0067	56	·0137
33	·0051	45	·0069	57	·0149
34	·0054	46	·0072	58	·0162
35	·0057	47	·0077	59	·0179
36	·0060	48	·0083	60	·0200

It will be observed that the rate of mortality between ages 40 and 45 is almost stationary—a feature I have retained, as it appears in both the experiences. It has been mentioned that Mr. Spens in 1850 read a paper to the Institute in which he stated his belief that there was no materially greater risk in the assurance for a year of a select life of from 40 to 45 than of a select life of from 20 to 25. His own observations, however, published in 1861, do not bear out this theory, as the unadjusted rate of mortality given above will show, although in the adjusted table which he gave (*J.I.A.*, x, 78), deduced from the same facts, he made the rate constant between ages 22 and 42, namely ·0045. He seems, however, to have been misled by the light mortality at the higher ages, the unadjusted rate for the groups whose central ages are 55 and 60, being ·0065 and ·0118 respectively. But the numbers at risk in these groups are 306·5 and 169 only, and I had rejected them as being too small—a course which is justified by the higher rates in the larger experience of the Gotha office. Possibly, however, the rate I have adopted for age 60, ·0200, may be considered a little too high; but I did not feel justified in making it lower, as I have gone on the principle of allowing the facts to speak for themselves. I have not given the rates for ages below 25 or above 60 on account of the paucity of numbers in the observations.

We may now compare the rates with those obtained by Mr. Sprague from the Institute statistics in the manner described by him *J.I.A.*, xxi, 229, and referred to by me on page 102.

TABLE 7.

First Year of Insurance—Adjusted Rate of Mortality.

Age at Entry	Sprague's Select	My Select
25	·0045	·0031
30	·0045	·0040
35	·0049	·0057
40	·0055	·0066
45	·0064	·0069
50	·0079	·0095
55	·0104	·0128
60	·0144	·0200

It will be seen that, even with the great labour expended on making calendar years coincide with policy years, the results can hardly be considered satisfactory, being too high at the younger ages and too low at the older ages. I do not think, therefore, that it is practicable to get accurate results from experiences arranged according to calendar years. Nor is the point immaterial. In the discussion which followed the reading of Mr. Sprague's paper mentioned above, Mr. King stated that the differences in the rate of mortality for the first year would have no appreciable effect on annuity-values. "I find that if we "double the q_x of the first year of my tables, we reduce the "value of an annuity of £1 by about 1*s*. 9*d*. only, and by "about 10*d*. if we add 50 per-cent to q_x. But, further, I do "not think our efforts to ascertain the mortality in the first "insurance year will be of much avail, because I doubt if it be a "determinate quantity. After lives have been insured some years, "I think the rates of mortality among them will be found to be "nearly identical in all sound offices; but not so the mortality "of the first year. That will depend on the management of the "particular office, and will vary widely. In days gone by, when "medical examination was less strict, the mortality of the first "year must have been heavier than now." Mr. Sprague, however, pointed out in reply, that although the rate of mortality in the first insurance year has but a small effect on the annuity-value, it will have a much more important effect on the policy-value at the end of the first year. "Mr. King states that, when the first "year's q_x is doubled, the value of an annuity of £1 is reduced "by about 1*s*. 9*d*.—a sum equal to about ·6 per-cent of the "annuity (age at entry 45, 3 per-cent interest). But I find that "the corresponding change in the value of the policy at the end

"of the first insurance year, is no less than 20 per-cent of its "value. We therefore must not conclude that, because a change "in the mortality of the first year produces only a small effect on "the annuity, it is therefore of little consequence."

I think that I have accurately determined the mortality in the first insurance year, and that future investigations, if conducted on right principles, will confirm the results. I think also I have shown that the mortality even in the first insurance year will, in well-managed offices transacting an ordinary business, be the same.

It will be interesting now to compare the rates I have obtained for the mortality among recently-selected lives with those of the general population, and this is done in the following table:

TABLE 8.

Age	Rate of Mortality: My Select First Insurance Year	Rate of Mortality: Population of Germany	Percentage of (2) to (3)
(1)	(2)	(3)	(4)
25	·0031	·0085	36·5
30	·0040	·0093	43·0
35	·0057	·0110	51·8
40	·0066	·0136	48·5
45	·0069	·0168	41·1
50	·0095	·0215	44·2
55	·0128	·0279	45·9
60	·0200	·0382	52·4

The percentages in the last column would not have been altered much if the English table had been used instead of the German, except at the youngest and oldest ages; but they would have run more regularly if my rate of mortality between ages 40 and 45 had not been kept stationary. In the experience of the New York Mutual, a table is given showing the exposures, deaths, and rates of mortality in the first year of insurance; and it is stated that "if, now, the mortality rates for the whole American popula- "tion were known, the absolute effect of selection could easily be "ascertained. But these rates are not known, and resort was had "to Farr's English Table No. 3, males, for the population of "England and Wales. This gives a low rate of mortality for that "population; but whether the rate for the United States is below "or above this, there are no means to determine. Comparing the

"company's rate for the first year of insurance with this standard, "it is found that under the age of 40, the former is only 40 per-"cent of Dr. Farr's; from ages 40 to 60 this percentage advances "slowly, and at the latter age reaches 50. A similar table was "constructed for the experience of the Twenty English Offices, "and showed, when compared with the same table, nearly the "same result for the younger ages, but a lower percentage than "40 for the higher. As the observations were few at the older "ages, it would seem probable that the rate at all ages of life will "ultimately be brought down to 40 per-cent of the English table "of Dr. Farr, and that this may be considered the measure of "careful selection in the first year of insurance." It will be seen, however, from my table, that with a correct rate of mortality for the first year of insurance and a table of mortality applicable to the population from which the insured are selected, only one of the percentages is below 40, and that the others range from 41 to about 52 per-cent.

IX.—Rate at which the Mortality among recently-selected Lives increases during the 10 years after Insurance.

The gradual wearing out of the effect of selection has been exhaustively treated by Mr. Sprague in his paper, "On the Rate of Mortality among Assured Lives" (*J.I.A.*, xv, 328), and I cannot do better than reproduce his remarks here:—"If we "consider how the medical selection of the lives proposed for "assurance operates, we shall see that there is nothing impossible "in the supposition that its effect may endure and be traceable, "for any number of years up to 20, 30, or 40, or even to the "extreme limit of life. Among the population at large, or among "a very large number of persons chosen by lot, there will be "persons in every possible state of health. Some will be on their "death-beds, suffering under diseases which will certainly kill "them within a few hours, or it may be days, or weeks. Some "will be in the last stage of lingering illnesses, such as consump-"tion, of which they will certainly die within the course of two "or three years. Others, again, will be suffering under the effects "of acute diseases, such as fevers; or from the effect of accidents; "from which they may either entirely recover within a few weeks

"or months, or recover only with impaired constitutions and "diminished prospects of life, or from which, on the contrary, "they may die. There will, lastly, be some who are suffering "from chronic diseases, such as heart disease; which will certainly "on the average shorten their lives, but will not prevent indi-"viduals among them from attaining to extreme old age. In "fact, it is easy to conceive that a body of such lives may "throughout be subject to a much heavier mortality than that "prevailing among the population at large; and yet, that if "sufficiently numerous, they may not all die before the extreme "age in the Table of Mortality is reached. The original medical "selection will, more or less, completely weed out from the "persons proposed for insurance those who belong to any of the "above classes. In this way it is undeniable that the assurance "office will escape many premature deaths, by the weeding out of "persons suffering from acute diseases. And it seems very "probable that the effect of the medical selection, through weeding "out the persons labouring under chronic diseases, will have the "effect of slightly reducing the rate of mortality even to the "extremity of life. Whether its effect, supposing it to exist to "the extremity of life, would be of sufficient magnitude to be "traceable; or for how many years it would be traceable; are "questions with regard to which *a priori* reasoning can give no "assistance. We must deduce the answers from careful examina-"tion of observed facts; and the more numerous the facts we can "obtain, the more satisfactorily shall we be able to answer these "questions.

"Let us now trace the progress of a large body of assured "lives, on the supposition that none of the members are with-"drawn from observation otherwise than by death. We shall "then see that there is a constant tendency for the constitution "of the body to assimilate itself to that of a body selected by lot "from the class of persons to which the members belong. A very "short time will suffice to introduce into the body acute diseases "and accidents, which run their course rapidly towards death or "recovery. In other words, assured lives may die of acute "diseases, such as fevers, or from accident, within a short time "from the date of the assurance; and the experience of every "assurance company could probably furnish examples of this kind. "A few years will suffice to develop in some of the persons who "were passed as first-class lives, consumption and other diseases,

"of which they will die within two or three years. As time "progresses, a larger proportion of the surviving members will "be suffering from chronic diseases, until ultimately, after the "lapse of a sufficient number of years, the constitution of the "body, supposed to be still sufficiently large, would be precisely "the same as that of a body of the same number of members "selected by lot from the class of society to which the assured "lives belong.

"Much light would be thrown upon this subject by a properly-"conducted examination of the causes of death among insured "lives; if it could be ascertained what diseases or what classes of "diseases are most fatal in the early years of the policy, and in "each successive interval of, say, five or ten years; if, in other "words, the mortality from each class of diseases were compared "with the standing of the policies. Several elaborate nosological "tables have been published by different companies; and in some, "an attempt has been made to compare the deaths from each "disease with the numbers living at various ages; but the enquiry "now suggested appears to me to be likely to be attended with "far more useful results.

"To use Mr. Higham's expressive phrase, the above described "body will ultimately consist of 'mixed' lives, some being in full "vigour and health, others in fair health, some in feeble, and "some in very bad health. The average vitality of the members "being now less than that of a number of recently-selected "persons of the same ages, they are sometimes inaccurately "spoken of as 'deteriorated' lives; but it cannot be too strongly "stated, or too carefully borne in mind, that the deterioration is "only *on the average*. Many of the lives may be in even better "health than formerly; and the great proportion of the whole "would probably be still regarded as select lives, insurable at "the ordinary rate."

The numbers at risk in the Scottish Amicable experience after the first few years are very small, and we have therefore only the Gotha experience left with which to determine the rate at which the mortality increases during the ten years after insurance. Some actuaries object to the mortality experience of an individual office on account of its very limited usefulness; but I think that very little objection can be taken to the Gotha experience for determining the rate of mortality within so short a period, especially when at the outset it agrees almost exactly with the

experience of a well-managed British office. The numbers at risk in the Gotha experience, moreover, are so large, and the rates run so regularly, that I have great confidence in it. It is also confirmed to a greater or less extent by other experiences to which considerable weight should be attached after the first year, namely, the New York Mutual and the Institute. I imagine, however, that the rates deduced from it would only apply to carefully-selected lives. I have therefore graduated the facts in the manner already described, and the rates obtained and their differences are given in the following table, along with the expected and actual deaths and their difference:

TABLE 9.—*Gotha Experience—Males.*

Year of Insurance (1)	Adjusted Rate of Mortality (2)	$\Delta \times 10^4$ (3)	Expected Deaths (4)	Actual Deaths (5)	Difference (4)−(5) (6)	Accumulated Difference $\Sigma(6)$ (7)
			Age at Entry 25			
1	·0031	23	20·1	20	+ ·1	+ ·1
2	·0054	12	30·9	31	− ·1	·0
3	·0066	5	33·6	34	− ·4	− ·4
4	·0071	3	32·5	39	− 6·5	− 6·9
5	·0074	2	30·3	25	+ 5·3	− 1·6
6	·0076	2	28·3	25	+ 3·3	+ 1·7
7	·0078	2	26·6	28	− 1·4	+ ·3
8	·0080	2	25·1	35	− 9·9	− 9·6
9	·0082	2	23·7	17	+ 6·7	− 2·9
10	·0084		22·0	18	+ 4·0	+ 1·1
			273·1	272		
			Age at Entry 30			
1	·0040	18	65·3	65	+ ·3	+ ·3
2	·0058	9	86·6	91	− 4·4	− 4·1
3	·0067	6	91·7	91	+ ·7	− 3·4
4	·0073	5	92·3	92	+ ·3	− 3·1
5	·0078	4	90·9	91	− ·1	− 3·2
6	·0082	4	87·8	83	+ 4·8	+ 1·6
7	·0086	4	84·5	90	− 5·5	− 3·9
8	·0090	3	81·9	71	+ 10·9	+ 7·0
9	·0093	3	79·6	89	− 9·4	− 2·4
10	·0096		75·4	72	+ 3·4	+ 1·0
			836·0	835		

TABLE 9.—*Gotha Experience—Males*—(continued).

Year of Insurance (1)	Adjusted Rate of Mortality (2)	$\Delta \times 10^4$ (3)	Expected Deaths (4)	Actual Deaths (5)	Difference (4)—(5) (6)	Accumulated Difference $\Sigma(6)$ (7)
			Age at Entry 35			
1	·0057	9	102·3	102	+ ·3	+ ·3
2	·0066	6	109·9	112	− 2·1	− 1·8
3	·0072	5	111·5	103	+ 8·5	+ 6·7
4	·0077	4	111·5	111	+ ·5	+ 7·2
5	·0081	5	109·5	104	+ 5·5	+12·7
6	·0086	5	108·6	109	− ·4	+12·3
7	·0091	6	106·6	116	− 9·4	+ 2·9
8	·0097	6	106·0	91	+15·0	+17·9
9	·0103	7	106·0	120	−14·0	+ 3·9
10	·0110		104·5	106	− 1·5	+ 2·4
			1,076·4	1,074		
			Age at Entry 40			
1	·0066	11	95·4	96	− ·6	− ·6
2	·0077	10	104·4	91	+13·4	+12·8
3	·0087	9	110·8	111	− ·2	+12·6
4	·0096	8	115·4	119	− 3·6	+ 9·0
5	·0104	7	117·5	137	−19·5	−10·5
6	·0111	8	117·5	122	− 4·5	−15·0
7	·0119	9	117·4	118	− ·6	−15·6
8	·0128	10	118·5	113	+ 5·5	−10·1
9	·0138	10	120·7	118	+ 2·7	− 7·4
10	·0148		119·6	113	+ 6·6	− ·8
			1,137·2	1,138		
			Age at Entry 45			
1	·0069	25	61·9	60	+ 1·9	+ 1·9
2	·0094	12	78·9	66	+12·9	+14·8
3	·0106	11	83·8	99	−15·2	− ·4
4	·0117	12	87·2	88	− ·8	− 1·2
5	·0129	13	90·7	77	+13·7	+12·5
6	·0142	15	94·5	100	− 5·5	+ 7·0
7	·0157	16	97·7	113	−15·3	− 8·3
8	·0173	17	100·5	101	− ·5	− 8·8
9	·0190	18	104·0	108	− 4·0	−12·8
10	·0208		104·8	91	+13·8	+ 1·0
			904·0	903		

TABLE 9.—*Gotha Experience—Males*—(continued).

Year of Insurance (1)	Adjusted Rate of Mortality (2)	$\Delta \times 10^4$ (3)	Expected Deaths (4)	Actual Deaths (5)	Difference (4)−(5) (6)	Accumulated Difference $\Sigma(6)$ (7)
			Age at Entry 50			
1	·0095	50	56·5	57	− ·5	− ·5
2	·0145	20	81·3	84	− 2·7	− 3·2
3	·0165	15	87·0	92	− 5·0	− 8·2
4	·0180	15	89·6	87	+ 2·6	− 5·6
5	·0195	17	91·3	86	+ 5·3	− ·3
6	·0212	19	93·5	91	+ 2·5	+ 2·2
7	·0231	23	95·0	87	+ 8·0	+ 10·2
8	·0254	26	96·9	102	− 5·1	+ 5·1
9	·0280	29	99·8	104	− 4·2	+ ·9
10	·0309		100·7	100	+ ·7	+ 1·6
			891·6	890		
			Age at Entry 55			
1	·0128	75	39·9	40	− ·1	− ·1
2	·0203	30	59·3	65	− 5·7	− 5·8
3	·0233	25	63·9	63	+ ·9	− 4·9
4	·0258	20	66·8	71	− 4·2	− 9·1
5	·0278	22	67·8	65	+ 2·8	− 6·3
6	·0300	26	68·3	61	+ 7·3	+ 1·0
7	·0326	31	68·5	67	+ 1·5	+ 2·5
8	·0357	37	69·9	65	+ 4·9	+ 7·4
9	·0394	45	71·6	84	−12·4	− 5·0
10	·0439		72·1	69	+ 3·1	− 1·9
			648·1	650		
			Age at Entry 60			
1	·0200	80	26·8	32	− 5·2	− 5·2
2	·0280	30	34·8	26	+ 8·8	+ 3·6
3	·0310	25	36·3	40	− 3·7	− ·1
4	·0335	28	36·7	31	+ 5·7	+ 5·6
5	·0363	33	37·4	35	+ 2·4	+ 8·0
6	·0396	38	37·7	54	−16·3	− 8·3
7	·0434	49	37·6	39	− 1·4	− 9·7
8	·0483	63	38·8	35	+ 3·8	− 5·9
9	·0546	78	39·9	35	+ 4·9	− 1·0
10	·0624		41·5	41	+ ·5	− ·5
			367·5	368		

I have not attempted to make the totals of the expected deaths agree with the totals of the actual, as it seemed undesirable to do this at the end of an arbitrary period; but it will be seen that they do not differ to any appreciable extent. Great care, however, has been taken that the rates in the last few years for each age at entry should be accurate. For this purpose the rates were taken out for a period of at least five years longer; and, in addition, the observations up to the sixth year of insurance, the time when selection seemed to be nearly exhausted, were excluded, and the rate of mortality for each age, irrespective of the length of time for which the lives had been assured, was calculated from the remainder to indicate the ultimate rate (see page 156). This rate, after being grouped in one or two cases to avoid accidental irregularities, is shown by the line of crosses on the Gotha diagram, where the adjusted rates for the first 10 years are represented by the lines of dots, and the rate of mortality in the general population by the broken line. Having obtained the adjusted rates for the first 10 years after insurance, I calculated the ratios for the various ages at entry that the rate of mortality in the second year of insurance bears to that in the first, the third to the second, and so on. This is a severe test, but I think the regularity of the ratios in the following table show that my results stand it very well:

TABLE 10.

Age at Entry x	$\frac{q_{[x]+1}}{q_{[x]}}$	$\frac{q_{[x]+2}}{q_{[x]+1}}$	$\frac{q_{[x]+3}}{q_{[x]+2}}$	$\frac{q_{[x]+4}}{q_{[x]+3}}$	$\frac{q_{[x]+5}}{q_{[x]+4}}$	$\frac{q_{[x]+6}}{q_{[x]+5}}$	$\frac{q_{[x]+7}}{q_{[x]+6}}$	$\frac{q_{[x]+8}}{q_{[x]+7}}$	$\frac{q_{[x]+9}}{q_{[x]+8}}$
25	1·74	1·22	1·08	1·04	1·03	1·03	1·03	1·03	1·02
30	1·45	1·16	1·09	1·07	1·05	1·05	1·05	1·03	1·03
35	1·16	1·09	1·07	1·05	1·06	1·06	1·07	1·06	1·07
40	1·17	1·13	1·10	1·08	1·07	1·07	1·08	1·08	1·07
45	1·36	1·13	1·10	1·10	1·10	1·10	1·10	1·10	1·10
50	1·53	1·14	1·09	1·08	1·09	1·09	1·10	1·10	1·10
55	1·59	1·15	1·11	1·08	1·08	1·09	1·10	1·10	1·11
60	1·40	1·11	1·08	1·08	1·09	1·10	1·11	1·13	1·14

The next step was to interpolate so as to get the adjusted rates of mortality during the 10 years after insurance for each age at entry, and this was done by means of the formula used for the first year of insurance. The following are the rates thus obtained with some slight adjustments:

TABLE 11.—*First* 10 *Years after Insurance—Adjusted Rates of Mortality.*

Age	Year of Insurance									
	1	2	3	4	5	6	7	8	9	10
25	·0031	...	...	...	...	...	...	...	...	...
26	·0031	·0054	...	...	...	...	...	...	...	...
27	·0032	·0054	·0066	...	...	...	...	...	...	...
28	·0034	·0055	·0067	·0071	...	...	...	...	...	...
29	·0037	·0056	·0067	·0073	·0074	...	...	...	...	...
30	·0040	·0057	·0067	·0073	·0076	·0076	...	...	...	...
31	·0044	·0058	·0067	·0073	·0078	·0078	·0078	...	...	...
32	·0047	·0059	·0067	·0073	·0078	·0080	·0080	·0080	...	...
33	·0051	·0061	·0067	·0073	·0078	·0082	·0082	·0082	·0082	...
34	·0054	·0062	·0068	·0073	·0078	·0082	·0084	·0084	·0084	·0084
35	·0057	·0064	·0969	·0073	·0078	·0082	·0086	·0088	·0088	·0088
36	·0060	·0066	·0070	·0074	·0078	·0082	·0086	·0090	·0091	·0091
37	·0062	·0068	·0072	·0075	·0078	·0082	·0086	·0090	·0092	·0093
38	·0064	·0070	·0074	·0077	·0079	·0082	·0086	·0090	·0093	·0095
39	·0065	·0072	·0077	·0080	·0081	·0084	·0087	·0091	·0094	·0096
40	·0066	·0075	·0081	·0084	·0085	·0086	·0089	·0092	·0095	·0097
41	·0066	·0077	·0084	·0088	·0089	·0090	·0091	·0094	·0097	·0099
42	·0066	·0079	·0087	·0092	·0094	·0095	·0096	·0097	·0099	·0101
43	·0066	·0081	·0090	·0096	·0099	·0100	·0101	·0102	·0103	·0105
44	·0067	·0084	·0092	·0099	·0104	·0106	·0107	·0108	·0109	·0110
45	·0069	·0088	·0095	·0102	·0108	·0111	·0113	·0114	·0114	·0115
46	·0072	·0094	·0100	·0106	·0112	·0116	·0119	·0121	·0122	·0122
47	·0077	·0102	·0106	·0111	·0116	·0121	·0125	·0128	·0130	·0130
48	·0083	·0111	·0115	·0117	·0121	·0126	·0131	·0135	·0138	·0139
49	·0089	·0122	·0126	·0127	·0129	·0133	·0138	·0143	·0147	·0148
50	·0095	·0131	·0138	·0139	·0139	·0142	·0147	·0152	·0156	·0158
51	·0101	·0145	·0151	·0151	·0151	·0153	·0157	·0161	·0165	·0168
52	·0107	·0156	·0165	·0165	·0165	·0166	·0169	·0173	·0177	·0180
53	·0113	·0168	·0178	·0180	·0180	·0180	·0183	·0186	·0190	·0193
54	·0120	·0179	·0192	·0195	·0195	·0196	·0198	·0201	·0205	·0208
55	·0128	·0191	·0205	·0211	·0211	·0212	·0214	·0218	·0222	·0225
56	·0137	·0203	·0219	·0226	·0227	·0229	·0231	·0235	·0240	·0244
57	·0149	·0216	·0233	·0242	·0244	·0246	·0249	·0254	·0260	·0265
58	·0162	·0230	·0247	·0258	·0261	·0264	·0267	·0273	·0280	·0287
59	·0179	·0245	·0262	·0274	·0278	·0282	·0286	·0293	·0301	·0309
60	·0200	·0261	·0277	·0289	·0295	·0300	·0306	·0313	·0323	·0332
61	...	·0280	·0293	·0305	·0312	·0319	·0326	·0335	·0345	·0357
62	...	...	·0310	·0320	·0329	·0338	·0347	·0357	·0369	·0383
63	...	...	...	·0335	·0346	·0357	·0368	·0380	·0394	·0410
64	...	...	...	...	·0363	·0376	·0389	·0404	·0420	·0439
65	...	...	...	...	...	·0396	·0412	·0429	·0449	·0470
66	...	...	...	...	...	...	·0434	·0455	·0479	·0504
67	...	...	...	...	...	...	...	·0483	·0511	·0540
68	...	...	...	...	...	...	...	...	·0546	·0580
69	...	...	...	...	...	...	...	...	...	·0624

Several of the above rates, perhaps, do not run quite so regularly as might be desired; but it is an exceedingly difficult matter to obtain rates for select lives which shall be free from all irregularities, and at the same time faithfully reflect the prominent

features of the mortality to which they are subject. I have not merged the initial rates into an ultimate, nor is it desirable in this instance to do so, in view of the remarks I made on page 105. I fear this is a point which has been overlooked by some writers, and that for some ages at entry at least the true rate of mortality after (say) 10 years will be very different from the ultimate obtained by excluding the observations for the first five or six years. This, however, does not open up a pleasant prospect to the actuary, as it would render still more complicated some of the elaborate tables relating to select lives which we at present possess. How far it may be desirable to ignore it in practice is a question which it is unnecessary for me to consider on this occasion.

In the New York Mutual Experience the gradual wearing out of the effect of selection is discussed, and the conclusions arrived at are—"(1) that the advantage of selection diminishes at all "ages with the duration of the policy; (2) that it decreases very "rapidly among those who insure at the younger periods of life; "and (3) that it decreases more slowly at middle life and among "older insurants, and probably never entirely disappears." These conclusions are confirmed to a great extent by the above table. Taking ages 35 and 60, for instance, it will be seen that the advantage of selection decreases rapidly at the former age, and more slowly at the latter age. They are also confirmed, with one or two exceptions, by the following table, in which are given for quinquennial ages at entry the percentage of my adjusted select rates of mortality to those of the general population of Germany:

TABLE 12.

Age at Entry	Percentage of Adjusted Rate of Mortality for Select Lives to that of Population in Year of Insurance									
	1	2	3	4	5	6	7	8	9	10
25	36·5	62·8	75·9	79·8	81·3	81·7	82·1	81·6	80·4	79·2
30	43·0	61·1	68·4	71·6	73·6	74·5	74·8	75·0	74·4	73·3
35	51·8	57·4	60·0	61·6	61·8	63·2	64·1	65·5	66·9	68·3
40	48·5	54·2	58·8	62·3	64·6	66·1	67·6	69·2	71·1	72·5
45	41·1	53·4	57·3	60·3	63·2	66·0	69·5	73·0	76·0	78·8
50	44·2	64·2	69·6	72·0	73·9	76·0	78·0	80·9	83·8	86·6
55	45·9	68·6	74·2	77·2	77·9	78·5	79·5	81·1	82·9	85·7
60	52·4	68·3	70·3	70·5	70·9	71·7	72·8	75·1	78·7	83·2

It will be seen from the above table that even in the tenth year of insurance the rate of mortality is considerably below that of the general population.

I think it will be interesting to give here the unadjusted rate of mortality already mentioned as having been deduced from the Gotha experience after the observations for the first six years had been excluded, designating it Gotha $H^{M(6)}$, and to compare it with the $H^{M(5)}$ adjusted table. A column showing the rate of mortality in the general population has been added.

TABLE 13.

Age	Gotha$^{M(6)}$ Unadjusted	$H^{M(5)}$ Adjusted	Population of Germany	Age	Gotha$^{M(6)}$ Unadjusted	$H^{M(5)}$ Adjusted	Population of Germany
35	·0084	·0100	·0110	53	·0195	·0199	·0250
36	·0094	·0103	·0115	54	·0206	·0209	·0264
37	·0094	·0107	·0120	55	·0213	·0222	·0279
38	·0096	·0111	·0125	56	·0238	·0235	·0296
39	·0095	·0112	·0131	57	·0268	·0251	·0314
40	·0100	·0113	·0136	58	·0279	·0266	·0334
41	·0098	·0113	·0142	59	·0321	·0284	·0357
42	·0106	·0116	·0148	60	·0313	·0306	·0382
43	·0107	·0118	·0154	61	·0373	·0329	·0410
44	·0108	·0123	·0161	62	·0385	·0356	·0441
45	·0127	·0129	·0168	63	·0401	·0385	·0475
46	·0128	·0137	·0176	64	·0467	·0417	·0512
47	·0138	·0144	·0185	65	·0487	·0446	·0552
48	·0145	·0153	·0194	66	·0567	·0478	·0596
49	·0150	·0163	·0204	67	·0615	·0510	·0643
50	·0156	·0171	·0215	68	·0622	·0544	·0694
51	·0172	·0180	·0226	69	·0720	·0581	·0750
52	·0190	·0188	·0237	70	·0717	·0628	·0811

It will be observed that the rate of mortality in the Gotha$^{M(6)}$ is, with one exception, less than in the $H^{M(5)}$ up to age 55, notwithstanding that the first six years' observations have been excluded in the former, as against five (or, more correctly, $4\frac{1}{2}$) in the latter, and afterwards greater; while it is throughout very much less than that of the general population.

In order that the reader may judge of the effect of my select rates of mortality on premiums, I give in the following tables the net single and annual premiums for the insurance of £100 for various ages at entry and terms of years, calculated according to my tables at $3\frac{1}{2}$ per-cent interest; also the same premiums according to Mr. Sprague's Select Mortality Tables, and the H^M, extracted from Mr. R. P. Hardy's Valuation Tables:

TABLE 14.—*Single Premiums.*

Age	One Year			Five Years			Ten Years			Age
	H^M	Sprague's Select	My Select	H^M	Sprague's Select	My Select	H^M	Sprague's Select	My Select	
25	·640	·435	·290	3·096	3·515	2·596	6·021	6·835	5·489	25
30	·746	·435	·377	3·598	3·420	2·798	6·973	7·231	6·022	30
35	·848	·473	·560	4·176	3·744	3·153	8·017	7·891	6·653	35
40	·996	·531	·647	4·784	4·165	3·803	9·569	9·261	8·345	40
45	1·178	·618	·696	6·001	5·037	4·539	12·048	11·568	10·566	45
50	1·541	·763	·928	7·695	6·392	6·790	15·666	14·808	15·333	50
55	2·032	1·005	1·237	10·351	8·698	9·405	21·257	20·120	20·845	55
60	2·867	1·391	1·932	14·635	12·558	12·589	29·047	27·636	27·211	60

TABLE 15.—*Annual Premiums.*

Age	Five Years			Ten Years			Age
	H^M	Sprague's Select	My Select	H^M	Sprague's Select	My Select	
25	·671	·762	·561	·721	·821	·654	25
30	·782	·741	·605	·839	·869	·719	30
35	·909	·812	·683	·970	·951	·797	35
40	1·045	·905	·826	1·165	1·122	1·007	40
45	1·317	1·097	·988	1·485	1·415	1·285	45
50	1·701	1·399	1·491	1·966	1·837	1·909	50
55	2·313	1·919	2·085	2·741	2·558	2·667	55
60	3·332	2·808	2·831	3·923	3·658	3·601	60

Looking at the rates for five and ten years, it will be seen that up to age 45 my rates are lower than either Mr. Sprague's or the H^M, and that after that age they are similar to Mr. Sprague's, which are lower than the H^M.

X.—WHETHER THE RATE OF INCREASE IN THE MORTALITY DURING THE 10 YEARS AFTER INSURANCE DEPENDS ON THE RATE OF DISCONTINUANCE OR NOT.

Two ways occurred to me in which this problem might be solved with the materials available: (1) by observing the rate at which the mortality increases in lives among whom there are no withdrawals, and (2) by comparing the rate of mortality in one office with that in another in which there was a larger number of withdrawals. The mortality in the first few years, however, among lives in which there are no withdrawals, such as the Government annuitants, differs so much, on account of there being no medical examination, from that which prevails among assured

lives that, notwithstanding that I had obtained some valuable results from which deductions might have been made, I had to abandon the first method in the present investigation; for, according to the terms of the syllabus, it is necessary to show whether the rate of increase in the mortality during the 10 years after insurance depends on the rate of discontinuance or not.

Of the experiences available, we have now only one left which is arranged according to policy years, and it is therefore necessary to fall back upon experiences made up by calendar years, in order that we may ascertain whether the rate of increase depends upon the rate of discontinuance or not. No objection can be taken to our doing so, as it matters little for our present purpose how the experiences are arranged, so long as the principle adopted in each is the same. In the following table I give the unadjusted rates of mortality for the 10 years after insurance for quinary groups of ages at entry in the Equitable, Institute, and New York Mutual (denoting them, as before, by the central age of the group), with the rates of discontinuance in the first two. Only partial statistics of the rate of discontinuance are published in the New York Mutual Experience, and these are given in Table 17. I have assumed that they have been obtained by taking the ratio of the withdrawals to the exposed to risk of death, as this seems to be the course usually adopted in America, and I have therefore treated the Institute statistics in the same way. I have tabulated the rate of mortality for the first year in the Equitable under year 0, but this will not affect the argument I shall afterwards base upon it.

Table 16.

Year of Insurance	Unadjusted Rate of Mortality			Unadjusted Rate of Discontinuance	
	Equitable	Institute	New York Mutual	Equitable	Institute
	Age at Entry 25				
0	·0030	·0024	·0038	·002	·031
1	·0100	·0062	·0060	·071	·083
2	·0080	·0075	·0064	·111	·060
3	·0099	·0079	·0069	·065	·049
4	·0094	·0091	·0082	·071	·040
5	·0076	·0087	·0082	·062	·035
6	·0107	·0093	·0076	·054	·030
7	·0110	·0090	·0101	·055	·038
8	·0099	·0093	·0071	·065	·019
9	·0117	·0107	·0091	·038	·015
10	·0134	·0117	·0066	·018	·020

TABLE 16—(*continued*).

Year of Insurance	Unadjusted Rate of Mortality			Unadjusted Rate of Discontinuance	
	Equitable	Institute	New York Mutual	Equitable	Institute
	Age at Entry 30				
0	·0026	·0045	·0032	·001	·027
1	·0060	·0050	·0054	·051	·065
2	·0071	·0073	·0062	·078	·048
3	·0087	·0087	·0061	·063	·041
4	·0091	·0105	·0078	·055	·033
5	·0106	·0097	·0092	·046	·025
6	·0113	·0106	·0100	·042	·024
7	·0110	·0098	·0089	·053	·034
8	·0105	·0111	·0064	·054	·021
9	·0098	·0103	·0085	·026	·015
10	·0120	·0126	·0077	·026	·015
	Age at Entry 35				
0	·0030	·0046	·0050	·001	·025
1	·0117	·0066	·0059	·039	·062
2	·0147	·0086	·0059	·060	·042
3	·0108	·0097	·0089	·056	·038
4	·0110	·0111	·0079	·056	·029
5	·0103	·0104	·0088	·047	·027
6	·0111	·0103	·0093	·039	·022
7	·0114	·0108	·0076	·050	·032
8	·0101	·0109	·0083	·053	·019
9	·0118	·0105	·0091	·025	·015
10	·0115	·0124	·0102	·035	·012
	Age at Entry 40				
0	·0039	·0035	·0042	·001	·025
1	·0105	·0071	·0066	·038	·060
2	·0106	·0091	·0068	·066	·044
3	·0106	·0107	·0086	·049	·034
4	·0106	·0129	·0091	·046	·027
5	·0121	·0117	·0099	·031	·026
6	·0108	·0149	·0101	·027	·020
7	·0116	·0137	·0103	·056	·032
8	·0138	·0141	·0115	·051	·018
9	·0166	·0168	·0140	·024	·016
10	·0157	·0172	·0123	·018	·013

TABLE 16—(*continued*).

Year of Insurance	Unadjusted Rate of Mortality			Unadjusted Rate of Discontinuance	
	Equitable	Institute	New York Mutual	Equitable	Institute
Age at Entry 45					
0	·0055	·0057	·0078	·001	·016
1	·0123	·0093	·0065	·039	·058
2	·0128	·0119	·0099	·074	·042
3	·0124	·0142	·0098	·049	·035
4	·0149	·0145	·0104	·041	·030
5	·0171	·0152	·0093	·035	·025
6	·0183	·0143	·0116	·043	·023
7	·0178	·0192	·0141	·056	·037
8	·0211	·0201	·0153	·055	·014
9	·0181	·0195	·0168	·036	·014
10	·0210	·0202	·0234	·012	·016
Age at Entry 50					
0	·0116	·0066	·0090	·001	·018
1	·0178	·0092	·0102	·038	·062
2	·0227	·0138	·0112	·067	·042
3	·0241	·0173	·0124	·052	·035
4	·0197	·0179	·0120	·055	·030
5	·0203	·0217	·0146	·028	·026
6	·0272	·0241	·0174	·030	·022
7	·0234	·0209	·0181	·047	·040
8	·0222	·0225	·0179	·057	·014
9	·0265	·0274	·0183	·023	·018
10	·0309	·0319	·0318	·018	·012

TABLE 17.

Ages at Entry	Ratio of Withdrawals to Exposures during the			
	1st Five Years		2nd Five Years	
	New York Mutual	Institute	New York Mutual	Institute
20–24	·013	·074	·038	·036
25–29	·078	·052	·032	·025
30–34	·062	·044	·030	·024
35–39	·058	·044	·028	·024
40–44	·053	·040	·028	·023
45–49	·050	·039	·025	·022

The New York Mutual withdrawals may, like the Institute, contain endowment assurances which have matured, but this will not affect the first 10 years to any appreciable extent.

The unadjusted rates of discontinuance in the Institute experience agree with those given by Mr. King in Table G of his paper, "On the Rate of Mortality amongst Assured Lives" (*J.I.A.*, xix, 381). I would have used the rates there given, and saved myself the labour of calculating them, had it not been that I was unable to reproduce some of the figures in Tables K and L of that paper. He says the exposed to risk and discontinued were deduced in the manner the similar tables of exposed to risk and died were deduced for the H^{M} and other Institute tables, as explained on page 18 of the mortality experience; but certainly year 0 has not been dealt with in this way. As far as I can see, about one-quarter of the died has been deducted in deducing the rate of discontinuance for that year, instead of a half, as one would have anticipated from his description.

Mr. King pointed out the extraordinary fatality of year 7 to policies through discontinuance. In the discussion which followed the reading of the paper, Mr. Bailey explained this by saying that, "when life insurance was first practised in this country, in "the early days of the Royal Exchange and the London Assur- "ance, from 1720 for several years onwards, the whole of the "policies were term policies. In the early days of the Equitable "the great bulk of the policies were the same; and formerly the "premiums quoted in prospectuses generally, were premiums "for one and seven years and the whole term of life, and no "others." It will be seen that the high rates of discontinuance in the Equitable for years 7 and 8 entirely corroborate this statement.

Even the unadjusted rates of mortality for quinary groups of ages at entry given in Table 16, indicate very well the law they follow. Comparing the mortality in the Equitable with that in the Institute, it will be seen that, leaving out of account year 0, the rate in the former is as a rule greater at the outset than in the latter, and afterwards has a tendency to approximate to it. In the New York Mutual, the mortality in the year 0 is, except at one age, greater than in the Institute, while afterwards it is almost invariably less—considerably less. If the facts were graduated, these features would be more apparent; but in treating this part of the subject I have preferred not to introduce anything to which exception might be taken, and have therefore rather adopted the

plan of adding together several of the groups, so as to remove irregularities and obtain a steadier progression. In the following table, the groups having the central ages at entry 25, 30, ... 40, are added together:

TABLE 18.

Year of Insurance	Unadjusted Rate of Mortality			Unadjusted Rate of Discontinuance	
	Equitable	Institute	New York Mutual	Equitable	Institute
0	·0031	·0038	·0040	·001	·027
1	·0094	·0061	·0059	·049	·068
2	·0102	·0081	·0063	·077	·049
3	·0100	·0092	·0076	·059	·041
4	·0100	·0108	·0082	·056	·032
5	·0103	·0101	·0090	·046	·028
6	·0110	·0111	·0093	·040	·024
7	·0112	·0107	·0091	·054	·034
8	·0111	·0113	·0081	·055	·019
9	·0124	·0118	·0099	·027	·015
10	·0131	·0133	·0091	·025	·015

On referring to the table of average ages on page 138, we see that, with one or two unimportant exceptions, the average ages in the Equitable and New York Mutual are, within the above limits, slightly greater than those in the Institute, so that one would naturally expect the mortality to be, if anything, a little higher. My reason for excluding the entrants at the higher ages, is because the average ages do not agree so well there. The rate of mortality in the Equitable after the first few years, however, is very similar to that in the Institute, while in the New York Mutual it is very much less after the first two years, notwithstanding that the rate of discontinuance in both cases is higher—in the case of the Equitable very much higher. If I am wrong in my surmise as to the manner in which the Equitable experience has been made up, and the rates in Table 18 should each be one line lower down, then the rate of mortality after the first few years would be lighter than the Institute. The low rate of mortality in the New York Mutual may, however, be due partly to the lighter mortality which prevails in the population of America at these ages, and partly to the fact that endowment assurances form a very large proportion of the business. That experience also contains about 3 per-cent of females, but this would probably have the effect of slightly increasing the rate. In the introduction to the experience it is stated, that "of the 9,866 entrants at ages 20 to 24 inclusive,

"3,045 retired within the first five years, or more than 30 per-"cent of the whole number, and this percentage is much greater "than at the older ages. Among these retirants the larger "number were, doubtless, sound lives; and if so, their departure "reduced the vitality of those who remained below the limit to "which it would otherwise have fallen. The very great number "of withdrawals at these early ages makes the increase of "mortality more marked than at other periods of life; and "although it would be impossible to give its exact measure, "it is probably a considerable part of the whole cause at these "younger ages." It has been shown, however, by Mr. Makeham (*J.I.A.*, xii, 305) that there is a marked increase in the rate of mortality at these younger ages in experiences where there are no withdrawals. So far as I am aware the above extract is the only instance in the experiences in which the effect of withdrawals on the rate of mortality during the early years of insurance, is brought to the test of actual figures.

The results here shown, however, are contrary to the opinion held by several eminent actuaries that withdrawals exercise a detrimental effect on the rate of mortality. The first to publish this opinion was Mr. Higham, in his paper "On the Value of Selection as exercised by the Policyholder" (*J.I.A.*, i, 179). He there says: "On comparing the probabilities of living a year "in the separate classes with those of Mr. Farr, given in the "reports of the Registrar-General, one is struck with the fact "that assured lives are, for some time after selection, much better "than the community at large, but that after awhile they "become much worse. If the comparison be made with a table "representing the chances of life in the better classes of society, "to which assured lives generally belong, their inferiority after "a short period of assurance will be still more strongly exhibited. "There can be nothing in their constitution or circumstances to "account for this; and it can arise from no other cause than *the* "*selection which the assured exercise against the companies* by "dropping policies on healthy lives and retaining those on lives "which have become bad or doubtful. Various writers who have "noticed the inferiority of assured lives taken in the mass, have "hastily concluded that the care which the companies exercise in "the admission of applicants fails of its purpose, through the "incompleteness of the information they obtain, and the occasional "success of dishonest speculators. But if this were so, it would "be apparent from the outset; whereas, in the early years after

"entry, the reverse is the case. I conclude, therefore, that if we "could trace the after-history of *all* who appear upon our books, "we should find that they really were, as a whole, the select lives "we supposed them to be on their admission;—that those who "dropped their policies were those who permanently enjoyed "good health; while those who had fallen into bad health were "careful to retain them.

"I am not aware that attention has previously been called to "this point, but it is one deserving of very serious consideration, "especially by those who are accustomed to regard lapsed "assurances as a source of profit. It might be so if the policies "to be dropped were settled by lot; but seeing that a power "of selection is exercised in opposition to the interests of the "company, there is every reason to fear that a loss instead of a "gain is the result. It is true, a measure of profit is shown "upon the immediate transaction, inasmuch as the company are "enabled to dispose of their reserve in respect of the policies "surrendered. But this is so obviously done at the expense of the "future, that one can hardly hear a company congratulating "themselves on their large profits from discontinued policies, "without being reminded of the man in the fable, who cut open "his goose to secure the golden eggs."

Mr. Makeham refers to the subject in his above-mentioned paper: "But it is a well-known fact that many lives are prematurely withdrawn from observation by the discontinuance or "surrender of the policy, and such withdrawals necessarily "consist of *select* lives—that is, of course, select at the time "of withdrawal—for, as previously observed, the deteriorated and "doubtful lives do not withdraw. This constant draining of the "better class of lives must necessarily have the effect of materially "increasing the rate of mortality among the lives which remain "upon the books. . . . ."

Mr. Sprague subsequently took up the point in his paper, "On the Rate of Mortality among Assured Lives", already quoted from, and gave several interesting tables. He arranged the Institute H^M statistics for each year of assurance in two vertical columns, showing the numbers at risk and the deaths for each age at entry. The expected number of deaths for each age at entry and in each year of assurance were then calculated, and the results entered by the side of the actual deaths. The expected deaths were calculated separately from three tables of mortality, namely, the Seventeen Offices' Experience, the Peerage

Males as adjusted by Mr. Berridge, and the H^M Table. In this way the total actual and expected deaths for each year of assurance were obtained for those insured between ages 15 and 75, the observations beyond these ages being excluded on account of the paucity of numbers. The following table shows the results according to the H^M table given in Table F of the paper, grouping the years of insurance so as to avoid irregularities and bring out the law which the facts follow:

TABLE 19.

Year of Insurance (1)	Number at Risk (2)	Actual Deaths (3)	Expected Deaths H^M (4)	Percentage of (3) to (4) (5)
0	63,644·5	290	650·32	44·6
1	116,565	891	1,246·93	71·5
2	103,312·5	1,028	1,156·45	88·9
3– 5	252,291·5	3,177	3,104·08	102·35*
6–10	288,416	4,515	4,295·06	105·12
11–15	170,529	3,589	3,300·41	108·74
16–20	96,560	2,614	2,483·78	105·24
21–25	52,939·5	1,877	1,796·85	104·46
26–30	26,232·5	1,210	1,168·66	103·54
31–63	18,468	1,250	1,224·62	102·07

Mr. Sprague says: "If we now consider the way in which "the withdrawal of healthy lives must affect the rate of "mortality among the persons remaining under observation, "we shall see that it may be expected to produce precisely "the effect shown in the above tables. We shall find that "the effect produced upon the rate of mortality by the "withdrawal of a number of healthy persons, will be precisely "the same as would be produced by the addition of a number "of unsound lives. To prove this, let us suppose 1,000 "persons insure their lives at the same time, and that they "are divided into two bodies of 500 each; then, after the lapse "of a year, a certain proportion of the lives in each body, say, for "example, to fix the ideas, 5 per-cent of the original number, "*i.e.*, 25, will have had diseases developed in them, which render "them no longer insurable at the ordinary rate, while the other

* This percentage is not the same as that in Table F above mentioned. I observed that the percentages in Table E of the paper were inconsistent with those in Table F; and on examining the figures for the first 10 years, I found that in the former table the percentage for the fifth year of insurance should be 107·3 instead of 108·4, and that in the latter the percentage for the years of insurance 3–5 should be 102·35 instead of 98·64.

"survivors of each body may be regarded as still insurable at the "ordinary rate. If, now, we suppose that at the end of the year "all these insurable lives in one of the bodies lapse their policies, "there will remain assured of the one body only the 25 lives now "uninsurable, and of the other body all the survivors of the "original 500. Doubling these numbers, the rate of mortality is "of course unaltered, and we see that the rate of mortality "among the persons still remaining under observation out of the "original 1,000, will be the same as if to the survivors of the "original 1,000 there were added after the lapse of a year, 50 "unsound lives. To express the result in a more general form, "we may say that the withdrawal of the good lives increases the "proportion which the bad lives bear to the whole. Now, the "greatest number of withdrawals take place within a few years "from the date of the policy, and after the lapse of, say, 15 "years, they become so few as to produce no appreciable effect "upon the mortality. In order, therefore, to trace the effect of "the withdrawals upon the rate of mortality, we may suppose "that in each year of insurance up to the 15th, a diminishing "number of unsound lives is added to those surviving out of the "original number insured. These unsound lives will experience "a very much heavier mortality than the mixed lives which "survive from the original number insured; but as their numbers "are by supposition continually recruited by fresh additions "during the early years of the insurance, there will be a rapidly "increasing number of deaths among them, which will have the "effect of causing the rate of mortality among the whole body to "increase faster than it would otherwise do. But when we come "to the subsequent years of the insurance, when the unsound "lives receive no new additions to their numbers, the heavy rate "of mortality prevailing among them will cause their number to "bear a continually diminishing ratio to the whole, and these "unsound lives will consequently exercise each year a less "influence on the rate of mortality. As far as they are con-"cerned, there will therefore be a tendency for the rate of "mortality to improve; but as the survivors of the original 500 "in the above illustration are probably yearly getting, on the "average, worse, there is a tendency on that account for the "general mortality to get worse. We have thus two forces "now operating in opposite directions, and it depends on which is "the more powerful whether the mortality will improve or get "worse; and there will be found no difficulty in explaining the

"whole of the observed results, by the combined operation of "these two causes.

"There can, I think, remain no doubt after a study of the "figures in the above tables, that the withdrawals do exert a very "great influence in increasing the rate of mortality. Mr. Higham, "in the passage quoted above (page 331), draws this conclusion "very fairly from the circumstance that the mortality among "insured lives after the lapse of some years from the date of "selection, is much heavier than that among the population at "large. It might be objected, however, that this reasoning is not "conclusive, inasmuch as it is conceivable that the mortality "among insured lives might be ultimately greater than that "among the population at large, in consequence of their consisting "mostly of persons who work harder than the average, and thus "exhaust their vital energies prematurely. But no such argument "can apply to explain the phenomena shown above, namely, first "the increase of the mortality and then its subsequent diminution.

"The foregoing observations not only demonstrate that the "withdrawals produce a powerful effect in increasing the rate of "mortality among the lives remaining under observation, but "enable us to see clearly the way in which this effect is produced, "and, to a certain extent, to measure its magnitude. It is obvious "that the greater the number of withdrawals, the greater will be "the effect produced upon the mortality; and when any special "circumstance causes an unusually large number of withdrawals, "it may be expected to produce a most disastrous effect upon the "mortality of the remaining lives. A conspicuous instance of "this is afforded in the transfer of the business of one office to "another of inferior credit; and to this cause I believe may be "principally attributed the recent lamentable failure of the Albert "Life Office. On the other hand, it may be expected that those "offices which have the fewest withdrawals, will experience the "most favourable mortality. Mr. Higham's remarks upon these "points (pp. 191-2) appear to me extremely judicious." We are not concerned in the present enquiry with the phenomena shown above, namely, first the increase of the mortality and then its subsequent diminution, but, as already mentioned, only with the first 10 years—a limitation which has not rendered easier the treatment of a difficult subject.

Mr. King, in his paper already mentioned, grouped the H^{M} statistics into three great classes for what may be called respectively the young entrants, ages at entry 20–35; the middle-aged, 40–50;

and the old, 55–60; and gave a comparison of the rates of mortality and discontinuance for various years of assurance. He says: "All the curves rapidly rise during the first three years, and "this is clearly due to something in addition to the mere lapse of "time since medical examination. The numerous discontinuances "at starting have a tangible though not a very great effect; and "when they cease to operate with the same force, there is at year "4 a manifest tendency in the curves to fall. The discon-"tinuances remain for a year or two nearly constant, but chronic "diseases begin to tell, and the curves again gradually ascend till "they reach their second maximum, which is attained earlier for "the old ages than for the young. Discontinuances at length "practically cease, and the curves take a downward bend. For "the older ages at entry they seem at the last, when the lives "are all advanced in years, to take again an upward direction, "but this, I think, is due to the character of the standard table "at the late years of life, and not to the discontinuances."

I do not see why the fact that all the curves rapidly rise during the first three years is clearly due to something in addition to the mere lapse of time since medical examination. I have shown that the rate of mortality for select lives in the first year of insurance is only about 45 per-cent of the general population; and as there is a constant tendency for them to assimilate themselves to the body from which they are drawn, this will of itself cause a rapid rise in the first few years. With regard to the maximum at year 4, Mr. Sprague, in a few explanatory remarks appended to the discussion, says: "I am of opinion that, even assuming this "maximum to have a real existence, it is not of sufficient impor-"tance, either as regards magnitude or duration, to be worthy "of preservation in the adjustment of the results." Some actuaries, I believe, hold that the rate of mortality has a tendency to fall after the numerous discontinuances in the early years of insurance; but I think it is clear from an examination of the rate of mortality in the separate and collective groups for various ages at entry already given that there is no such tendency during the first 10 years—indeed, by a very small amount of grouping in the latter we can secure a steady progression.

I think, however, that were I to stop here the investigation I have made into this part of the subject could hardly be considered conclusive, having regard to the results obtained. Objections may be taken to the experiences I have compared; for instance, that the conditions of assurance have changed very much since the

Equitable Experience was published, and that there is some doubt as to how it is made up; or that the low rate of mortality in the New York Mutual may be mainly, if not entirely due to the other two causes mentioned. I therefore again searched the experiences mentioned in the syllabus, but was unable to find any statistics which would prove entirely satisfactory. I then directed my attention to other experiences; and on looking through the mortality experience of the Scotch life assurance offices which contributed their experience to the Institute, I was at last rewarded by the discovery of what I may fairly term a nugget. I observed that the withdrawals were less numerous there than in the Institute; and on page 9 of the experience there is published an index to a number of manuscript tables, in Nos. 1, 2, and 3 of which are given for healthy lives (males, with and without profits) the discontinued, died, and existing, arranged according to ages at entry and at exit. On enquiry I ascertained that these tables were deposited in the library of the Faculty of Actuaries. The idea I had first of all was to subtract the figures there given from those in the Institute experience, and thus obtain the figures relating to the English offices; but on considering the matter I came to the conclusion that it would not be correct to do this, as any duplicate policies would be eliminated when the cards were combined. This was rather unfortunate; but we can still have a good comparison between the mortality in the ten Scotch offices, and the combined experience of the Scotch and English offices, forming the Institute already given. The average ages are almost identical in the two experiences, except at the older ages, as will be seen from the following table:

TABLE 20.

Central Age of Group	Average Age of Group	
	Ten Scotch	Institute
25	25·19	25·19
30	29·97	29·98
35	34·89	34·90
40	39·84	39·84
45	44·80	44·83
50	49·76	49·80

The following table gives the rates of mortality and discontinuance for each of these groups during the first 10 years of insurance:

Table 21.

Year of Insurance	Unadjusted Rate of Mortality		Unadjusted Rate of Discontinuance		Year of Insurance	Unadjusted Rate of Mortality		Unadjusted Rate of Discontinuance	
	Ten Scotch	Institute	Ten Scotch	Institute		Ten Scotch	Institute	Ten Scotch	Institute
Age at Entry 25					Age at Entry 40				
0	·0032	·0024	·008	·031	0	·0043	·0035	·005	·025
1	·0067	·0062	·071	·083	1	·0075	·0071	·049	·060
2	·0075	·0075	·048	·060	2	·0099	·0091	·034	·044
3	·0080	·0079	·036	·049	3	·0100	·0107	·029	·034
4	·0088	·0091	·032	·040	4	·0138	·0129	·024	·027
5	·0094	·0087	·028	·035	5	·0118	·0117	·023	·026
6	·0095	·0093	·020	·030	6	·0149	·0149	·017	·020
7	·0088	·0090	·021	·038	7	·0150	·0137	·020	·032
8	·0094	·0093	·016	·019	8	·0127	·0141	·016	·018
9	·0104	·0107	·013	·015	9	·0176	·0168	·011	·016
10	·0124	·0117	·017	·020	10	·0159	·0172	·010	·013
Age at Entry 30					Age at Entry 45				
0	·0048	·0045	·007	·027	0	·0060	·0057	·004	·016
1	·0042	·0050	·052	·065	1	·0088	·0093	·045	·058
2	·0072	·0073	·040	·048	2	·0110	·0119	·036	·042
3	·0086	·0087	·035	·041	3	·0139	·0142	·026	·035
4	·0107	·0105	·028	·033	4	·0135	·0145	·024	·030
5	·0099	·0097	·019	·025	5	·0166	·0152	·018	·025
6	·0113	·0106	·017	·024	6	·0157	·0143	·015	·023
7	·0089	·0098	·020	·034	7	·0179	·0192	·020	·037
8	·0105	·0111	·018	·021	8	·0174	·0201	·011	·014
9	·0095	·0103	·012	·015	9	·0174	·0195	·008	·014
10	·0116	·0126	·012	·015	10	·0215	·0202	·013	·016
Age at Entry 35					Age at Entry 50				
0	·0054	·0046	·006	·025	0	·0095	·0066	·010	·018
1	·0065	·0066	·049	·062	1	·0094	·0092	·045	·062
2	·0098	·0086	·034	·042	2	·0128	·0138	·034	·042
3	·0096	·0097	·030	·038	3	·0120	·0173	·027	·035
4	·0122	·0111	·022	·029	4	·0158	·0179	·024	·030
5	·0108	·0104	·022	·027	5	·0235	·0217	·020	·026
6	·0113	·0103	·017	·022	6	·0252	·0241	·015	·022
7	·0110	·0108	·017	·032	7	·0226	·0209	·021	·040
8	·0106	·0109	·015	·019	8	·0213	·0225	·011	·014
9	·0096	·0105	·013	·015	9	·0290	·0274	·010	·018
10	·0128	·0124	·009	·012	10	·0293	·0319	·009	·012

The rates of mortality for the separate groups, however, are so similar that it is not possible to draw any conclusion from them; but by combining them for central ages at entry 25–40, as was

previously done with the other observations, the features of the experiences become more apparent:

Table 22.

Year of Insurance	Unadjusted Rate of Mortality		Unadjusted Rate of Discontinuance		
	Ten Scotch	Institute	Ten Scotch	Institute	Percentage of Ten Scotch to Institute
0	·0045	·0038	·007	·027	25·9
1	·0061	·0061	·056	·068	82·4
2	·0085	·0081	·039	·049	79·6
3	·0090	·0092	·033	·041	80·5
4	·0113	·0108	·027	·032	84·4
5	·0104	·0101	·023	·028	82·1
6	·0116	·0111	·018	·024	75·0
7	·0107	·0107	·020	·034	58·8
8	·0107	·0113	·017	·019	89·5
9	·0113	·0118	·012	·015	80·0
10	·0130	·0133	·012	·015	80·0

It will be seen that there is only a slight increase in the rate of discontinuance in year 7 in the ten Scotch offices experience, which again confirms Mr. Bailey's explanation previously given. According to Messrs. Higham, Makeham, Sprague, and King, the rate of mortality in the ten Scotch offices should be less than that in the Institute, as the withdrawals are less numerous in the former—roughly speaking about 20 per-cent less. But the reverse is the case: the rate of mortality up to year 7 is, with one exception, greater than in the Institute or the same. It is true that the rate after that is less, and I thought that possibly the turn had come; but on taking out the figures for year 11, I found the rate of mortality in the ten Scotch to be ·0132, against ·0125 in the Institute. I think, however, that in the former experience there is a tendency in the rate of mortality to fall.

For the purpose of testing the startling results here shown, I calculated the expected deaths in the Scotch experience in the same manner as was done by Mr. Sprague for the Institute experience, using the H^M rates of mortality published by him on page 337 of his paper, although the rates for ages 15 and 16 there given differ from those in the Institute table. The following table gives for each year of insurance the numbers exposed to risk, the actual and expected deaths, and the percentage of the former to the latter:

Table 23.—*Scotch Experience.*

Year of Insurance (1)	Number at Risk (2)	Actual Deaths (3)	Expected Deaths HM (4)	Percentage of (3) to (4) (5)
0	40,717	201	400·08	50·24
1	73,994	539	759·93	70·93
2	65,536	653	703·24	92·86
3	58,770·5	630	659·52	95·52
4	52,657	682	618·81	110·21
5	47,548	621	585·57	106·05
6	43,297	610	558·41	109·24
7	39,402·5	562	532·87	105·47
8	35,575	510	504·48	101·09
9	31,996·5	492	476·63	103·22
10	28,507	483	446·14	108·26

In the following table the years of insurance are grouped in the same manner as was done by Mr. Sprague, his figures being also given for comparison:

Table 24.

Year of Insurance	Number at Risk		Actual Deaths		Expected Deaths		Percentage of Actual to Expected	
	Ten Scotch	Institute	Ten Scotch	Institute	Ten Scotch	Institute	Ten Scotch	Institute
0	40,717	63,644·5	201	290	400·08	650·32	50·2	44·6
1	73,994	116,565	539	891	759·93	1,246·93	70·9	71·5
2	65,536	103,312·5	653	1,028	703·24	1,156·45	92·9	88·9
3– 5	158,975·5	252,291·5	1,933	3,177	1,863·90	3,104·08	103·71	102·35
6–10	178,778	288,416	2,657	4,515	2,518·53	4,295·06	105·50	105·12

It will be seen that the figures in the above table confirm the previous results, namely, that the ratio of the actual to the expected deaths is, with one exception, greater in the ten Scotch offices' experience, where the withdrawals are less numerous, than in the Institute; and at the same time show that the method I have adopted of grouping together various ages at entry to ascertain the rates of mortality and discontinuance, is legitimate. They also confirm the results obtained from the other experiences examined, namely, the Equitable, Institute, and New York Mutual; and I think there can be no doubt that the figures I have given show that where the withdrawals are most numerous in the early years of insurance, there the mortality is lightest. The difference, however, between the two experiences upon which

I lay most stress, namely, the ten Scotch and the Institute, is small; and it will be observed that it gradually becomes less, until in years of insurance 6–10 it almost vanishes. If it had been possible to separate the Scotch observations from the English, probably the difference would have been greater, and then I should have said that the rate of mortality *does* depend upon the rate of discontinuance, and that the withdrawals are, on the average, slightly worse than those who remain; but, as matters stand, and seeing that we are dealing with the observations for 10 years only, I shall content myself with saying that in ordinary circumstances the rate of increase in the mortality during the 10 years after insurance is independent of the rate of discontinuance. I do not think that any exception can be taken to the statistics on which this conclusion is based; and I question whether it is possible to get materials more suited for the purpose, unless in the direction I have indicated. It must be borne in mind, however, that the figures I have given relate to observations collected more than 25 years ago; and as policies now find a much more ready market than previously, observations relating to the last quarter of a century might not show so great a difference as my results would lead one to anticipate.

XI.—Outline of the Best Methods to be pursued in the future Collection and Compilation of a Mortality Experience.

I have discussed the various methods of arranging the data as I have gone along, and indicated which I prefer, and it only now remains for me to bring the various steps together. Lives are undoubtedly the best material for deducing the rate of mortality; but as I have shown, the difference is not great when policies are used, except at old ages. Amounts, however, may be used where the numbers at risk are very large, or where it is desired to ascertain whether the mortality has been heaviest among policies for large or small amounts. The method I have adopted to ascertain this is an easy one, but if it is desired to investigate the point more fully the sums insured may be divided into classes arranged according to amounts, care being taken to transfer those who have increased their insurances from one class to another, or to exclude them altogether. If it is found that the mortality is considerably greater among policies of large amounts, then this

circumstance should be allowed for in estimating the liability of the office.

In many insurance offices, cards are now written for policies as they are issued; and in view of their being combined for the purpose of deducing the rate of mortality among insured lives, it is desirable that there should be as much uniformity as possible in the form of card used. I would therefore suggest the following form as being suitable for general adoption.

No.........

Class................. £..............

Life..

English, Scotch, or Irish....................................

Occupation...

Date of	Day, Month, Year	Age at Entry, Duration	
Birth	.........:.........: 18......		
Entry	........:.........: 18......	....	....
Exit ( ) . .	........:.........: 18......	....	....

Rated up.......................for...............................

....

Cause of Death

Remarks

As it may be desirable at some future time to investigate the mortality in a particular district, the place of residence may be inserted after the nationality.

The exact dates of birth, entry, and exit, may be inserted in figures. The office age at entry in the case of British offices will be age next birthday; and this may be used without any correction,

or the exact age may be obtained by subtracting the calendar year of birth from the calendar year of entry, and inserted in the blank to the right. In the case of American and German offices it will, as a rule, be age nearest birthday, and this will no doubt be sufficient for all purposes. It is becoming more common now, however, among British offices to issue policies at half-years of age. The experience should be arranged according to policy years, each policy being traced to its anniversary; and when the cause of exit is surrender, the duration should be stated to the nearest month, or to two decimal places. A simpler method, perhaps, is to reckon the fraction in the last policy year as 0 if less than half a year, ·5 if half a year, and 1 if more than half a year. If selection is to be taken into account, it will not be necessary to make any allowance for the dating back of policies, as what has happened in the past may be expected to occur in the future. It will, however, require to be allowed for if it is desired to ascertain the true rate of mortality for the first year of insurance, or if the mortality table is to be formed in the ordinary way, that is to say, taking all the lives of the same age together, irrespective of the length of time for which they have been insured.

The cards should be divided into males and females, and then into healthy and rated up or exposed to extra risk; and whenever a healthy life incurs any special risk for which an extra premium is payable, it should no longer be kept under observation or at least transferred. They should also be divided into classes such as whole-life and endowment-assurance, otherwise the latter class, when they have arrived at maturity, will be reckoned as discontinued. The short-term insurances should be kept separate, and the opportunity might be taken to form a table for children's endowments. If the observations are sufficiently numerous, the whole-life class might be sub-divided into with and without-profit and limited-payment policies, and also arranged according to various classes of sums insured. The numbers at risk when broken up in this way, might be small, and in that case it would be sufficient to tabulate the materials for quinary groups of ages at entry, and thus save space. Duplicate policies should be eliminated by arranging the cards according to date of birth. Care should be taken when titles are given, as they sometimes change; and when the christian name and the date of birth is the same for females, enquiry should be made, in case they have married and simply changed the surname. It would be better, perhaps, to eliminate duplicate cards

before the policies are divided into separate classes, otherwise the same life may be in more than one class; and this would prevent their being afterwards combined without first going to the labour of eliminating these duplicate policies.

The cards should then be sorted into (1) existing, discontinued, and died; (2) age at entry; (3) duration of policy; and tabulated according to year of insurance under the respective ages at entry. They could then also be arranged according to age attained under the respective ages at entry, if desired.

If it is desired to form the mortality table in the ordinary way, the formula for obtaining the exposed to risk, using the same notation as before, would be:

$$\begin{aligned} E_x &= \Sigma n_x - \Sigma(d_{x-1} + w_{x-1} + e_{x-1}) - Cw_x \\ &= \Sigma(n_{x-1} - f_{x-1}) + n_x - Cw_x. \end{aligned}$$

where C represents the time which elapses on the average between the date of withdrawal and the anniversary of the policy, and x is the age at entry, however arrived at, plus the duration in complete years. Messrs. King and Hardy's formula, given *J.I.A.*, xxii, 200, may be used to adjust the results.

If, however, selection is taken into account, the results should be adjusted by the graphic method, as that is the only method at present known which works satisfactorily in the early years of insurance.

The above is a necessarily brief description of the methods which I think should be adopted to deduce the rate of mortality from the experience of an insurance office; but it seems to me that in this way all questions concerning the lives could be answered, except one or two, such as, for instance, the question of the mortality among those who surrender their reversionary bonuses for cash as compared with those who allow them to remain. It is questionable, however, whether an office would be willing to publish all the information here asked. Individual offices do not seem to care to give information as to their rate of discontinuance, but possibly the results arrived at in this essay may induce them to be less reticent on this point.

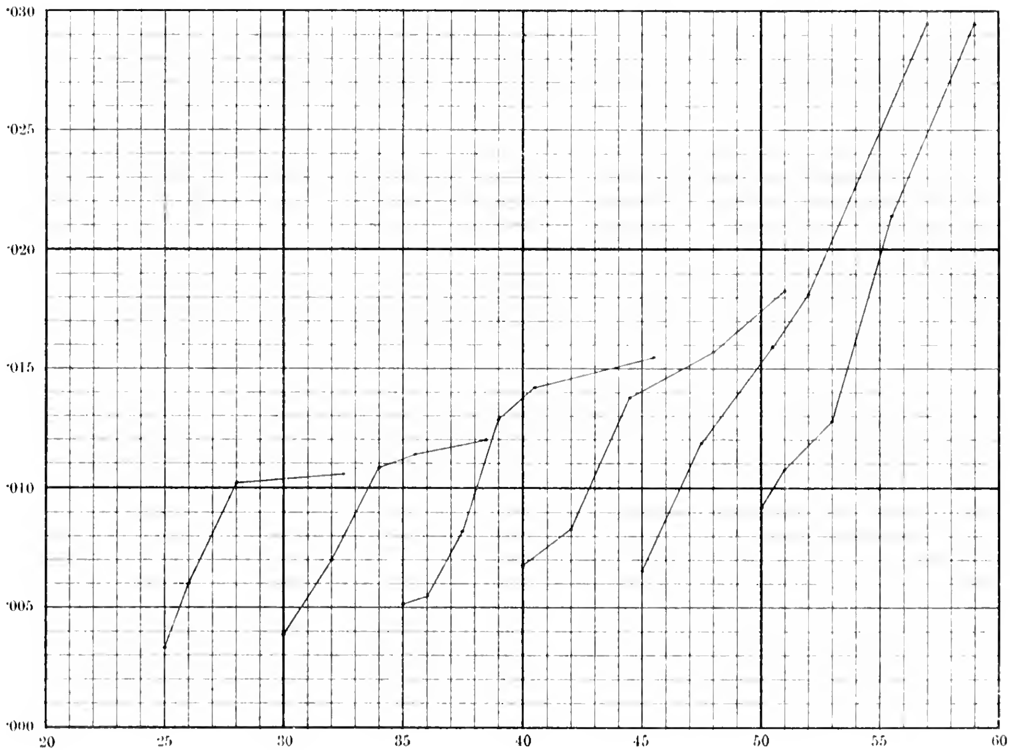
DISCUSSION.

The PRESIDENT (Mr. B. Newbatt), in inviting discussion, said that it might, perhaps, be charged against Mr. Chatham that his historical disquisition was too long, and that much of the preliminary matter might have been omitted. Remembering, however, that it was a prize essay, which by the nature of things was supposed to be *totus in toto*,

DIAGRAM No. 1A.

The abscissa represents the age, and the ordinate the probability of dying in a year.

Scottish Amicable—Unadjusted Rate of Mortality after grouping.



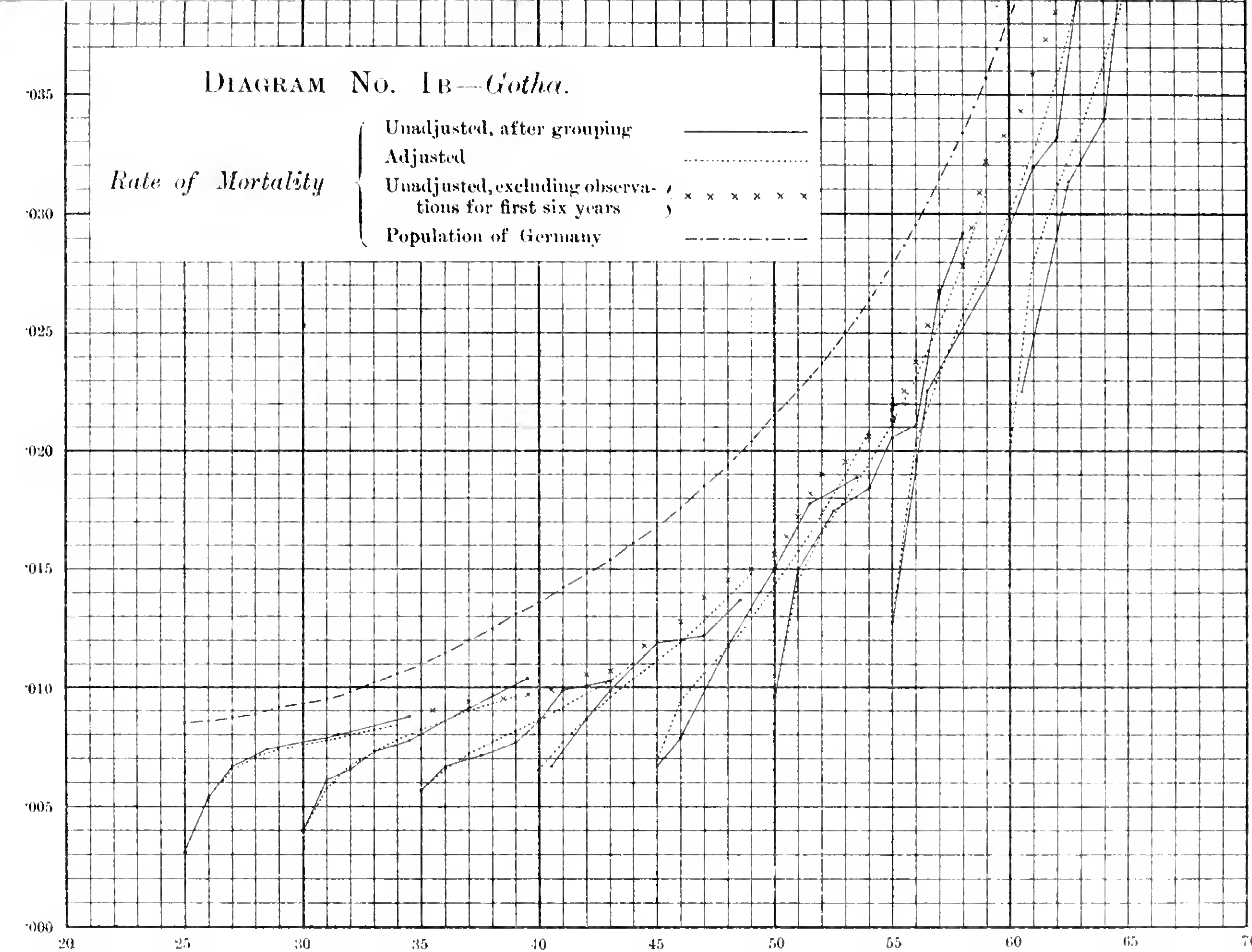


DIAGRAM No. IB—Gotha.
Rate of Mortality
Unadjusted, after grouping
Adjusted
Unadjusted, excluding observations for first six years
Population of Germany
·035
·030
·025
·020
·015
·010
·005
·000
20
25
30
35
40
45
50
55
60
65
70

DIAGRAM No. 1c.

Equitable—Unadjusted Rate of Mortality after grouping.

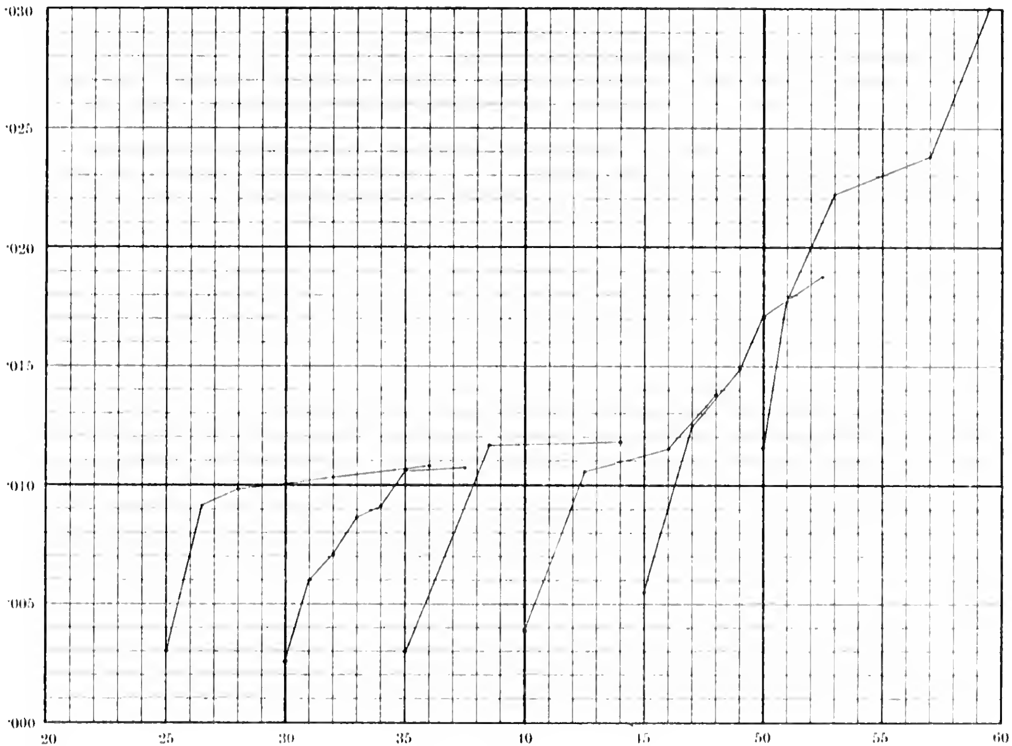


DIAGRAM No. Id.

Institute—Unadjusted Rate of Mortality after grouping.

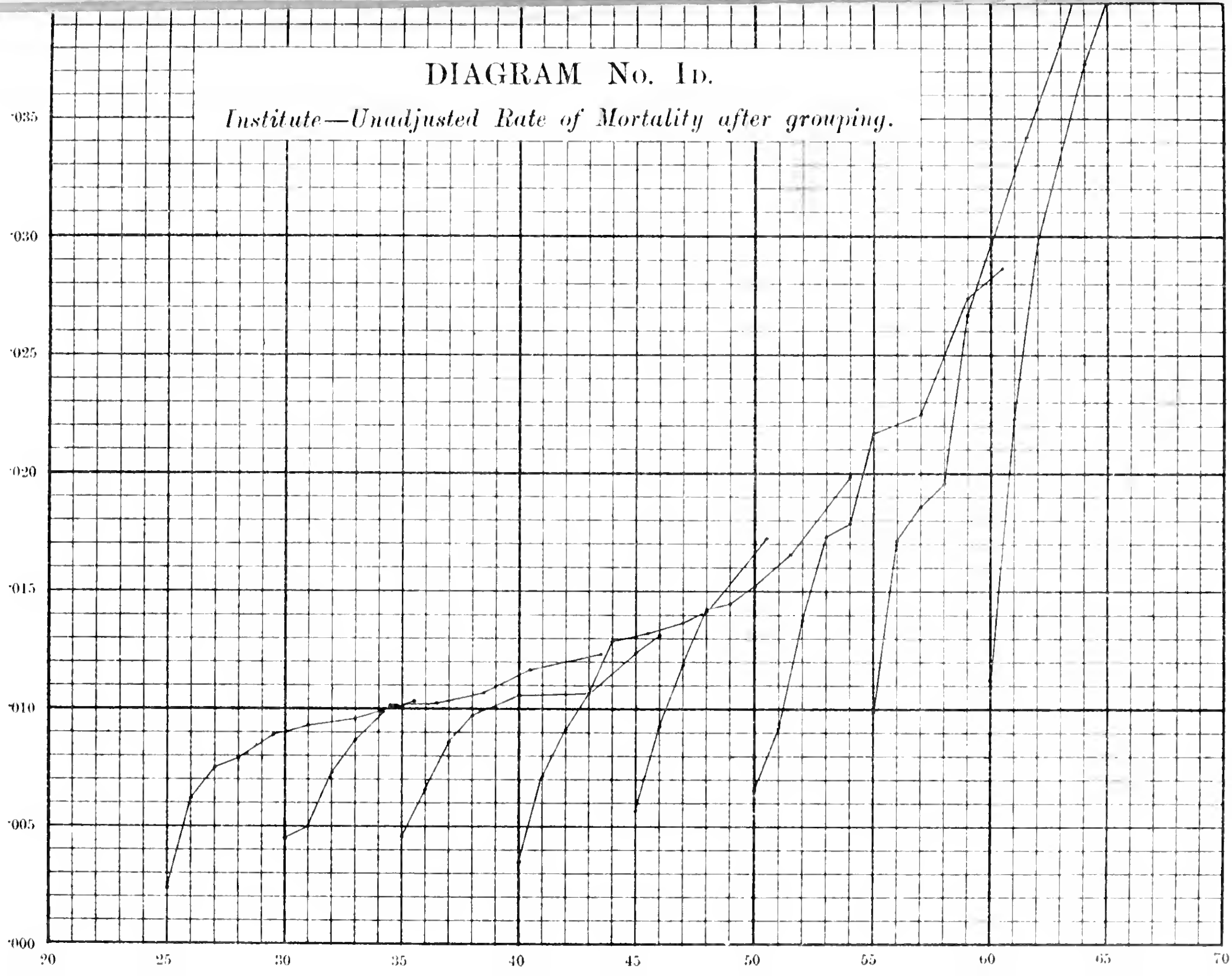


DIAGRAM No. 1E.

New York Mutual—Unadjusted Rate of Mortality after grouping.

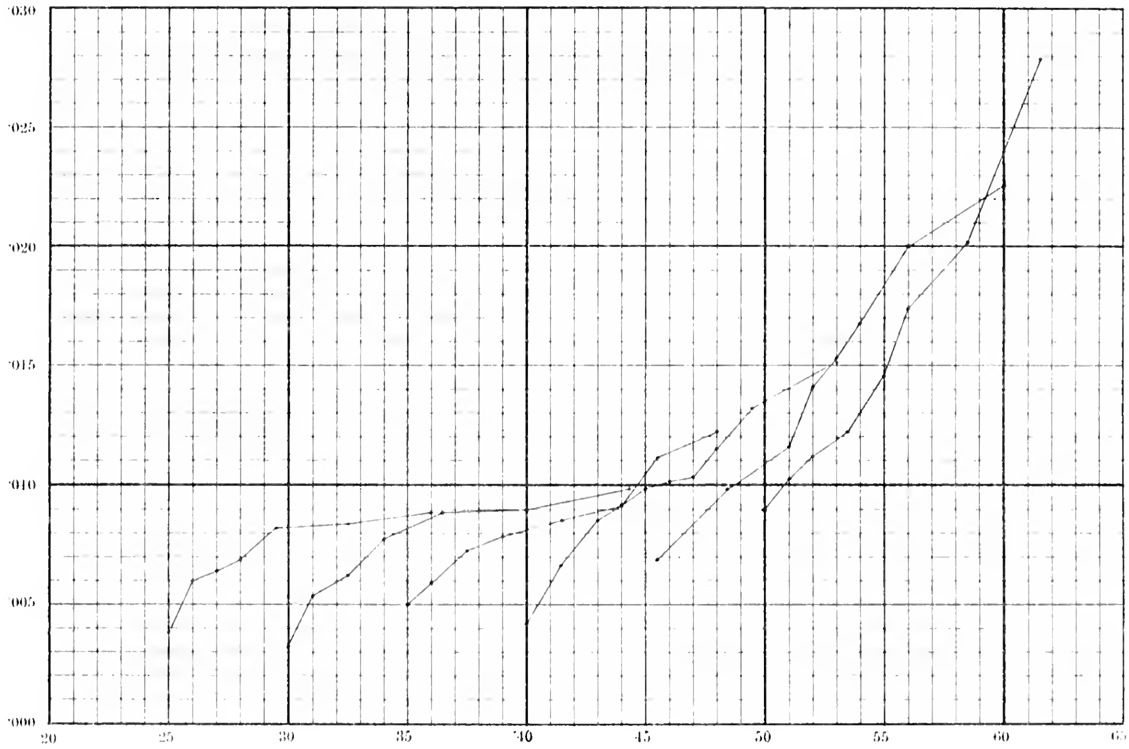
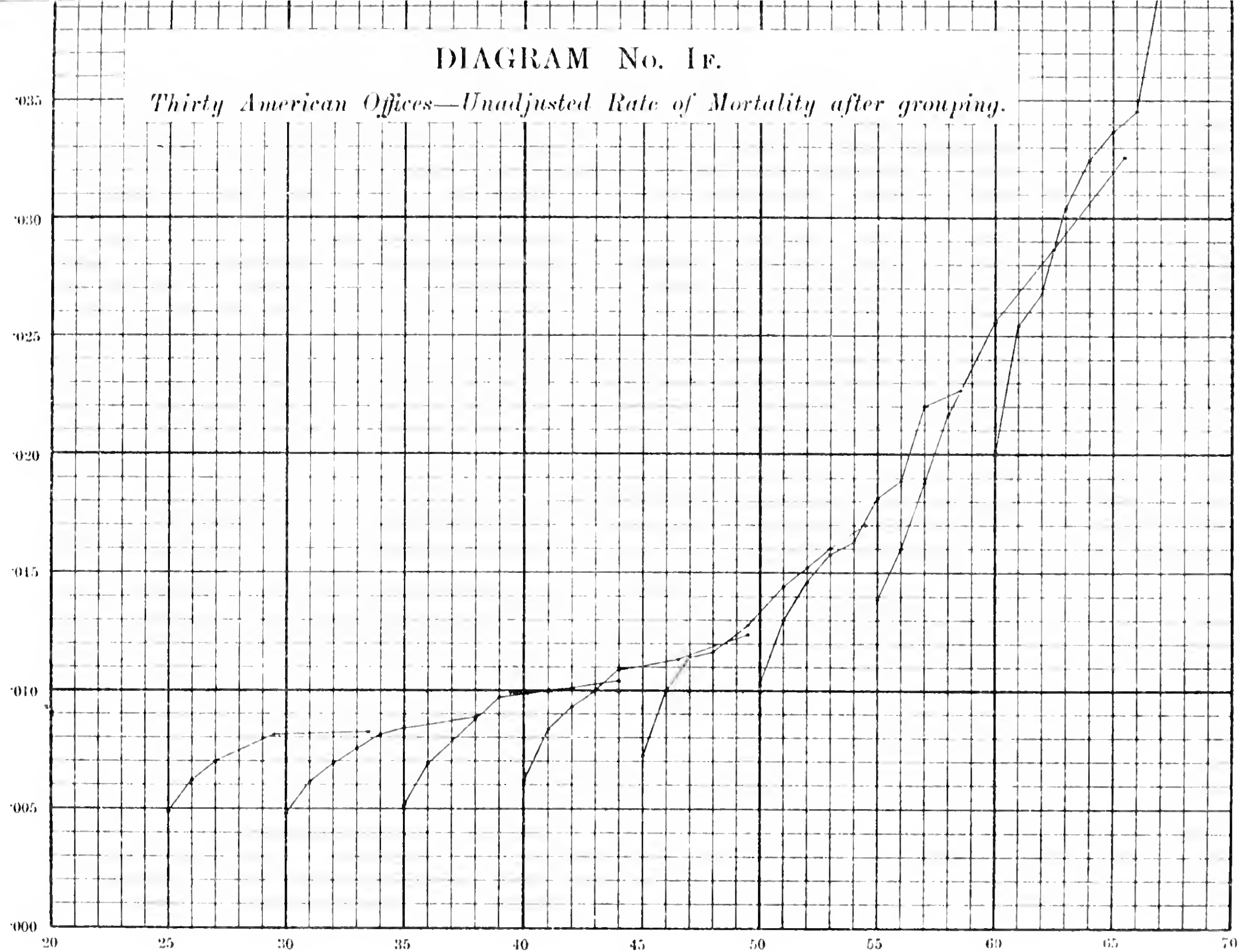


DIAGRAM No. 1F.

Thirty American Offices—Unadjusted Rate of Mortality after grouping.



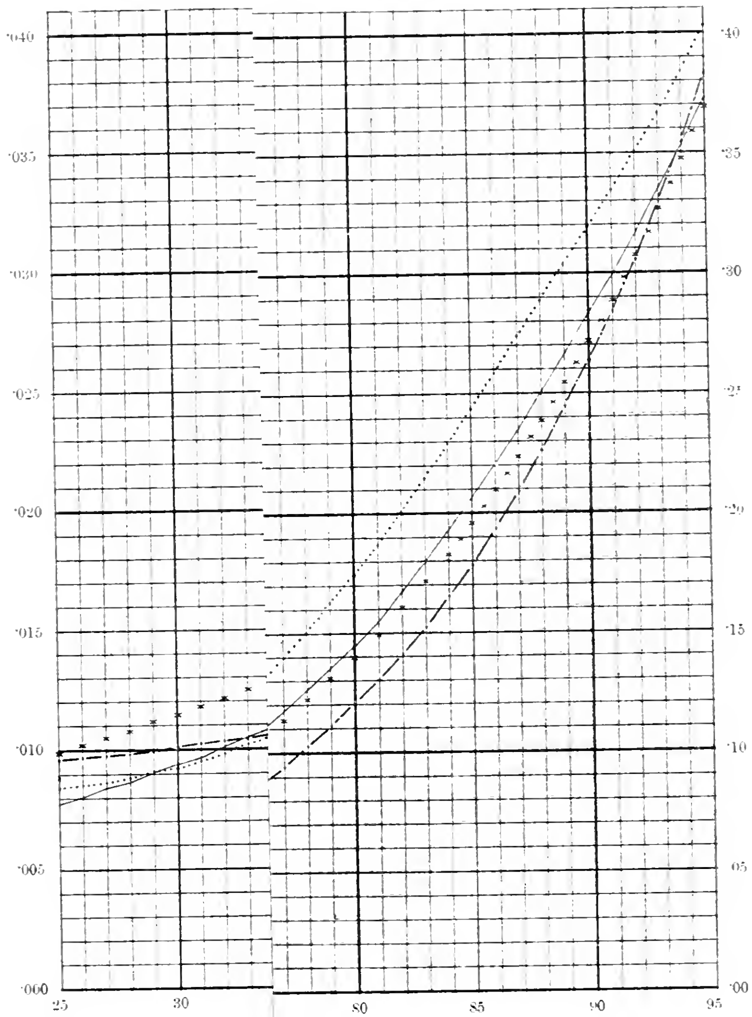
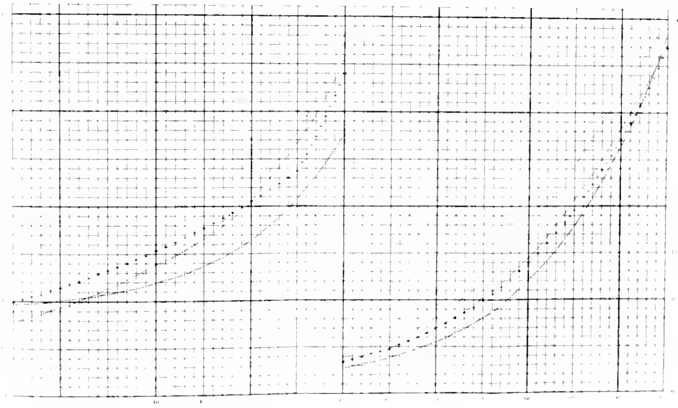

·040
·035
·030
·025
·020
·015
·010
·005
·000
·40
·35
·30
·20
·15
·10
·00
25
30
80
85
90
95

DIAGRAM No. 2

Rates of Mortality in | England
Scotland
America
Germany

The abscissa represents the age, and the ordinate the probability of dying in a year



he doubted if that objection could be sustained. As to the chief result of Mr. Chatham's investigation, which, if he were right or not right, he was justified in calling startling, he would maintain an attitude of reserve. Mr. Chatham was conscious that he made a large demand upon them in submitting for acceptance his conclusion that withdrawals did not affect adversely, and might even affect beneficially, the rate of mortality among the remaining lives, in disregard not only of the personal authority of Mr. Higham, Mr. Makeham, Mr. Sprague, and Mr. King, but of that great compilation, the New York Mutual Table. He doubted whether the whole truth in this matter was before them, and would like to know the opinion of Mr. McClintock, who had, of course, an especial familiarity with that Table. In the fact that the arguments and deductions under heading 9 (owing to the Institute Tables having been formed so that they would not lend themselves to the investigation) had to be drawn from foreign sources—from the Gotha office—all would see an urgent reason for that new mortality investigation which so many desired.

Mr. T. B. Sprague said he owed so much to the Institute of Actuaries that he was anxious to do what he could to serve it. With that object, he asked the permission of the Council to offer a prize for the best essay upon the subject which headed Mr. Chatham's paper, and it was a great gratification to find the offer had the result of producing three good essays. It was a pleasant surprise to find that his assistant, Mr. Chatham, was the successful candidate. He would like to make a few suggestions regarding the essay. Mr. Chatham mentioned the system adopted by the American actuaries, and called by Mr. Meech the method of final series, but did not offer his own opinion of it, which he might very suitably have done. He (Mr. Chatham) had remarked that he was somewhat hampered by the restriction of the investigation to 10 years after selection, but spoke of having obtained certain valuable results from the experience of subsequent years. He hoped Mr. Chatham would see his way at some future time to give them the benefit of those results. His (Mr. Sprague's) opinions on various points discussed in the essay have been quoted at considerable length by Mr. Chatham, and he had no wish to alter anything in the quotations given. Although there was a great deal in this essay that was old, it was useful to have it all brought together. Another useful thing was the comparison of the different ways in which mortality experiences have been taken out and could be taken out. He protested against the phrase "current age" as incorrect. It was intended to mean "age next birthday." Of course there was no similar objection to the phrase "current year of age." Lastly, with regard to the principal conclusion of the paper—that the withdrawals did not influence the mortality—he noted with satisfaction that Mr. Chatham said his investigation was not conclusive. He wished to draw special attention to the undeniable fact which appeared in the new tables now examined by Mr. Chatham, as well as in those formerly examined, namely, that for any age at the time of observation (or age attained) the rate of mortality first increased up to a maximum with the duration of the assurance, and then diminished. That

undeniable fact could only be explained and accounted for by the lives which withdrew being on the whole better than those which remained. They did not yet know what would be the mortality amongst insured lives if there were no withdrawals—that was an object he did not despair of seeing accomplished. If they were satisfied that they had the correct rate of mortality for healthy lives, not subject to withdrawal, they might calculate premiums upon that basis, and thus be able to charge lower premiums than at present. The necessary reserves and the surrender-values for policies on healthy lives would be smaller than at present, but larger reserves would have to be made for policies on bad lives, and if, in consequence of a large number of surrenders and lapses occurring, the lives insured in an office contained an unusually large proportion of bad lives, a larger reserve would have to be made for its liabilities than that given by any method now in use. The reserve would, in fact, depend partly on the discontinuances, and an approximation to the truth would be obtained by reserving at each valuation all the profit supposed to have been made by lapses and surrenders during, say, the previous five years.

Mr. George King said that, as regards the materials for a mortality experience, Mr. Chatham had decided in favour of lives rather than of policies, and if they required what he might call a mixed mortality table—a mortality table which ignored the ages at entry and simply brought all lives of equal present age together—he thought Mr. Chatham was right. His own opinion was that, in the future, select mortality tables would be the order of the day. They would want more accurate estimates of human mortality than the ordinary mixed tables such as the H^M mortality tables supplied, and would adopt in some form tables of the kind supplied by Mr. Sprague in his "Select Tables." That being so, they should take policies and not lives as the basis of their investigation, because when a fresh policy was taken out a new medical examination was undergone, and the life became once more a select one, and must be treated in that way under a new series. There was only one exception to this rule, namely, where simultaneous policies were effected, such as when re-assurances were effected by a company. As to the question of amounts, he thought discrimination would have to be exercised. It was shown by the experience of one English company that amounts gave a more favourable mortality than lives, whereas in an American company it was the reverse. If they were to investigate the mortality experience of a single company having large policies effected for family purposes, marriage settlements, and such like, they would have a light mortality, whereas in the case of a company having large policies under loan transactions they would have a heavy mortality. In future mortality experiences they must neglect calendar years and adopt policy years, and devise some means of doing that with great accuracy. Mr. Chatham seemed to believe that his (Mr. King's) method dealt with calendar years, but that was a mistake. It was quite true every policy was not followed out from the moment of its creation to its termination, but the averages were so taken that they really got policy years. The average was so arranged as to make the policy years harmonize with the financial years of the company. He was glad to have an opportunity of replying to a

criticism Mr. Sprague passed a good many years ago upon one of his papers. He (Mr. King) had pointed out that the change in the annuity-value was very insignificant if they varied the method of dealing with the year 0, and Mr. Sprague said it made a great deal of difference in the policy-value in the first year, but he (Mr. King) maintained that the valuation of policies in the first year was of little consequence in the reserves of an office, and he still submitted that he was right in saying it was not of much consequence financially how year 0 was treated. He now came to what was really the pith of the paper, where Mr. Chatham had prided himself on having discovered a "nugget." If this matter were closely looked into he thought that would be found to be fallacious. He would point out, first, as a cause of disturbance, the different average age of the companies. The Scottish companies included in the experience had an average age of $39\frac{3}{4}$ years, while that of the English companies was $51\frac{1}{4}$ years. The experience of the English companies was drawn from almost a generation preceding that of the Scottish, and it was difficult to say what the effect of that was either upon mortality or selection. Mr. Chatham had quoted from a speech by Mr. Bailey to the effect that in the ancient days of the London Assurance Corporation, and other companies of the same class, included in the English experience, there were a great many term policies. Now, these went to swell the discontinuances in the early years of insurance, but produced no effect upon the mortality, and, therefore, such policies should be entirely neglected in investigating selection. Everyone knew of many cases where a policy had been kept up by friends for the benefit of a policyholder in bad health, and there was a counterbalance in selection where the assured became of bad habits, and was not able to keep up his policy. At the present time they had not the means of investigating those two conflicting forces. Mr. Chatham had suggested two ways of doing so, but there was a third, namely, by a company investigating the different classes of policies on its own books, and seeing what the effect of selection was amongst those classes. One effort in that direction had been made by Mr. G. F. Hardy, who analyzed the mortality experience of the company with which he used to be connected, and showed that those policies where bonus was taken in cash gave a better rate of mortality than where the bonus was taken in reversion. That afforded distinct evidence of selection by the manner in which bonus was taken. Unfortunately that company was not quite suitable for the purpose, because it allowed the option only in the case of the first bonus. At the end of his paper, Mr. Chatham threw out hints as to the best methods to be pursued in taking out a new experience. He (Mr. King) hoped an opportunity would be afforded of discussing the question fully, but he would mention that he differed very considerably from Mr. Chatham's views. Mr. Chatham spoke of the average moment of birth, and he (Mr. King) had occasion recently to investigate that point. He examined the registrar's report of the statistics of Scotland for the year 1888. There the births were given for each calendar month, and by assuming the lives were born in the middle of each month he found the average moment of the birth of the whole population was almost exactly

10 o'clock at night on the 30th June, so that they were not far wrong in assuming that the average was midnight between the 30th June and the 1st July.

Mr. P. Newman said that the Gotha office had done for Germany what had to be done in the future for England. A good standard for calculating premiums and reserves, and particularly for the daily work of the offices where options were allowed, was needed. With regard to the effect of withdrawals on the mortality experience, he should like to know whether Mr. Chatham's contention was that the information was insufficient, or whether he had satisfied himself that the rate of discontinuance did not affect the rate of mortality. His own opinion was that Mr. Chatham's "nugget" was not of any value, but that, speaking generally, only the healthy lives withdrew, and that the mortality of the remaining lives must be affected in consequence.

Mr. G. H. Ryan said he would first refer to one of the main divisions of the paper, namely, that of mortality experience, a question which at the present time was a burning one. Mr. King had given the meeting some reasons for thinking that in some cases the observations should be based upon policies, but he (Mr. Ryan) had come to the conclusion that it was far better to base their experience on lives. In dealing with amounts it was true they dealt with the financial element rather than the mortality element, but when they considered that mortality altered but slowly, if at all, whereas the character of insurance business was apt to experience great changes, they would recognize in that a strong reason for preferring experience based on lives to experience based on amounts. Mr. King had pointed out that in cases of re-assurance the same policies appeared in several companies, and it was also not an unusual thing for a man to propose at the same time to three or four companies. In that way, they would get an unavoidable and perhaps considerable re-duplication, and this was his main objection to using policies instead of lives. He had already, in a short note in the *Journal* (*J.I.A.*, xxvi, 256), stated why in his opinion policy years should be preferred to calendar years, and he thought that not only could the rate of mortality in the first year but also that in all subsequent true years of insurance be ascertained with greater accuracy than under the system of calendar years, which really gave the rate for years 0 to $\frac{1}{2}$, $\frac{1}{2}$ to $1\frac{1}{2}$, and so on. As regards the method of deducing the experience from the raw material, he raised the objection that in Mr. Chatham's new formula, and also in the similar formulas previously given by Mr. G. F. Hardy and Mr. Rothery and by Mr. King, the unfortunate quantity x seemed to be a variety of things at the same time. Sometimes it was the age next birthday, sometimes age last birthday, and sometimes the mean age. He would make a passing observation on a singular omission which Mr. Chatham had made. He (Mr. Chatham) gave the three principal methods of graduation as Woolhouse's, Gompertz's and the graphic systems. But why did he leave out Mr. Makeham's method, a method to which he himself referred in the closing part of his paper as the one which might be used to adjust the final results of a mortality experience? If they required to test exactly the mortality which a certain body of lives had experienced in the past, and to

compare that statistically with other bodies of facts, then the graphic method would perhaps show the law which those facts held within themselves more truly than any other method; but when, for the calculation of premiums and reserves, they wished to graduate rough results, he would suggest that Mr. Makeham's method of graduation, or some other method which relieved the figures of their irregularities, should be adopted. He contended that in getting out a mortality experience they should throw aside not only rated-up cases, but all cases where any extra premium whatever was charged. Referring to the final conclusions of the paper, he remarked that Mr. Chatham was disinclined to think that the rate of discontinuance exercised any injurious effect upon mortality during the first ten years of insurance, and, indeed, his figures went to show that the rate of discontinuance favourably affected the rate of mortality. In considering those two views—the opinions of Mr. Higham, Mr. Makeham, Mr. Sprague and others, on the one hand, and of Mr. Chatham on the other, he felt inclined to say, appropriating a famous phrase, "I am on the side of the angels." He unhesitatingly believed that the rate of discontinuance did adversely affect the rate of mortality. The reasons which went to render Mr. Chatham's comparisons of little real weight were those to which Mr. King had called attention—chiefly, the inclusion of term policies in the facts analyzed. If they wished to obtain a true idea of the rate of discontinuance, they must distinguish between withdrawals in which some option was exercisable, and withdrawals in which no option was exercisable. Following out that idea they should rigidly exclude term policies and endowment-assurance policies, because a fictitious weight would be given to withdrawals if they included either one or the other——[The PRESIDENT: You mean such of them as have run their course.]—Mr. RYAN: For such purposes it would be better to exclude them altogether, and to confine the experience entirely to whole-life insurances. Finally, he would point out, in connection with the experience of the 10 Scotch offices included in the Institute results, that there were really 11 Scotch offices—the Northern, which most of them looked upon as a Scotch office, being ranked among English offices.

Mr. ACKLAND thought that the wisest course in future investigations would be to base their experience upon lives, and although the remarks of Mr. King as to the desirability, in some respects, of taking policies when they were investigating a select rate of mortality, had considerable weight, yet he thought the difficulties in practice would be very great, and he should deplore a return to the adoption of policies as a basis, as in the case of the Seventeen Offices' Table. It was to him a matter of regret that in the preparation of the H^M Table, those who conducted that work should have proceeded on the very defective lines of investigating by calendar years. Mr. Sprague had shown (*J.I.A.*, xxi, 233) how policy years might, by an ingenious method of interpolation, be deduced from calendar years, and he had obtained results which were generally regarded as very valuable, but Mr. Chatham appeared to be unable to adopt those results in his paper, and had restricted his enquiries to those investigations which were based upon policy years. The present practice of taking lives at the next following quarterly age might materially affect future

investigations, and less difficulty would arise in regard to the assumptions made as to age at entry. With regard to the methods of graduation, where they were investigating a select rate of mortality which tended to increase with the policy duration, such methods as Finlaison's (or those suggested by Mr. J. A. Higham) were not suitable, and it might be that the graphic method was the only one which would give satisfactory results. Having had some little experience in the graphic method of graduation, he himself hoped that ultimately some method would be found where so much would not be left to the skill of the individual computer. Mr. Chatham had referred to the method he (Mr. Ackland) suggested of dealing with the initial ages of the mortality table where Woolhouse's method appeared to fail, and he rightly spoke of that method as somewhat empirical. At the same time the method had at least this advantage, that it depended upon the original facts at those entry ages, whereas the method proposed by Mr. Woolhouse in this respect was practically independent of the original facts at those ages. He would endorse what had been said as to the advisability of separating endowment assurances from whole-life assurances, as it was generally recognized that the mortality differed, being more favourable in the case of endowment assurances. Temporary assurances, or, generally speaking, those under which a minimum premium was payable, usually tended to show a relatively higher mortality. It was a little disheartening for them to have to go to Germany and the Gotha company to find a basis upon which to investigate the rate of mortality during the first ten years of assurance. In comparing the Scotch companies with the English, they would have to consider the relative mortality of Scotland and England, and the differences between the experiences. The method adopted in some American experiences of deducing the rate of discontinuance upon the assumption that those who died during the year were exposed to risk for the full year, appeared to him to be radically wrong. Speaking generally on the question of discontinuances and their effect on the rate of mortality, he was reluctant to accept the conclusions at which Mr. Chatham had arrived. He had found, in dealing with a large body of facts extending over many years' experience, that the effect of a high rate of secession was practically to eliminate the effect of medical selection, and bring out at the outset a rate of mortality closely approximating to that of the $H^{M(5)}$ Table. That was a very significant conclusion, and one which seemed to indicate that the discontinuances had an unfavourable effect on the rate of mortality.

Mr. G. F. Hardy thought it would be interesting if Mr. Chatham would say something more as to why he had rejected the rates of mortality for the early years of assurance deduced from the Institute experience by Mr. Sprague. He could quite conceive that the difficulty of determining the exact fraction of a year represented by "year 0" did vitiate the result which had been obtained for that year, but he did not see why that error should continue to affect the results for the remaining years of insurance to any appreciable extent. As to the conclusion to which Mr. Chatham had arrived with respect to the effect upon the mortality of the withdrawals, he (Mr. Hardy) did not think that conclusion followed upon Mr. Chatham's figures,

comparing the Twenty Offices' and the Scotch experience. On any reasonable assumption as to the class of lives withdrawing, the difference in the rates of mortality in the two experiences ought to be very small. The additional lives withdrawing in the Twenty Offices' experience represented about 6 per-cent of the lives at risk at the close of year 10; and if the rates of mortality among the withdrawals were so much as 33 per-cent less than among the remaining lives, the value of q_x would only be increased by about 2 per-cent of its amount by the extra withdrawal rate. The effect of the whole of the withdrawals, on the same assumption, would be, however, to increase the value of q_x by 10 per-cent. If the relative rates of mortality for the Scotch and Twenty Offices were compared, it would be found that the Institute was decidedly lower in years 0 to 2 of assurance; in years 3 to 5 there was less difference; while for years 6 to 10 they were almost identical. This equalizing of the rates he thought due to the heavier rate of withdrawal in the Institute experience. He believed the materials existed for investigating this matter more completely. He congratulated Mr. Chatham on having brought together such a valuable mass of materials, and on having arranged them with so much care and effect.

Mr. A. W. Tarn suggested that an index of the authorities quoted should be given at the end of the paper.

Mr. Chatham, in reply, said that he noted with satisfaction that not one voice had been raised in defence of calendar years. As regards the method of "final series", he would say that it made no difference whatever when those who entered at any given age were kept separate, but it was altogether different when the mortality table was formed in the usual way by adding altogether all those of the same age, irrespective of the length of time for which they had been insured. The rate of mortality for those of the same age, but who had been insured 1, 2, 3, &c., years, attained a maximum and then diminished; and therefore if the average duration of the policies was short the rate would be very light. That was the case with the American experience. He hoped to comply with Mr. Sprague's request to publish the results obtained from lives among whom there were no withdrawals, taking up the subject of discontinuance at the point where it was of necessity left in the paper. So far as the first ten years of insurance were concerned, his conclusion was clearly stated, namely, that the rate of mortality was not injuriously affected by the withdrawals. For the first time in the history of life assurance the effect of withdrawals had been brought to the test of actual figures. Mr. King had referred to the average ages of the English and Scotch offices included in the Institute experience. Only one very old office, the London Assurance, contributed its experience, and assuming the observations to have been continued to 1863, and taking the differences between that year and the dates of establishment given on page 13 of the Institute *Mortality Experience*, the ages of the offices were as under:

Scotch	.	48, 40, 40, 39, 38, 32, 26, 25, 25, 22 years.
English	.	143, 66, 49, 42, 39, 39, 28, 27, 19, 18 „

The average ages of the offices, omitting the London Assurance, were as follows:

19 Offices	35 years.
10 Scotch	34 „
9 English	36 „

Even with the London Assurance included, the average age of the 20 offices was only 40 years, and it was therefore a mistake to suppose that the Institute experience was of much larger average duration than was the Scotch portion of it. Mr. King pointed to the results obtained by Mr. G. F. Hardy as to the options exercised in the case of bonuses; but he could not see that this would help in determining the effect of the option exercised by withdrawals. He was desired by Mr. R. P. Hardy to say that during his great experience of friendly societies he had found that where the withdrawals were heaviest the mortality was lightest. That was much more to the point, because in collecting societies the same option was exercised, although in a lower stratum of society. Mr. G. F. Hardy said that according to his last table the Institute lives were better at the outset than the 10 Scotch, and gradually came up to them, and from this he reasoned that withdrawals had an injurious effect on the rate of mortality. He thought he had clearly pointed out in his paper that no stress whatever was to be laid on year "0"; and that this would have prevented Mr. Hardy from falling into this mistake. He found that five out of the 10 Scotch offices closed their books at various periods before 30 June, whereas only one of the English offices did; also, that only three of the Scotch offices closed their books on 31 December, whereas five of the English did. The exposed to risk in the Scotch offices therefore would be under-estimated, and therefore the percentage of the actual deaths to the expected would be too large in the Scotch and too small in the Institute. They must, therefore, leave year "0" out of account altogether; and doing so, he drew exactly the opposite conclusion to Mr. Hardy's from the figures, considering that the Institute lives were slightly worse at the outset, but that the difference was so small as to be hardly appreciable. Mr. Ryan wished the subscript x to be defined. He would find it accurately defined throughout his (Mr. Chatham's) paper, which was more than could be said of his own paper on formulas for exposed to risk. He thought that Mr. Sprague's suggestion of "age next birthday" instead of "current age" should be adopted. He had not discussed Mr. Makeham's method in his paper, as he looked upon it as a development of Gompertz's. He recognized the great advantages which Messrs. King and Hardy's method of applying Mr. Makeham's method possessed when complicated benefits had to be calculated, and if the mortality table was formed in the usual way, he should be inclined to adopt it; but not if selection had to be taken into account—at any rate not until he had first graduated the table by the graphic method.

[Mr. Woolhouse's reply to Mr. Sprague's note, on p. 59 *ante*, has been received, and will appear in the July number of the *Journal*.—ED. *J.I.A.*]

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On the Practice and Powers of Assurance Companies in regard to the Investment of their Life Assurance Funds. By A. G. MACKENZIE, F.I.A., F.F.A.

[Read before the Institute, 26 January 1891.]

THE subject of Life Assurance Investments does not seem to have been discussed for a long time at the meetings of the Institute. Two valuable and instructive papers, bearing the honoured names of Mr. Samuel Brown and Mr. Arthur H. Bailey, were read to the Institute respectively on 25 January 1858 (*J.I.A.*, vii, 241) and 24 February 1862 (*J.I.A.*, x, 142), but they appear to be the only contributions of importance, dealing with the question in its entirety, to be found in the *Journal*. Notwithstanding the lapse of time which has taken place since the latter date, the principles of investment as defined by these two distinguished authorities are still very generally applicable to present needs; and the articles themselves have lost none of their charm for the student of life assurance finance, characterized as they are by soundness of judgment and by felicity of expression.

It may here be convenient to recapitulate in a condensed form the five cardinal canons of investments which were enunciated by

Mr. Bailey. They were (1) the primary necessity of securing the safety of the capital, (2) that the highest practicable rate of interest consistent with such security should be obtained, (3) that the smaller proportion of the funds be held in readily-convertible securities to provide for current demands, (4) that the larger proportion may safely be invested in securities not readily convertible, and (5) that as far as practicable the capital should be employed to aid the life assurance business. That these principles have met with general acceptance is shown by the extent to which they have governed the practice of assurance companies in the past, although the manner in which they have been acted upon has necessarily varied with altering circumstances from time to time. Thirty years of the busiest and most changeful century which the world perhaps has known have had their effect, and the changes of the last few years seem the greatest and the most rapid of all. The value of broad acres in England has depreciated; the Three per Cents. have lost a twelfth of their sweetness and all of their simplicity; Parliament has repeatedly enlarged the field of investments open to trustees; and the immense demand for and limited supply of securities formerly thought most suitable for insurance companies have had the effect of diminishing to a perilous extent their earning-powers, and has made some of them no longer desirable for our purposes. The keen competition for new proposals in *fin de siècle* insurance has, moreover, led to an increase in working expenses, which, showing as little abatement at present as the thirst of the policyholder for large bonuses and special advantages, makes it all the more important that a satisfactory rate of interest should be maintained. What wonder, then, that many companies have noted the example of a paternal Government, and that while some are seeking to extend their powers of investment, others are widening their practice in regard to powers which they already possess. To the institutions with which most of us are officially connected this question of the rate of interest is a vital one; and it is only in the absence of a senior member of the Institute taking the initiative, and with the hope of prefacing an interesting discussion in which many members of far greater experience than I can claim may join, and from which I expect to learn much more than I can communicate, that I have had the temerity to submit the present paper to the notice of the Council.

In accepting the responsibility, I must ask for indulgence. The subject is a prosaic one, I shall have to go over much beaten

ground, and I fear I shall not be able to avoid using some of those "platitudes" to which the President referred in the brilliant address which opened the present session, though I hope they may not be considered "perfunctory." The censure passed on a Scotch preacher that he never told his audience anything they did not know already may, I am afraid, be applicable to much of what I have to say. I do not forget that many of those whom I am addressing are daily dealing in a practical form with my subject, and I cannot hope to place anything of a particularly novel character before them. I have attempted, however, to glean information from many sources; and I trust that the "unconsidered trifles" which I have managed to pick up in the form of a few statistics may at least clear the ground for the discussion to follow.

The rate of interest earned on life assurance funds during the last generation, so far as it is possible to ascertain it, shows a very small annual decrease, the regularity of which, however, leads to a considerable diminution over a series of years. In his very interesting paper on "The Progress of Life Assurance Business in the United Kingdom during the last Fifty Years", recently printed in the *Journal* (*J.I.A.*, xxviii, 442), Mr. David Deuchar shows that, according to the answers returned by the companies to Question 9 of the 6th Schedule of the Life Assurance Companies Act of 1870, the average rates of interest returned by the companies were as follows, in each year: from 1870 to 1875 inclusive, 4·5 per-cent; from 1876 to 1880 inclusive, 4·4 per-cent; in 1881, 4·3 per-cent; in 1882, 4·4 per-cent; in 1883 and 1884, 4·3 per-cent; and in 1885, 4·2 per-cent. I have extracted the figures, so far as given in the Board of Trade Returns up to the present time, for the years 1886, 1887, and 1888, and find that the average rate of interest returned in these years was 4·22, 4·23, and 4·2 respectively.

I am not satisfied, however, that the average rate of interest, as deduced from these answers, gives us the best idea which we can obtain of actual results.

Consider the different ways in which the 9th Question of the 6th Schedule of the Act may be, and is, answered.

A statement is requested of the average rate of interest at which the life assurance fund of the company was invested at the close of each year during the period since the last investigation.

This has been generally, and I think rightly, construed to mean the average rate of interest which the fund at the close of

the year is expected to yield. If so, it is not an entirely satisfactory basis of comparison, since earnings, as most of us are painfully aware, may differ very considerably from expectations; and something is to be said in favour of those offices who hold that the best compliance with the spirit of the question is made by showing, not an estimate, but the actual rate of interest realized on the fund during the year which has closed. But, even supposing that the former and more general construction were acted upon by all offices, there would still be abundant room for divergences in the answers given.

For instance, the rate of interest assumed to be earned on reversions and reversionary interests may be calculated in any of the various ways pointed out by Mr. Sprague in his masterly *Treatise on Life Assurance Accounts*. These assets may be assumed to produce no immediate income at all, or to produce a certain fixed rate which is capitalized, or the rate of interest may be held to be the difference in their actuarial value from year to year. Again, and as Mr. Sprague also points out, the practice of companies owning their offices is different. Some of them credit themselves with interest as a set-off against the amount of rent which they would otherwise have to pay, and which they include on the other side in their expenses of management; while others ignore the item in their accounts altogether, making them up as if they were living rent-free. But the most frequent difference in practice appears to be in regard to outstanding premiums and interest, cash in hand and on current account, office furniture, and agents' and other miscellaneous balances. The *spirit* of the Act seems to require a statement of the return on the full amount of the life assurance funds whether bearing interest or unproductive, but the *letter* has been sometimes interpreted to allow a return only upon the amount which is actually invested. Then there is an additional difference of treatment in regard to income tax—some offices deducting it from the rate of interest quoted, and others making no allowance for this impost.

Taking only the last two divergences, let us see their effect. Suppose an office to have assets to the extent of £1,000,000; to be earning an average of $4\frac{1}{4}$ per-cent, subject to deduction of income tax, on its invested assets of £950,000; and to have thus an uninvested balance of £50,000. It may firstly return its average rate at £4. 5*s*., ignoring the tax, perhaps because it does not think the deduction is called for in the question; perhaps

because the rate of the income tax for the next year cannot be known until the following March or April, while the tax itself may be altogether repealed, of which there was a strong probability in 1874, by a much-to-be-desired Chancellor of the Exchequer. Or it may return its interest less the current rate of income tax, which would in an answer given at the present time reduce the quoted rate from £4. 5*s.* to £4. 2*s.* 10*d.* Or it may take the interest expected to be received during the year on its £950,000 of invested assets, that is £40,375, and thus show an average rate of £4. 0*s.* 9*d.* on its whole life assurance fund. Or it may even be courageous enough to deduct the income tax from the latter rate and give its return, as I think it ought to be shown, as £3. 18*s.* 9*d.* The rates which may be quoted in this hypothetical case, which is not at all a strained one, may thus differ by 6*s.* 3*d.*, or $\frac{5}{16}$ per-cent.

While there is no rule laid down there will no doubt be always a temptation to an actuary, for even actuaries are not insensible to the value of that present-day commercial factor called "appearances", to adopt the method which shows the highest rate of interest, as an evidence of the prosperity of his company and of the skill of his directors in investment. I am glad to know that this is often resisted, but the examination of a great many returns has led me to the belief that it is not unfrequently yielded to; and the necessity seems to arise in the interest of fair comparison, and in order that good men should not be tempted, that the meaning of the question should be authoritatively construed, so that all answers thereto may at least be framed upon a common and clearly-defined basis.

But even were this done the average of the separate average rates returned by the companies, upon which I understand that Mr. Deuchar's figures are based, may well differ somewhat from the average rate which is being earned upon the combined funds of all the companies, and the funds of offices differ so much in their proportions, that a slight deviation from the average is, indeed, to be expected in the general result. This being so, I have thought it well to make an investigation in a different way. In the endeavour to arrive at a more correct idea as to what rate of interest, after deduction of income tax, has lately been earned on the aggregate funds of all companies, I thought at first of taking, subject to certain necessary adjustments, the mean amount of the funds appearing in the Revenue Accounts published in the Board of Trade Returns in each year from 1883 to 1890 inclusive, and,

for comparison, the mean amount in Mr. Sprague's Summary of the Returns in 1872, that is one half of the total funds at the beginning and end of the years, and of calculating the percentage borne thereto by the net interest received. Mr. G. H. Ryan has recently, however, very kindly called my attention to the report of a paper by Mr. G. F. Hardy on "Compound Interest Tables and Formulas", read before the Actuarial Society of Edinburgh on 4 December 1890, in which Mr. Hardy points out, that whereas a calculation upon the basis which I have described will show the instantaneous rate or force of interest at which the funds have been invested, assuming increase or decrease in the funds to be uniform throughout the year, yet the effective rate of interest is more truly shown by the formula

$$\frac{2I}{A+B-I}$$

in which I represents the amount received for interest during the year, A the fund at the beginning of the year, and B the fund at the end of the year. In other words, half the interest received during the year is deducted from the mean amount of the funds, and the proportion borne thereto by the interest received is a sufficiently accurate representation of the effective rate for all practical purposes. I have therefore adopted this method, and the results should give, I think, a very near approximation to the rate of interest which the life assurance funds have actually earned, the amount of other funds appearing in the Revenue Accounts, and upon which it may be assumed interest is being earned at a somewhat similar rate, not exceeding 3 to 5 per-cent of the total. I have also made a separate calculation, after giving effect to the net profits and the funds arising from re-valuations, sales of securities, or other causes, and I may mention that each year under review shows a satisfactory net profit, no year showing a net loss, from these causes. The results are as follows:

Board of Trade Returns published in	Appertaining mainly to Year	Average Rate of Interest as ascertained by Averages in Returns	Actual Average Rate of Interest realized	
			Exclusive of Profits on Investments	Inclusive of Profits on Investments
1872	1870	4·5	4·51	4·53
1883	1881	4·3	4·29	4·43
1884	1882	4·4	4·28	4·4
1885	1883	4·3	4·24	4·32
1886	1884	4·3	4·24	4·35
1887	1885	4·2	4·23	4·31
1888	1886	4·2	4·13	4·15
1889	1887	4·2	4·14	4·2
1890	1888	4·2	4·13	4·23

In whichever way, therefore, we look at it, we see a considerable fall in the average rate of interest. Confining our attention to the fourth column, which seems to me to give the truest indication of the movement which has been going on, we see a fall in the rate of interest of 4*s*. 6*d*. per-cent from 1870 to 1881, and a further fall of 3*s*. from 1881 to 1888, the rate of interest having practically fallen in 18 years from $4\frac{1}{2}$ to $4\frac{1}{8}$ per-cent.

What does this difference of 7*s*. 6*d*. per-cent between the years 1870 and 1888 mean? It means a difference of about £600,000 per annum on the produce on the life assurance and annuity funds as ascertained by the Returns of 1890, a sum equal to over 4 per-cent of the total premium income of the companies, to over 80 per-cent of the total commission paid by them, to over 40 per-cent of their other expenses of management. It would probably cause a difference in reversionary bonus of 30*s*. per-cent on every participating policy in existence over a single quinquennium, while the difference upon a series of years between the results of investing the life assurance funds of the country at £4. 10*s*. and at £4. 1*s*. 8*d*. would be simply enormous.

The gravity of the results arising from a fall in the rate of interest is a sufficient apology for the remainder of my paper, which seeks to ascertain in what way the companies are endeavouring to meet and resist such a movement in the future, the decline which has already taken place having assuredly been less than it would otherwise have been on account of the steps already taken by many offices in this direction.

As practice is a safer guide than theory, I first propose to look

at the actual facts of the case, so far as they can be ascertained. The actuaries who wrote on this subject thirty years ago had not access to such statistics as are now open to the investigator. Mr. Samuel Brown, in his paper of 1858, tabulates the investments of four well-known offices, but in this case the material supplied by him was probably too restricted to give a very correct average. Now, the store-house of information supplied by the Board of Trade Returns is available for our purpose, and although there is a decided want in some instances for more definite classification, still a fairly reliable idea can be obtained from it of the manner in which the vast accumulations of our life assurance companies are invested.

I have already alluded to Mr. Sprague's summary of the Returns published in 1872. The first Blue Book in which an official summary is appended to the Returns is that published in 1882. The last Blue Book available for my purpose was issued in 1890. These will as nearly as possible show the positions of the companies at the end of the years 1870, 1880, and 1888, and the intervals between these periods are long enough to give indications of the changes in the manner of investing funds which have been taking place. The aggregate classification in the official summaries, though very comprehensive, as to which I shall make a few remarks later on, is as well arranged as the different groupings employed by the various offices seem to render possible, with the exceptions that the amount of mortgages on property out of the United Kingdom is not given separately and that the number of offices investing in each class might advantageously have been supplied. In Mr. Sprague's summary for 1872, loans on rates and rent-charges are included in mortgages, but I have gone through the separate Returns for that year and have extracted the former items wherever separately mentioned, in order to place them under a separate heading for purposes of comparison.

Taking the summaries, then, as given in the Returns published in 1872, 1882, and 1890, and having extracted the additional matter which I required from the Returns themselves, I find that, exclusive of agents' balances, outstanding premiums and interest, and certain small and quite unimportant balances, the assets of 110 offices according to the Returns of 1872, of 99 offices according to the Returns of 1882, and of 95 offices according to the Returns of 1890, were invested as follows:

Table for all Offices.

Class of Investment	Number of Offices investing in each Class, according to Returns of			Amount of Invested Assets, according to Returns of			Percentage of Assets invested in each Class, according to Returns of		
	1872	1882	1890	1872	1882	1890	1872	1882	1890
				£	£	£			
Mortgages within the United Kingdom	92	91	92	50,428,729	69,570,461	71,785,674	47·50	46·70	39·28
Mortgages out of the United Kingdom	20	17	21	729,816	1,216,618	6,292,371	·69	·82	3·44
Loans on Rates (and Rent-Charges)	46	58	71	10,606,507	19,354,056	21,701,631	9·99	12·99	11·89
				61,765,052	90,141,135	99,779,676	58·18	60·51	54·61
Loans on Companies' Policies	87	90	84	5,269,363	7,157,500	8,801,757	4·96	4·80	4·82
British Government Securities	73	61	54	8,214,559	4,886,923	5,526,669	7·74	3·28	3·02
Indian and Colonial Government Securities	55	70	78	5,255,753	7,058,688	12,527,740	4·95	4·74	6·85
Foreign Government Securities	25	36	36	1,228,494	4,112,007	3,588,218	1·16	2·96	1·96
Railway and other Debentures	72	71	75	10,112,626	10,569,636	16,809,493	9·81	7·09	9·20
Shares and Stocks	42	62	70	2,730,245	7,938,771	12,470,045	2·58	5·34	6·82
Land and House Property and Ground Rents	96	90	87	4,691,432	7,279,364	11,830,769	4·42	4·89	6·47
Life Interests and Reversions	47	49	52	1,689,322	2,687,290	3,268,031	1·59	1·80	1·79
Companies' own Shares	17	15	14	381,032	565,936	461,133	·36	·38	·25
Loans on Personal Security	70	56	56	1,806,924	1,803,641	1,208,959	1·70	1·21	·66
Cash, Deposits, &c.	...	...	...	2,709,013	4,473,945	6,479,706	2·55	3·00	3·55
Total	110	99	95	106,153,815	148,974,836	182,755,226	100·00	100·00	100·00

Though this table tells us a good deal it cannot be considered a very safe indicator of the actual investments of life assurance funds for a very evident reason. The shareholders' capital of all proprietary companies, with one exception—the fire and marine funds of companies who do that class of business in addition to life assurance, as well as the reserve and other funds—and the profit and loss and other balances of the companies are comprised in the assets given, and amount to from $\frac{1}{5}$ to $\frac{1}{6}$ of the entire total dealt with at each period. In order to meet the business purposes of fire and marine insurance companies, a different class of securities must often be desirable for the employment of funds appertaining to these branches, while shareholders' funds are generally invested on less rigid principles than are the accumulations forming the life funds. Although there are 25 companies out of the 95 under consideration in the 1890 Returns which transact other business than life assurance, only five out of these make separate statements in these Returns of the assets belonging to each department of their operations, and only five, again, of the proprietary offices make separate statements of the investments of their shareholders' and of their life assurance funds.

I find that there were 21 life assurance offices in existence in each of the years dealt with in the Returns for 1872, 1882, and 1890, in which no extraneous funds are shown. Twenty of these are mutual offices, the remaining one is a proprietary office, doing another class of business also, but showing the assets of its life assurance fund separately. Though the description, therefore, is not technically correct, I propose to refer to them, for convenience, throughout as the 21 mutual offices. I have thought that their returns would probably give a truer idea of the facts which it is desired to elicit, and I have therefore extracted the necessary statistics applicable to them alone, for each of the three years under observation.

The figures, then, for the 21 mutual offices, are as follows:

Table for 21 Mutual Offices.

Class of Investment	Number of Offices investing in each Class, according to Returns of			Amount of Invested Assets, according to Returns of			Percentage of Assets invested in each Class, according to Returns of		
	1872	1882	1890	1872	1882	1890	1872	1882	1890
				£	£	£			
Mortgages within the United Kingdom . .	21	21	21	16,163,862	24,851,250	28,144,737	47·93	52·99	46·40
Mortgages out of the United Kingdom . .	3	3	5	5,076	95,200	3,310,299	·02	·20	5·46
Loans on Rates (and Rent-Charges) . .	13	19	20	5,576,314	9,550,997	9,073,149	16·54	20·36	14·95
				21,745,252	34,497,447	40,525,185	64·49	73·55	66·81
Loans on Companies' Policies	21	21	21	2,596,866	3,338,873	4,252,124	7·70	7·12	7·01
British Government Securities	17	12	10	2,169,824	968,527	1,738,330	7·32	2·07	2·87
Indian and Colonial Government Securities .	13	11	15	1,143,407	1,042,140	3,161,065	3·39	2·22	5·21
Foreign Government Securities	4	2	5	38,622	15,734	42,107	·11	·03	·07
Railway and other Debentures	18	17	18	2,167,986	2,155,005	2,781,145	7·32	4·60	4·59
Shares and Stocks	7	9	11	624,159	1,369,170	2,039,384	1·85	2·92	3·36
Land and House Property and Ground Rents .	20	20	19	1,536,500	1,938,366	3,626,473	4·56	4·13	5·98
Life Interests and Reversions	11	11	12	312,374	738,508	969,363	·93	1·57	1·60
Loans on Personal Security	5	5	5	114,518	78,629	144,087	·34	·17	·24
Cash, Deposits, &c.	...	...	...	670,533	760,376	1,373,960	1·99	1·62	2·26
Total	...	...	...	33,720,014	46,903,075	60,653,223	100·00	100·00	100·00

It may here be convenient for me to say a word or two on the movements to be observed upon an examination of the results given in the two tables.

And first in regard to the mutual offices. The mortgages on property within the United Kingdom have increased very considerably in amount, yet the percentage invested in this way as compared with the total invested assets shows a slight decrease when 1890 is compared with 1872, notwithstanding the fact that there is an increase in percentage between 1872 and 1882. The same movement is observable in loans on rates and rent-charges, a heading which also includes municipal bonds of all descriptions. The greatest increase is in mortgages out of the United Kingdom, the percentage invested herein having risen from ·02 to 5·46, and the amount from £5,076 to £3,310,299. Loans on policies show a slight decrease in percentage. British government securities show a marked decline both in percentage and amount, though the lowest point touched in both is in 1882. Indian and colonial government securities show a marked increase, particularly when 1890 is compared with 1882. Debentures show a considerably decreased percentage in 1882 when compared with 1872, but the percentage in 1890 is much the same as in 1882. Shares have increased from 1·85 per-cent in 1872 to 2·92 per-cent in 1882, and to 3·36 per-cent in 1890; and in amount from £624,159 held by seven offices in 1872 to £2,039,384 held by 11 offices in 1890. Land and house property shows a large increase, apparently mainly due to the purchase of ground rents. Life interests and reversions show an increase in percentage from 1872 to 1882, and remain nearly stationary between 1882 and 1890. Foreign government securities and loans on personal security show a considerable decline in percentage since 1872, though the percentage in both cases is slightly higher in 1890 than it was in 1882. Cash, deposits, &c., show an increase in 1890 over 1872, though a decrease in percentage took place between 1872 and 1882.

Turning to the more imposing, but perhaps less instructive, figures for all the offices, the only items in which the movements are very similar to those in the mutual offices are loans on policies, British government securities, land and house property, and life interests and reversions. The decline in the percentage of mortgages within the United Kingdom, from 1882 to 1890, is more marked; the increase in mortgages out of the United Kingdom is scarcely so great; in loans on rates, &c., there is an increased

percentage over the whole period, there is a slight increase instead of a decrease in foreign government securities, probably explainable on account of companies doing fire business abroad having had to invest more largely therein: debentures show a slight increase in percentage in 1890 as compared with 1882; the increase in shares and stocks is greater, only 12 out of 110 offices investing therein in 1872 against 70 out of 95 offices in 1890: and cash and deposits, &c., show a larger proportionate increase and a steady increase at each period.

I now propose very briefly to take each heading in the Board of Trade Returns separately, and to make a few remarks on such facts in the comparisons as seem to me to call for notice, and also to try to ascertain what kinds of investment are likely to be available in the future if anything approaching to the present rate of interest is to be maintained. I shall assume that it is desirable in such a case to endeavour to obtain $4\frac{1}{4}$ per-cent or a little more, subject to deduction of income tax, upon the actually invested funds, so that a clear net return of at least 4 per-cent upon the total funds may be relied upon. I do not forget that the present rate of, say, $4\frac{1}{2}$ per-cent is to a great extent assisted by the fact of some of the assets standing in a good many balance sheets at less than their market value: a very conservative precaution, the result of which will be, however, that upon a re-valuation, unless changes in the investments are made, the rate of interest will of course decline still further, and it is also worthy of notice that the rate now prevailing is greatly influenced by that which is still obtained on loans, now in the course of gradual repayment, which were negociated on more favourable terms than are now prevailing.

Mortgages on Property within the United Kingdom.—For all the offices these investments were in 1872, say, $47\frac{1}{2}$ per-cent; in 1882, $46\frac{3}{4}$ per-cent; and in 1890, $39\frac{1}{4}$ per-cent. For the mutual offices they were in 1872, say 48 per-cent; in 1882, 53 per-cent; and in 1890, $46\frac{2}{5}$ per-cent of the total investments.

This is a very comprehensive and elastic heading. It includes not only mortgage on freehold, leasehold, and other descriptions of land and house property, but loans on all kinds of securities, including life interests and reversions. It is, unfortunately, quite impossible to learn from the Blue Books what proportions such mortgages bear to one another—it being quite optional to an office to lump all its mortgages together or to classify them, and I think some amendment of rule in this respect would be desirable. It would be very interesting, for example, to learn what amount is

now lent upon land alone in the United Kingdom. There is undoubtedly good reason to believe that there has been a considerable decline of late years in the proportion of mortgages of this description. The serious fall in the value of agricultural land, to the extent of probably over 30 per-cent on the average in Great Britain, and to a still greater extent in Ireland, must have been a cause of anxiety to those offices who used to lend the greater part of their funds upon this class of property, although there is a prevailing belief that their margins, at least in Great Britain, are still as a rule amply sufficient, and that in most doubtful cases the offices have already grappled with the difficulty, and have either written down their securities or provided for the risk by way of reserves. In Mr. Archibald Day's presidential address for 1886-7 (*J.I.A.*, xxvi, 161), he makes special and interesting reference to the position of companies in respect to their Irish mortgages, and the general tone of his remarks thereon, although not exactly optimistic, is nevertheless on the whole re-assuring. Opinions will always be much divided as to the future probable value of land in Great Britain, whilst in Ireland such securities, for reasons that I need not enter into, will probably for the present be severely left alone. Apart from value, the rate of interest to be derived from such investments is not now so attractive as it was before Lord St. Leonard's Act (22 & 23 Vic., cap. 35) opened them generally up for the investment of trust funds, nor will mortgages on freehold house property well let and of a good class and condition be usually found to yield a sufficient return. Apparently, mortgages on long leasehold property have recently come into more favour, as, not being available for the investment of trust funds, they give a higher rate, the precaution being usually taken to limit the advance to a fixed term of years. In lending on all kinds of property some offices have availed themselves of the guarantees of the various companies which have recently come into existence for the assurance of the repayment of mortgages. This of course means a reduction of the rate of interest by the amount of the premium required, and until these loan insurance companies have successfully stood the test of much longer time, and have practically proved the value of their guarantees, many offices will no doubt prefer to retain the whole interest and to be their own insurers. A difficulty with which it seems to me such companies have to deal, considering the present very limited number which do this class of business, is the adequate re-assurance of the risks which they undertake; and before doing business with such an office I think it would

be prudent to enquire as to its practice in this respect, and to ascertain what amounts it retains at its own risk upon its insurances. The *Loans on Life Interests and Reversions* seem to have very largely increased. The amount appearing separately under this heading in the 1872 Returns is £1,214,835, and in the 1890 Returns £5,965,749, and as certain companies do not state these loans separately, the amounts invested in this way on both occasions was probably much larger. This class of loans is so well known to all actuaries that I propose to say very little about them. In their nature they doubtless rank among the very best securities for life assurance funds, especially when they bring new policies to the offices, but recent competition seems to have had the effect of cutting down both rates of interest and margins of security; and as the demand for the better class of them is limited, and the number of offices supplying them is greater than formerly, the placing of any large sums of money in such investments on profitable terms is a matter of increasing difficulty. There remain the *Loans on Miscellaneous Securities.* Nine offices in 1872 stated these loans separately to the amount of £990,156, and 24 offices in 1890 stated them at £1,197,127. This increase is not large, but in 1872 this class of loan was mainly confined to offices doing fire as well as life assurance business. Only 3 purely life offices in 1872 showed separate loans on securities for a total of £100,921, whereas 17 purely life offices state them in 1890 at £547,169. Although always requiring care and special knowledge, there seems no reason why a share of the smaller proportion of the funds required to meet possible contingencies should not be invested when terms are favourable in lending on select Stock Exchange securities. In times of dear money such balances can usually be more advantageously invested in this way than through deposits with banks, and it may be remembered that this method of investment was recommended to the attention of offices by Mr. Samuel Brown in 1858, in his paper already referred to, and by Mr. John Coles in his paper on "Railway Debenture Stock", read on 21 December 1868 (*J.I.A.*, xv, 1).

Mortgages on Property out of the United Kingdom.—The proportions invested in this way according to the Returns at our three periods were—for all the offices, in 1872, say, $\frac{2}{3}$ per-cent; in 1882, $\frac{4}{5}$ per-cent; and in 1890, $3\frac{2}{5}$ per-cent. For the mutual offices, in 1872, say, $\frac{1}{50}$ per-cent; in 1882, $\frac{1}{5}$ per-cent; and in 1890, $5\frac{1}{2}$ per-cent. The Returns in 1890 show £6,292,371 invested in these mortgages, but it is perhaps worth mentioning

that £3,444,269, or over 50 per-cent of this amount, appears in the balance sheets of three leading Scotch offices, who had then not less than 15 per-cent of their productive funds invested in this manner, while the proportion stated to be so invested in the balance sheets of two of them on 31 December 1889 (or a year later than the amounts given in the 1890 Blue Book) is still greater, being in one case nearly 22 per-cent of the total invested funds. These offices have thus gone for a great proportion of their investments, literally as well as figuratively, to "fresh woods and pastures new"; and the motto of our great southern colony, "Advance Australia", seems to have been interpreted by them as a financial demand rather than a patriotic exclamation. The desirability or otherwise of investing so large a proportion of a company's funds in these mortgages might well in itself furnish matter for a lengthy paper, and having no personal experience of them I prefer to leave the discussion of their merits to others. It will not be forgotten that Mr. Sprague, in his presidential address for 1884–5 (*J.I.A.*, xxv, 65), lends the weight of his great name to an unhesitating pronouncement in their favour. If offices are satisfied with the security, the rate of interest to be obtained certainly seems very attractive; and the fact that my cautious fellow-countrymen at the head of such skilfully directed institutions have embarked in them so eagerly will probably be accepted as *prima facie* evidence of their desirability. The manager of one of these companies has very kindly allowed me to state that in Australia they lend from $\frac{2}{5}$ to $\frac{1}{2}$ of the value of the property, as assessed by their own valuer, at a rate of interest varying from 5 to $5\frac{1}{2}$ per-cent, and that the security consists principally of freehold land, but that they also lend on first-class city properties. I am also given to understand that these loans are usually arranged for a fixed term of years, and that parties lending up to $\frac{2}{3}$ of the value of the security can obtain somewhat higher rates of interest. I will only state here that loans of this nature appear to require a very large measure of confidence in local representatives, doubtless abundantly justified in the case of the offices who do this business; that they seem most fitted for companies who have large funds and can enter into them to some considerable extent; and that the present as well as the probable future course of legislation in the colonies in regard to land and other property is a feature requiring careful consideration. Much important information bearing upon the last-mentioned point will doubtless be found in Mr. A. H. Turnbull's inaugural address to the Actuarial Society of Edinburgh for the

present session, which will shortly be published, but with the contents of which I am not at present acquainted.

Loans on Rates (and Rent-Charges).—Under this heading the proportions of invested assets were—for all the offices, in 1872, say, 10 per-cent; in 1882, 13 per-cent; and in 1890, $11\frac{9}{10}$ per-cent. For the mutual offices, in 1872, say, $16\frac{1}{2}$ per-cent; in 1882, $20\frac{1}{3}$ per-cent; in 1890, 15 per-cent. This class includes not only (1st) direct loans on rates, and (2ndly) rent-charges, but also (3rdly) municipal bonds of all descriptions, home, colonial, and foreign. The first and second classes, although the first is secured on rates and the second on land, have this in common, that they are usually well-secured loans repayable by instalments, and were much sought after by insurance companies in the good old days of 5 per-cent and upwards. The absence of competition from private lenders, for whose purposes they were seldom suited, gave such institutions a great advantage. Local authorities have now, however, little difficulty in placing their loans with the public in the form of bonds or stock on much cheaper terms than ruled formerly, and if companies wish to cultivate this class of investment, they will not find the rates offered to them very remunerative. In a case which recently came under my own observation, a local loan was made by a life assurance company to a somewhat obscure local body at $3\frac{1}{2}$ per-cent. The same movement has also affected the rate of interest on British corporation stocks. By the provisions of the Trust Investment Act of 1889, trustees may invest in nominal or inscribed stock of the corporation of any borough having, in 1881, a population exceeding 50,000, or of any county council, and such stock will not easily now be bought to yield much over 3 per-cent. The attention of some assurance companies has therefore been turned to the stocks of corporations abroad, and I find that in 1890 £1,710,431 was stated to be invested in Indian and colonial municipal bonds, £270,472 in United States municipal bonds, and £99,625 in foreign municipal bonds. The best class of these bonds in the colonies and the United States, when bearing on their face a high rate of interest, stand at considerable premiums, and being usually redeemable on a given date require a deduction from the interest by way of a sinking fund. Still there is considerable opportunity here, and such securities seem to me well worthy of consideration in the present dearth of eligible investments.

Loans on Policies.—Out of the total invested funds there was invested—for all offices, in 1872, say, 5 per-cent; and in 1882

and 1890, $4\frac{4}{5}$ per-cent. For the mutual offices, in 1872, say, $7\frac{3}{4}$ per-cent: in 1882, say, $7\frac{1}{8}$ per-cent; and in 1890, 7 per-cent. The proportions therefore remain much the same, and no great outlet for investment in these undoubted securities can be expected in this direction. It is a satisfactory indication of the circumstances of policyholders to find that the amount assured by the companies, which in the end of 1888 was estimated at considerably over £500,000,000, exclusive of bonus additions, were then only anticipated by way of loan to the extent of £8,804,757, or under £2 per £100 assured, particularly when we consider that the amount includes arrears of credit portions of premiums as well as sums actually advanced against surrender-values. I need say nothing more on this point, except that while most offices charge 5 per-cent on their loans, a few charge somewhat less; but that, as far as can be judged by the returns, the reduction in rate does not seem appreciably to increase the proportion lent by the latter companies, and that it would be clearly unwise to lend upon policies at any lower rate than the assets are producing on an average.

British Government Securities.—The proportions invested herein are: for all offices, in 1872, $7\frac{3}{4}$ per-cent; in 1882, $3\frac{1}{4}$ per-cent; and in 1890, 3 per-cent. For the mutual offices, in 1872, $7\frac{1}{3}$ per-cent; in 1882, 2 per-cent; and in 1890, $2\frac{7}{8}$ per-cent. Never a favourite investment, the recent reduction in the interest in these securities has put them, and I think deservedly, quite out of fashion.

Indian and Colonial Government Securities.—Proportions invested: for all offices, in 1872, say, 5 per-cent; in 1882, $4\frac{3}{4}$ per-cent; and in 1890, $6\frac{7}{8}$ per-cent. For the mutual offices, in 1872, $3\frac{2}{5}$ per-cent; in 1882, $2\frac{1}{5}$ per-cent; and in 1890, $5\frac{1}{5}$ per-cent. The remarkable advance between 1882 and 1890, which covers a period when the above securities stood at tempting prices, and could readily be purchased to pay over 4 per-cent, has probably before now received a check. At present, and notwithstanding their much increased liabilities, our colonies are usually able to place their loans at $3\frac{1}{2}$ per-cent; and as stocks of the best class cannot now be bought to yield this rate, while the premiums at which some of them stand seem to necessitate a provision for a sinking fund, the present time does not seem favourable for further investments in this direction.

Foreign Government Securities.—Proportions invested: for all offices, in 1872, say, $1\frac{1}{6}$ per-cent; in 1882, 3 per-cent; and in

1890, 2 per-cent. For the mutual offices, in 1872, $\frac{1}{4}$ per-cent; in 1882, $\frac{1}{5}$ per-cent; and in 1890, $\frac{1}{12}$ per-cent. These securities have thus been never seriously entertained by offices for the investment of their life funds—the bulk of the amount, £3,588,248, held in 1890 belonging to offices mostly engaged also in fire insurances which have found them necessary for their business purposes abroad. They give in many cases high rates of interest, but are in other respects unsuitable. The best European state securities yield a low return, and even they are subject to violent fluctuations in times of war or of international complications. On the Central and South American States, whose securities are dealt in on the Stock Exchange, recent events are a sufficient criticism. I might say of them, in the language of the medical examiner, that their family history is unsatisfactory, that their constitutions are uncertain, and that their assurances are not to be accepted.

Debentures, and Shares and Stocks.—Although under separate headings in the Board of Trade Returns and in my comparison, I think it convenient to treat these securities together. The proportions invested were as follows—*Debentures*: For all offices, in 1872, say, $9\frac{4}{5}$ per-cent; in 1882, $7\frac{1}{10}$ per-cent: and in 1890, $9\frac{1}{5}$ per-cent. For the mutual offices, in 1872, $7\frac{1}{3}$ per-cent; in 1882 and 1890, $4\frac{1}{5}$ per-cent. *Shares and Stocks:* For all offices, in 1872, $2\frac{1}{5}$ per-cent; in 1882, $5\frac{1}{5}$ per-cent; and in 1890, $6\frac{4}{5}$ per-cent. For the mutual offices, in 1872, $1\frac{4}{5}$ per-cent; in 1882, $2\frac{1}{10}$ per-cent; and in 1890, $3\frac{1}{7}$ per-cent. *Debentures* are therefore clearly less popular than formerly, and their place as investments seems to some extent to be taken by *Shares and Stocks*. Nor is the reason far to seek. The debentures and debenture stocks of 1872 were mainly confined to such as are now admissible as trustees' investments, and the difference in their earning powers during the last generation will be strikingly shown by a perusal of the well-known paper on railway debentures, by Mr. John Coles, to which I have already alluded, and of the discussion which followed thereon. It will be remembered that railway companies were taking steps about the time the paper was written (1868), for the substitution of debenture stock for their debentures, and much difference of opinion was expressed as to the propriety of assurance offices investing in such stock instead of in debenture bonds repayable within a fixed term of years. There may be some who still maintain that securities

P 2

where the return on a given date of the capital invested is not guaranteed, and where realization consequently depends upon the public market, are not fitted for assurance companies. It is a position which in theory is attractive; and if a company could without risk place all its funds in redeemable mortgages on favourable terms the theory would be unassailable. But with whatever vigilance the security of capital laid out at interest may be safe-guarded, its return intact cannot be made an absolute certainty; and I am disposed to think that in investing a proportion of life assurance funds in securities which, although not redeemable as regards the purchase-money, insure with as near an approach to certainty as possible a fixed return of interest thereon, the risk of loss is not greater than that of a failure in the payment of interest or capital in many investments of the redeemable type. Despite market fluctuations, which are not usually great when the rate of interest is fixed and well secured, a company may be content in times of low prices to hold its hand, draw its interest, and with confidence await the return of better markets. The value of the advice given by Mr. Coles in 1868 to life assurance companies to buy English railway debenture stocks at prices yielding from 4 to $4\frac{3}{4}$ per-cent has been abundantly justified by results, affording such offices as then invested in them a splendid profit. The £100 stock which could then usually be bought under par is now standing at about £30 premium—an advance which is not, however, an unmixed blessing to assurance companies, as it now precludes the investment of any large portion of funds in such high-priced securities, practically only yielding an investor 3 per-cent on his purchase-money. Nor can much more than this rate be obtained on the debenture stocks of the best British water, dock, and gas companies. If the companies wish to make fresh investments in debentures, at satisfactory rates of interest, they must therefore go to the debenture stocks of sound British commercial, financial, trust, or land companies, upon which they will find it difficult in ordinary times to get even 4 per-cent, or to the bonds of American, colonial, and foreign companies. This brings us to the stocks and shares, for there are many who would rather invest in the preference shares of first-rate companies than in the debentures of second-rate companies. In making a choice of such investments, there is full scope for the exercise of that judgment and skill which Mr. Bailey alludes to in his paper, and to discuss which debentures, stocks, or shares are most suitable

is quite beyond the limits of the present article. There seems to me this advantage in these investments over many others, namely, that there is a wider area of selection. But, at least as a general rule, I think that ordinary shares or stock of every description, whether fully paid-up or not, should be rigidly excluded. There are some companies which, to a small extent, hold the very best class of ordinary stock—that of the great trunk lines of the English railway system; but I cannot help feeling, even here, that, at least for life assurance funds, this custom is "more honoured in the breach than in the observance." A security with a dividend subject to a possible decrease, and consequently with a greatly-fluctuating price, is not, I think, to be desired—and how great these fluctuations are, and how often they are in the wrong direction, is sufficiently shown in the recent as well as in the entire history of railway enterprise. Still more necessary is it to avoid the ordinary shares of English commercial, gas, dock, financial, and trust companies; while the large liability upon the shares of English banks is a fatal objection to their being held by assurance companies. As regards preference shares or stock, it is certainly safer to avoid all new companies, and only to invest in them when the ordinary capital is proportionately large, when steady dividends thereon have been paid, and where there is nothing in the nature of the business carried on to render future profits uncertain. Even the preference shares of companies whose earnings may not improbably be affected by the future course of legislation should, I think, be avoided. Many breweries and distilleries, to quote a class in which a great deal of public money is now invested, are dependent for much of their profits upon "tied houses", and the fact cannot be overlooked that both political parties seem to agree that a reduction should be made in the number of public-houses, whilst one party has declared against any compensation being given in cases where licenses are not renewed. Perhaps the preference shares of the better class of trust companies are worth attention. Their formation has recently been overdone, and some of the newer trusts have lent themselves too readily to the business of promoting companies, underwriting new issues, and relieving financial houses of undigested securities; but the principles on which the older trust companies were formed, and on which they have acted, should make the preference stocks of some of them very safe

investments. I refer to those that publish lists of well-selected and well-averaged securities, and that have a large deferred capital intact, earning good dividends, as well as a reserve fund formed from past profits believed to be sufficient to meet future contingencies.

As a general rule it is probably unadvisable to invest in any shares which are not quoted and readily dealt in on the Stock Exchange. Occasionally, however, opportunities may arise for the purchase of shares of unusual merit and price which are not bought and sold in the open market. And before leaving this subject, I think the fact that several life assurance companies have, within the last year or two, purchased shares of the various descriptions in "*The New River*" is worth mentioning, as, although I have elsewhere confined my remarks to groups of investment and have not attempted to discuss the merits of particular securities, yet it seems to me that the exceptional value and the altogether unique character of the original New River shares, as well as the increasing interest which is being taken in them by public companies, call for a few special observations. It is well to note that the amount invested in these shares is placed in the Board of Trade Summaries under the heading of "Shares and Stocks", but I am not at all sure that this is correct, and for the following reasons. The Adventurers' and King's shares in the New River, or portions of them (for they can be divided into any fractions), are real estate, and pass by deed of conveyance in the same manner as other freeholds. They carry with them the privileges accompanying landed property, such as qualifications for the parliamentary franchise and the magistracy, and they are only permitted as investments to life assurance companies having power to purchase real estate. I think therefore that they should more properly be included under the next heading in the summary, namely, "Land and House Property and Ground Rents", but as they have been treated as shares I think it better to deal with them at this stage. I am indebted to a well-known firm, through whom I am informed the life assurance companies holding these shares have made their purchases, for some particulars of these properties. The history of the New River corporation reads like a romance. In consideration of the assistance which King James I rendered to Hugh Myddelton and his friends in the opening up of the enterprize to supply London with water drawn from the chalk springs

of the Hertfordshire hills, one half of the concern was assigned to His Majesty, and the undertaking was then divided into 72 equal parts or shares, 36 of which were called "the King's Moiety", and 36 "the Adventurers' Moiety." King Charles I sold his moiety to the second Sir Hugh Myddelton for a fixed payment to the Crown of £500 per annum. The net dividend on each original share for the year 1889 was £2,610, so that the shares sold by Charles I for an annual payment of £500 are now producing £93,960. The Adventurers' and King's shares have equal rights as to dividend, but each Adventurer's share carries the right to a seat at the board, with fees amounting, in the case of regular attendance, to about £300 a year. An entire King's share will usually be bought at a slightly less proportionate rate than its fractions can be purchased for, but the converse is true in the case of an entire Adventurers' share, for the reason given. There is considerable difference in the price at which these securities stand in the balance sheets of the various life assurance companies, the cost price being presumably that which is given, but the latest market price of an Adventurer's share may be taken roughly at about £110,000, upon which a dividend of £2,610 yields only $2\frac{3}{8}$ per-cent; or, supposing the company to have the full benefit of the fees of their representative on the board, $2\frac{5}{8}$ per-cent. The question arises, have the companies been buying gold too dear? If the present dividends were all they had to look to, the answer would unquestionably be in the affirmative. It is pointed out, however, that the revenue of the New River Corporation has never retrograded but has continuously advanced, that it has doubled itself within the last 21 years; the same rate of progression being shown in the dividends, which have advanced from £1,340 per share in 1868 to £2,610 in 1889. There are also occasional bonuses—that declared in 1866 having been £1,400 per share, and that in 1874 £1,008. If the dividend continues to increase at a similar rate, much higher prices may confidently be expected, and there seem solid grounds for anticipating at least a considerable advance. The New River Corporation has the exclusive right of supplying water to the City of London, part of Westminster, and the intervening district, and to all the northern suburbs of London. The rated value in the sphere of its operations amounts at present to nearly £12,000,000, and may be expected to increase with the growth and wealth of the Metropolis, while the dividend is not limited by Act of Parliament, like that

of the other water companies. The large freehold estates of the New River in Middlesex and Herts include a property of 50 acres at Clerkenwell, known as the Myddelton Square Estate, which is covered with buildings let at purely nominal ground rents, but, on the expiration of the leases in some 20 years' time, rack rents, amounting to about 30 times the income from the ground rents, will come into possession. If the dividend and price of these shares during the next 20 years increase at only half the rate of the last 20 years, this increase would more than compensate a purchaser for loss of present income. I think, therefore, that they are at least worth the consideration of such companies as can afford to hold them for a while, and whose funds are so well invested that their being held does not seriously affect the average rate of interest on their investments. For all companies they would have been desirable investments some 20 years ago, when portions of them could be bought to pay 5 per-cent at the then rate of dividend. But there is now a certain element of speculation as to the future about them, which affects the market price, and some may think that before any further purchases are made it would be well to await the outcome of the deliberations of Parliament affecting the future water supply of the Metropolis, and of proposals which may be made to acquire existing rights.

Land and House Property and Ground Rents.—Proportions invested: for all offices, in 1872, say, $4\frac{2}{5}$ per-cent; in 1882, $4\frac{9}{10}$ per-cent; and in 1890, $6\frac{1}{2}$ per-cent. For the mutual offices, in 1872, $4\frac{1}{2}$ per-cent; in 1882, $4\frac{1}{8}$ per-cent; and in 1890, 6 per-cent. The increase under this heading, which includes also office furniture, seems to be mainly derived from the purchase of *Ground Rents.* In Mr. Sprague's treatise on life insurance accounts he expressed his surprise that, according to the returns of 1872, ground rents were not more patronized as investments by life assurance companies. In these returns only seven offices mentioned that they held them, and only five stated their separate values, which amounted in all to £233,468. In the returns for 1890, on the other hand, 33 offices state them in their returns, and 32 offices their values, at amounts producing a total of £3,422,674. Much is said both for and against such investments at the present day. The best class cannot at present be bought to yield 4 per-cent to a purchaser; but it is considered possible that before long the purchase of ground rents may be placed among the securities authorized to trustees, which of itself

would probably lead to higher prices still. On the other hand, increased taxation is feared by some.

Life Interests and Reversions.—Proportions invested: for all offices, in 1872, say, $1\frac{3}{5}$ per-cent; in 1882 and in 1890, $1\frac{4}{5}$ per-cent. For the mutual offices, in 1872, say, 1 per-cent; in 1882 and in 1890, $1\frac{3}{5}$ per-cent. I do not propose to add to the remarks which I have made on loans on these securities under the heading of mortgages.

Companies' own Shares.—I omit any reference to these, as they are evidently not investments for life assurance funds.

Loans on Personal Security.—The proportions thus invested were: for all offices, in 1872, say, $1\frac{3}{4}$ per-cent; in 1882, $1\frac{1}{5}$ per-cent; and in 1890, $\frac{2}{5}$ per-cent. For the mutual offices, in 1872, $\frac{1}{5}$ per-cent; in 1882, $\frac{1}{6}$ per-cent; and in 1890, $\frac{1}{4}$ per-cent. The number of offices transacting these loans is considerably less in 1890 than in 1872, and a glance at the Returns of the latter year seems to indicate that more than one office since that date has personally lent itself out of existence. Under stringent rules and careful management this class of loan has, however, been found by several offices to be safe and profitable, and an assistance to the life assurance business. Still, it is not, I would submit, a fitting medium for the investment of the life funds themselves, and it is satisfactory to note the exceedingly small proportions of this item in the returns for 1890.

Cash, Deposits, Stamps, &c.—The proportions here were: for all offices, in 1872, say, $2\frac{1}{2}$ per-cent: in 1882, 3 per-cent; and in 1890, $3\frac{1}{2}$ per-cent. For the mutual offices, in 1872, 2 per-cent; in 1882, $1\frac{3}{5}$ per-cent; and in 1890, $2\frac{1}{4}$ per-cent. The amount of cash at bankers on current account and on deposit is not always separately given; but the amounts which are stated to be on deposit were according to the Returns of 1872, £1,403,822, and according to those of 1890, £4,022,261. This very considerable increase has doubtless been caused by the difficulty of finding suitable investments, while there are now numerous colonial and foreign banks who are willing to pay from 4 to 5 per-cent upon fixed deposits for a year or a short term of years. The chief thing to be considered in such cases is of course the financial position of the bank, upon which the amount of its uncalled capital has an important bearing. I am informed that some companies are sending deposits direct to banks in the colonies, which offer a higher rate of interest there than is given by their branches in this country.

Having gone through the various items as appearing in the Blue Books, I now propose to say a word or two on the powers of investment which the offices generally possess. For the guidance of my own directors, who thought it advisable to endeavour to obtain some additional powers, I collected some information on this point a few months ago, as the result of enquiries addressed to 77 British life assurance offices. I desire to acknowledge publicly, as I have already done privately, the great courtesy with which the vast majority of these offices were kind enough to respond. With five exceptions I received satisfactory replies from them, in many instances accompanied by copies of their deeds or charters.

Out of the 72 offices whose practice was thus made known to me, the directors of 35, or very nearly one-half of their number, possess practically unlimited powers of investment. Many companies thus prefer giving a full discretion to their boards than to pinning them down to a certain number of defined securities, and there is no doubt that, with whatever care the authorizing clauses may be drawn, there will always be scope in them for unwise investments, whereas when the powers are not defined it is none the less the duty of directors to act with prudence and judgment. The investments of these 35 offices, so far as can be gauged by their Returns, differ little in their general character from those of the offices whose powers are wide but at the same time carefully defined. In the cases of 14 out of the 35 offices, otherwise unlimited powers are subject to certain restrictions or exceptions. Taking them from my list, and describing the offices by numbers, these are as follows:

Office No. 1.—Exception, Shares with liability.
,, 3.—Subject to certain regulations as to number present at a Board Meeting. Exception, Personal Security.
,, 4.—Exception, Shares with liability.
,, 7.—Only exception, Foreign Securities.
,, 9.—Exception, anything with liability.
,, 11.—A four-fifths majority of the Board required for any investment.
,, 13.—Exception, anything with liability.
,, 15.—Exception, Shares with liability.
,, 17.—Exception, Company's own Shares and Shares with liability.

Office No. 25.—One or two exceptions.

„ 28.—Board must be unanimous. Exceptions, Ordinary Stocks, Personal Security, or anything where there is responsibility.

„ 29.—Exceptions, Company's own Shares and Shares with liability.

„ 31.—Exceptions, Company's own Shares and Shares with liability.

„ 33.—Exceptions, Advances outside limits of Great Britain and Ireland, and Loans on Personal Security.

Out of the 72 offices as to which I received information, there remain 37 offices that possess more or less extensive powers, set forth in the various instruments under which they act. On account of the great difference in the phraseology employed, as well as in the groupings of the investments named, I found it a somewhat complicated and difficult task to classify their powers; but I think that the attempt that I have made gives a fairly accurate view, although, in the cases of a few offices, I would have liked a little more definite information. Excluding all such investments as are available for trust funds, and investments which are generally open to life assurance companies, such as loans on policies and on rates, life interests and reversions, and house property, the following classes of securities seemed to me those as to which information was specially desirable:

1.—Colonial, Indian, and Foreign Municipal, &c., Securities.
2.—Deposits with Banks.
3.—Foreign Government Securities.
4.—Lands or Interest therein out of the United Kingdom.
5.—Leaseholds, Rent-Charges, Ground Rents, &c.
6.—Loans on personal Security.
7.—Shares and Securities of Public Companies.

In the Appendix to this paper I give details of powers of the offices under observation in regard to the above securities, and it may be mentioned that most of the offices have taken powers both to purchase and to lend upon the securities named.

The following offices with defined powers require, like offices Nos. 3, 11, and 28 already alluded to, the sanction of a certain

number or majority of the Directors in the selection of their investments.

Office	Investments require Sanction of
No. 46 .	Six Directors.
,, 47 .	Majority of Board.
,, 50 .	Five Directors present.
,, 53 .	Six Directors.
,, 57 .	Two-thirds of Directors, not being less than five.
,, 61 .	Two-thirds of Directors.
,, 65 .	Three-fourths of Board for most Securities.
,, 71 .	Seven Directors present.

I have now dealt, in a very brief and incomplete manner, but as fully as I could within the space at my command, with three important branches of my subject—the rate of interest enjoyed in the present and the past on life assurance investments, the classes in which these investments have been grouped at three different periods from 1870 to 1888 inclusive, and the existing powers of investments of life assurance offices. After consideration of the facts before us, I think it may justly be claimed that the funds of these offices are well and carefully invested, and that their policy-holders and the public may have confidence in the future of the great institutions which act to so large an extent as trustees of the thrift of the present generation, and of the wealth of generations yet to come. From the Board of Trade Returns we cannot gather as much as could be desired. I think that some improvement in classification might well be made, which, for instance, would show clearly the amount lent upon different groups of securities now placed together under the headings of mortgages on property, both within and out of the United Kingdom; which would separate from one another such different assets as loans on rates, British, colonial, and foreign municipal bonds, and rent-charges; which would show with greater precision the nature and amounts of the debentures, stocks, and shares held; which would treat ground rents under a special heading; and which would require the names of the banks or other institutions in which cash has been placed on deposit. If this were done, we would certainly have a better idea of the actual investments of life offices. But the Board of Trade Returns, notwithstanding some vagueness in detail, at least bear eloquent testimony to the general high character of the investments, by showing that the interest upon them is regularly paid, that the average rate earned is not less

than 4 per-cent, and that revaluations and realizations during a considerable time have yielded a greater or less profit in every year under observation. That the rate of interest has only fallen $\frac{3}{8}$ per-cent in 18 years is undoubtedly due to the change which has taken place in the classes of investments sought after, and in considering the future it would seem necessary, if the rate is not to drop very considerably from that now earned, that this contrast in classification as compared with the year 1872 should become still greater. I, for one, look forward to these changes, not altogether without anxiety, but altogether without alarm. The difficulty of finding suitable investments has become greater. This must be surmounted by increased diligence in search and discrimination in choice. The yield of gold is less than it was, in comparison with the ore which we manipulate. We must improve our machinery. Part of our mine has been worked out. We must sink new shafts and open up new lodes. The offices generally are acting upon such views. The range of investments has become wider, and in the large variety there is more scope for the operations of that law of average upon which our business is so largely founded. The tendency to value liabilities by lower rates of interest is a conservative precaution which cannot be too strongly commended, but it does not seem to me to relieve directors and managers of their responsibilities in endeavouring, so far as they can, to get as much more interest upon their investments as is consistent with security. There may be some who are of opinion that investments should still proceed upon former lines, and who would restrict the operations of life offices to investments somewhat similar to those which are now, but many of which were not formerly, allowed by law for the investment of trust funds. I humbly beg leave to differ from that view. It is a safe one, no doubt, as regards the security of the principal; but observing the fluctuations which take place in the value of real estate and of trustees' securities generally, I do not see that it is safer than a view which would permit investment in certain other good and sound securities which, from the very fact of their being closed to trust funds, command a higher rate of interest in the market. And if the hypothetical "trust fund rule" that I have mentioned is safe, it is also certainly an easy one to act upon; but I think that trustees' acts have been framed to guide and protect a large class of people who are not paid for the responsibilities which they undertake, while ease is an item which can well be left out of calculation

by those who are remunerated for their services. The actuary or the chief officer of a life assurance office can only act as an adviser. It is for him to point out what classes of security are suitable, what yield should be looked for, what margins should be required. But the selection and the decision lies with the directors. Fortunately, the boards of life assurance companies are usually composed principally of prudent business men whose success in carving out their own fortunes is proof of their ability to well invest the funds entrusted to their guardianship. The composition of the board and the connections of the office will often determine to a considerable extent the character of the investments, and will give prominence to such securities as they are able personally to influence on specially advantageous terms. The vast majority of directors and managers are alive, I believe, to the necessities which altered circumstances have created. The saying so frequently quoted, that "high interest means bad security", is generally true, no doubt, but among equally sound investments some may easily, from quite intelligible causes, yield considerably higher rates than others. There is a great deal in fashion, and it is well to anticipate it in investments. Those companies, for instance, which bought railway debentures, colonial securities, and Indian railway shares when they were novelties on the market have reaped rich harvests. "All can raise the flower now, for all have got the seed"; but the flower is more expensive now, and the blossom less bountiful. Still, to those who look for them, "bargains in investments" are still to be made. And, whatever may be the future of the rate of interest, I am convinced that those institutions will be best entitled to the confidence of the community who have men at their heads, both as directors and managers, who note the tendency of the times and are inspired by the spirit of the age, who do not cling to old rules when they have become effete nor adopt new rules without sufficient reason, who act with courage in the general but with caution in the particular, who make the best of their opportunities, "who know the seasons when to take occasion by the hand", and thus to conserve and increase the vast accumulations committed to their care, with Mr. Bailey's two first principles of investment constantly before their eyes—the necessity (first) of providing for the security of the capital, and (secondly) of obtaining the highest practicable rate of interest.

APPENDIX.

SOME RESULTS OF ENQUIRIES REGARDING POWERS OF INVESTMENT OF 72 LIFE ASSURANCE OFFICES.

1.—Municipal Securities—Colonial, Indian, and Foreign.

The following 24 offices—in addition to 33 offices having unlimited powers of investment, being 57 out of 72 offices—have greater or less powers to invest in the above securities.

Number of Office	May invest in these Stocks
No. 36	Anywhere.
,, 37	In India and Colonies.
,, 38	In Colonies, and wherever required for business purposes.
,, 41	In Colonies.
,, 42	Wherever guaranteed by any Government.
,, 44	In India and Colonies, and wherever required for business purposes.
,, 45	In India and Colonies.
,, 46	In India and Colonies, and wherever required for business purposes.
,, 49	In India and Colonies.
,, 50	Anywhere.
,, 52	In any British Colony.
,, 54	In India and Colonies.
,, 55	Wherever required for business purposes.
,, 56	,, ,, ,, ,,
,, 57	In India and Colonies, and wherever required for business purposes.
,, 60	In India and Colonies.
,, 61	,, ,,
,, 63	Colonies and United States.
,, 64	,, ,,
,, 65	,, ,, (3-4ths of Directors must approve).
,, 66	In India and Colonies, and wherever required for business purposes.
,, 68	In India, Colonies, and United States.
,, 69	In India and Colonies.
,, 71	With Indian or Colonial Government guarantee.

2.—DEPOSITS WITH BANKS, &c.

The following 18 offices—in addition to 35 offices having unlimited powers, being 53 out of 72 offices—have power to invest as follows:

Number of Office	Power to place Money on Deposit with
No. 36	And also to invest in Debentures of any Banks in United Kingdom, India, and Colonies.
„ 38	„ „ „
„ 40	Any Bank.
„ 41	Any approved Bank having place of business in London.
„ 44	Any Bank.
„ 48	Any Bank or Discount Company.
„ 49	And also to invest in Debentures of any Banks in United Kingdom, India, or Colonies.
„ 50	Any Bank in United Kingdom, India, or Colonies.
„ 53	Any Bank.
„ 54	Any Bank or Company carrying on business in Great Britain.
„ 58	As may be considered advisable.
„ 60	And also to invest in Debentures of any Bank or Company in United Kingdom, India, or Colonies.
„ 61	„ „ United Kingdom, India, or Australasian Colonies.
„ 63	„ „ United Kingdom, India, or Colonies.
„ 65	Any Company having Head Office in United Kingdom (3-4ths of Board must approve).
„ 66	Any Bank or Company carrying on business in United Kingdom, India, or Colonies.
„ 67	The Bankers of the Society.
„ 68	Or on Loan to any Bank.

3.—FOREIGN GOVERNMENT SECURITIES.

The following 26 offices—in addition to 33 offices having unlimited powers of investment, being 59 out of 72 offices—have greater or less powers to invest in the above securities.

Number of Office	May invest in Foreign Government Securities
No. 36	Anywhere, including any State in the United States of America.
„ 38	Anywhere, so far as required to carry on business, abroad.
„ 39	Anywhere.
„ 40	With British Government Guarantee.
„ 42	Anywhere.
„ 44	„
„ 45	Anywhere (but not more than 1-10th of total funds can be invested in Foreign Government Securities where there is no British Government guarantee, and any investment therein must be sanctioned by an Extraordinary Board).
„ 46	So far as required to carry on business, abroad.
„ 47	With British Government guarantee.
„ 49	Of France, Germany, Holland, and United States.
„ 50	Anywhere.
„ 52	Of any State fulfilling its public obligations.
„ 53	With British Government Guarantee.

Number of Office	May invest in Foreign Government Securities
No. 55	So far as required to carry on business, abroad.
„ 56	„ „ „ „
„ 57	Anywhere.
„ 59	With British Government Guarantee.
„ 60	Of any European State.
„ 61	Of France, Germany, Holland, and United States.
„ 63	„ „ „ „
„ 64	United States Bonds.
„ 65	Of France, Germany, Holland, and United States (3-4ths of Board must approve).
„ 66	Of France, Germany, Holland, and United States, and also, so far as required to carry on business, abroad.
„ 67	Of the United States.
„ 68	Anywhere.
„ 71	„

4.—Lands or Interest therein out of United Kingdom.

The following 17 companies—in addition to 33 companies having unlimited powers, being 50 out of 72 companies—may invest herein as follows:

Number of Office	May invest in Lands or Interest therein
No. 36	Anywhere.
„ 38	In Australia, New Zealand, or Canada, and, so far as necessary to carry on business, abroad.
„ 44	So far as expedient for carrying on business abroad.
„ 46	In United Kingdom, India, and Colonies, and so far as expedient for carrying on business, abroad.
„ 49	May purchase land to extent of 10 acres in all wherever required for business purposes, and may lend on lands or interest therein in United Kingdom, India, the Colonies, or the United States.
„ 50	In Australia, Canada, and India.
„ 55	So far as necessary to carry on business, abroad.
„ 56	„ „ „ „
„ 57	„ „ „ „
„ 58	In any British Colonies or Dependencies.
„ 60	In India or Colonies.
„ 61	In Canada and Australasian Colonies.
„ 63	Same powers as No. 49.
„ 65	In British Colonies or Dependencies (3-4ths of Board must approve).
„ 66	In Australia, New Zealand, or Canada—also, so far as necessary to carry on business, in India or any other Colony or Dependency.
„ 67	
„ 68	Anywhere.

5.—Leaseholds, Rent-charges, Ground Rents, and Income arising from and charged on Land.

The following 31 companies—in addition to 35 companies having unlimited powers, being 66 out of 72 companies—may invest in above, or some of them.

Number of Office	May Invest in Leaseholds, &c.
No. 36	Anywhere.
,, 37	In United Kingdom.
,, 38	In United Kingdom, and also, so far as necessary for business, abroad.
,, 43	In United Kingdom.
,, 44	In United Kingdom, and, so far as necessary for business, abroad.
,, 45	In United Kingdom.
,, 46	So far as necessary for business, abroad.
,, 47	In United Kingdom.
,, 48	,, ,,
,, 49	,, ,,
,, 50	In United Kingdom, India, and Colonies.
,, 51	In England only.
,, 52	In United Kingdom.
,, 53	,, ,,
,, 54	,, ,,
,, 55	In United Kingdom, and, so far as necessary for business, abroad.
,, 56	,, ,, ,, ,, ,,
,, 57	,, ,, ,, ,, ,,
,, 58	In United Kingdom and Colonies.
,, 59	Ground Rents and Rent-Charges in United Kingdom.
,, 60	In United Kingdom.
,, 61	,, ,,
,, 62	,, ,,
,, 63	,, ,,
,, 64	
,, 65	,, ,,
,, 66	In United Kingdom, and, so far as necessary for business, abroad.
,, 67	In United Kingdom.
,, 68	Anywhere.
,, 69	In United Kingdom.
,, 72	,, ,,

6.—Loans on Personal Security.

The following 20 companies—in addition to 32 companies having unlimited powers, being 52 out of 72 companies—can invest in the above.

Number of Office	Remarks
No. 36	Two or more Sureties required.
„ 37	
„ 38	
„ 39	
„ 42	
„ 44	Two or more Sureties required.
„ 48	
„ 49	
„ 50	
„ 53	
„ 55	Two or more Sureties required.
„ 56	
„ 57	
„ 63	
„ 64	
„ 66	
„ 67	A feature is lending to Trustees of Chapels, Schools, &c.
„ 68	May lend on Bills of Exchange or other Securities guaranteed by any Bill or Stock Broker in United Kingdom, approved by the Managers.
„ 70	
„ 71	

7.—Shares and Securities of Public Companies.

In addition to the 35 companies out of the 72 companies under notice which, with a few exceptions as to shares with liability, have unlimited powers to invest in the above, there are 35 other companies which have considerable powers to invest therein. It will be seen that the powers are extremely diverse, and, indeed, are scarcely alike for any two companies.

Number of Office	Can Invest in
No. 36	Debentures, Mortgages, Preference or Guaranteed Shares or Stock of any Railway, Water, Canal, Dock, Trust, Land, Mortgage, Gas or other Lighting Company in or out of the United Kingdom, or in the fully-paid ordinary Shares of any such Company which has paid a dividend averaging 3 per-cent for 5 years on its ordinary Capital, or in Stock of the Bank of Scotland, British Linen Company, or Royal Bank of Scotland, Company's own Shares.
„ 37	Any Securities, including ordinary Shares of any Company in the United Kingdom, India, or the Colonies.
„ 38	Any Securities, including ordinary Shares of any Railways in United Kingdom, India, or Colonies, paying dividends, also in Shares of Indian Railways with Government guarantee, or in the Shares of "five Scotch Banks", Company's own Shares.

Number of Office	May invest in
No. 39	Debenture Stock or Bonds, or fully paid-up Guaranteed or Preferred Stocks or Shares of any Railway Company in Great Britain, India, and Colonies, and in the ordinary Stocks of Indian Railways with a Government Guarantee.
„ 40	Mortgages or Debentures of any Company in United Kingdom paying dividends upon its ordinary Stock.
„ 41	Any Securities except ordinary Shares of any Company in United Kingdom, having paid for 2 years preceding a dividend on its ordinary Shares; or on the Stock of any Company leased or guaranteed by any other Company in United Kingdom. Also on the Securities of any Railway or Tram Company in the Colonies which has paid dividends for 5 years preceding on its ordinary shares.
„ 42	Stocks or Securities of any Railway with British, Indian, Colonial or Foreign Government guarantee. Certain English Railway ordinary Stocks.
„ 43	May lend on any approved Shares or Securities of Public Companies.
„ 44	Stocks or Shares with Indian or Colonial Government guarantee; Securities and Shares of any Company incorporated by Act of Parliament; also Securities of Companies abroad so far as may be expedient for carrying on business there; also Securities of any Railway, Water Works, or Gas Works, or other Company or body incorporated by law of Great Britain, or of any Foreign State for any public purpose, and having power to levy Rates, &c. Also all Securities except ordinary Shares of any Company in United Kingdom or Colonies chartered or incorporated by law.
„ 45	Any Securities except ordinary Shares of any incorporated Company in United Kingdom, India, or Colonies, and of any Railway Company in the United States; also any Stock guaranteed by any such Company, and any Shares whose undertaking is leased at a fixed rental to any incorporated Company in United Kingdom, India, or Colonies.
„ 46	Any Securities, including ordinary Stock or fully paid-up shares, of any Company in United Kingdom, India, or Colonies.
„ 47	Any Securities except ordinary Shares of any Canal, Dock, Gas, or Water Company which shall have paid dividends on ordinary Shares for 5 years, and of any Company in India with Indian Government guarantee. Also in Debentures of any chartered or incorporated Company in United Kingdom, India, or Colonies, having paid dividends for 5 years preceding; and in the Shares or Stock of any Company leased at a fixed rental to any incorporated Company in the United Kingdom.
„ 48	Any Securities, including fully paid-up Shares of any Railway, Canal, Dock, Gas, or Water Company in the United Kingdom, or Colonies and Dependencies, also Securities of any undertaking with interest guaranteed by the Governments of any of these places. Exception, Shares with any liability.
„ 49	Any Securities except ordinary Shares of any Company paying dividends on its ordinary capital; Stocks of any Company with interest guaranteed by British, Indian, or Colonial Governments; also the ordinary Stocks of any Railway, Canal, Water, Land, Dock, Navigation, or Harbour Company in United Kingdom, India, or Colonies, having paid dividends for 3 years preceding purchase. Stock of any bank in Scotland established by charter or Act of Parliament. No Shares with liability except Stock of Bank of

Number of Office	May Invest in
	Scotland. Can lend on own Shares, but cannot purchase them.
No. 50	Any Securities, including ordinary Stock or fully paid-up Shares of any Public Company duly constituted by law in Great Britain, India, Colonies, or any Foreign State.
„ 52	Any Securities, including ordinary Shares of any Company in United Kingdom, Colonies, any European State, or the United States, paying income on any Stock ranking later for dividend where such exists.
„ 53	Mortgages or Securities of chartered or incorporated bodies down to the Preference Shares of any Company incorporated by Act of Parliament, also in Stocks and Securities with Indian or Colonial Government guarantees.
„ 54	Same Securities as No. 36 except Stocks of the British Linen Company and Royal Bank of Scotland, and Company's own Shares.
„ 55	Any Securities except ordinary Stock or Shares of any incorporated Company paying dividends on ordinary capital, also in the Securities of any Company in so far as necessary for purpose of carrying on business.
„ 56	Any Securities except ordinary Stock or Shares of any incorporated Company which has for 3 years preceding paid dividends, also in Gas and Water Companies' Stocks, and ordinary Stock of certain Railways.
„ 57	Shares or Securities of any chartered or incorporated body in the United Kingdom, India, or Colonies, and of any Company or Trust investing its funds exclusively in Mortgages in the United Kingdom, India, or Colonies. Exception, any Shares with liability.
„ 58	Securities with Indian Government guarantee, and in the Shares or Securities of any Company incorporated by Act of Parliament paying dividends.
„ 59	Debentures, Bonds, and guaranteed Stock of any chartered or incorporated bodies paying dividends of 3 per-cent for 5 years preceding, also in any ordinary Shares with Indian Government guarantee.
„ 60	Stock or Debentures of any Company with dividends guaranteed by Great Britain, India, or Colonies, also in Securities down to Preference Shares of any Company paying dividends having agency or branch office in the United Kingdom, and in the ordinary Stock of any Company in the United Kingdom, India, or Colonies having paid dividends for 3 years preceding. Exception, any Shares with liability.
„ 61	Securities and Shares of any Railway, Canal, Water, Dock, or Harbour Company in United Kingdom, Canada, Australasian Colonies, or the United States, paying dividends on ordinary Capital. Exception, any Shares with liability.
„ 62	Debenture Stocks of any Company, also Securities, including ordinary Shares of Railways, guaranteed by Indian Government.
„ 63	Securities down to Preference Shares of any Company paying dividends on ordinary Capital, also in Stocks of any Company with Indian or Colonial Government guaranteed dividend, and in the ordinary Shares of any Railway, Canal, Water, Gas, Dock, or Harbour Company in the United Kingdom, India, or Colonies paying dividends. Exception, any Shares with liability.
„ 64	Indian Railway Guaranteed Stocks, Water Annuities, Debentures and Preference Stocks of British Railways, Company's own Shares.

Number of Office	May invest in
No. 65	In Indian and Colonial Guaranteed Railways, also in Securities, including ordinary shares of any Railway, Canal, Water, or Dock Company in the United Kingdom, India, or Colonies, paying dividends on ordinary capital, also in Debentures of any Company in United Kingdom, India, or Colonies, Stock of any Bank in Scotland established by charter or Act of Parliament. (Some of these Investments require the approval of 3-4ths of the Board).
,, 66	In the Securities down to Preference Shares of any Railway, Canal, Gas, Water, or Dock Company in the United Kingdom, incorporated by Act of Parliament, and paying dividends on ordinary Shares, Stock of Bank of Scotland, British Linen Company, and Royal Bank of Scotland.
,, 67	Railway Stocks with Indian Government guarantee, also in any Securities except ordinary Shares of any Gas, Water, Dock, Harbour, or Navigation Company, Board, or Trust, incorporated or chartered, having paid dividends for 5 years previous—provided 3-4ths of Board concur—Company's own Shares.
,, 68	Any Securities except ordinary Shares of any Parliamentary or Chartered or Joint-Stock or Trust Company in United Kingdom, India, Colonies, or any Foreign Country, including any of the States in the United States of America, also in the fully paid-up Shares of such Companies as have paid dividends averaging 3 per-cent on ordinary capital for 5 years, Stock of Bank of Scotland, British Linen Company, and Royal Bank of Scotland.
,, 69	Shares or Securities of any Company in the United Kingdom, India, or Colonies.
,, 70	Indian Railway Stocks and Securities, British Railway Securities down to Preference Stocks.
,, 71	Any Securities except ordinary Shares of any Railway, Canal, Dock, Water, Gas, or Land Improvement and Drainage Company, incorporated in the United Kingdom, also in any Securities or Shares of any incorporated Company in India or Colonies with Indian or Colonial Government guarantee.

Discussion.

The President (Mr. B. Newbatt) having asked the two gentlemen who had acted as referees to open the discussion,

Mr. G. Humphreys said that the subject of Mr. Mackenzie's paper was one which came home to all the senior members of the Institute, and which referred to matters that daily occupied their most thoughtful consideration. The author had discussed the ninth question of the sixth schedule of the Life Assurance Companies Act, and he concluded by saying, "and even supposing that the former "and more general construction were acted upon by all offices, there "would still be abundant room for divergences in the answers given." The different circumstances of offices would naturally lead them to give varied answers to the question with regard to the average rate of interest earned by the funds. He differed from the author as to the

way in which income tax should be treated with reference to interest and dividends. The author considered that the income tax should be deducted before proceeding to ascertain the rate of interest which the funds of the society had earned. It seemed to him (Mr. Humphreys) that the income tax was really no more virtually connected with the interest and dividends than any other imperial tax, such as, for instance, the house tax. Supposing, for the sake of argument, that there were no income tax, the companies would be compelled to provide contributions in some other form to enable the Government to carry on the work of the country. The wording of the Act seemed to require that the average rate of interest earned by the funds should be given without deduction. With regard to the ability of assurance companies to safely obtain higher rates of interest than might be expected, it must be borne in mind that there were several investments, notably life interests, in which the borrower would wish to avoid the risk of the loan being called in at an inconvenient time, which might be the case if the money had been obtained from a private source. Those loans would therefore bear a rather higher rate of interest.

Mr. G. H. Ryan said that Mr. Mackenzie's paper enabled them to grasp the effect of that decline in the productivity of capital which had been so marked a feature for some years past. He thought the decline in the rate of interest would be apt to have more serious consequences to companies transacting a large annuity business than to pure life offices, inasmuch as the annuity companies, not having to make provision for large bonuses in the future, had not perhaps felt themselves under the same necessity of strengthening their reserves so as to provide against the reduction in their interest earnings. No doubt the decline in the rate of interest since 1870 had been continuous and marked, but he doubted whether, if they could trace back further than 1870, they would find that it had been continuous for any long period of years previously. In fact, if they could get the returns of English offices of about 30 years ago, he thought they would find that their net rate of interest was greater—say in the years 1850 to 1860—than in the year 1890, notwithstanding the fact that in the interval the rate had risen and fallen. Since 1870 there unquestionably had been a considerable decline in the yield of money, but against that circumstance there was to be set the fact that life offices had very largely strengthened their reserves in the meantime. It was possible that if a strict account could be taken in 1870 and 1890 of the net contribution to profit from interest sources, they would find they were just as well off in 1890 as they were in 1870 in that respect. Referring to the table of life assets prepared by the author, there was one item which caused him some surprise, namely, the item of "Life Interests and Reversions." Comparing 1872 with 1890, there had been practically no movement in the investment of life assurance funds in life interests and reversions. In connection with that, it occurred to him to ask how companies treated the purchases of reversions and reversionary securities when the option of redemption was allowed during a few years, because if it were customary to treat them as purchases, he should certainly have expected the total investments in life interests and reversions to show a considerable

increase, instead of remaining practically stationary. It was satisfactory to observe that the investments in Consols had declined in the period reviewed by Mr. Mackenzie. In the old days, investments in Consols were needed for many purposes, but now they seemed of little further utility than to give, so to speak, a tone to the balance sheet, and 3 per-cent of the funds of a company in that particular security, considering its low yield, was perhaps as large a percentage as was necessary. In conclusion, he would draw attention to that portion of the paper in which Mr. Mackenzie had very fully summarized the objections to the present practice of classifying assets in life assurance balance sheets, and had recommended several alterations, all of which seemed to be in the right direction. If life offices could see their way to fall in with the suggestions now made, he was sure that a great benefit would be derived from the present paper.

Mr. John Coles said that Mr. Mackenzie had brought out one very important point, namely, that the decline in the yield of life office investments had amounted to only $\frac{3}{8}$ per-cent in the course of 18 years. Some explanation of that might be found in the fact that investments were, in many cases, allowed to stand at the original cost price. He was surprised at the smallness of this difference, because, in regard to leading English and Colonial securities, a much greater variation in the rates of interest had been apparent. As an investment for temporary purposes, he favoured deposits in a good bank. It was true there was unlimited borrowing power, but there was usually a very large amount of capital paid up, and the reserves were very large, and there were companies who would insure the deposits at very moderate rates. He strongly objected to many of the trust, loan, and land companies, because of their enormous borrowing powers, and he gave several illustrations of this point. Four or five years ago disaster overtook the East and West India Docks, and when the ordinary stock seemed to be in jeopardy, many were surprised to find there was no preference stock to break the fall upon the debenture stock. That was a rude awakening to many, but there was worse to come. It was found that the debenture bond itself was so drawn that the directors, after consulting a leading firm of city solicitors, thought themselves justified in taking the whole of their title deeds to a neighbouring bank, and borrowing half a million upon them. Unless these things were inquired into with the utmost care, it might be found that they were lending upon debentures where the title deeds had gone to the next bank, and that therefore they were only in the position of second mortgagees. The Scotch had invested very largely in the Australian Colonies. Now it was an open secret that a very large bank, doing business in Australia, was, at the present time, farming large quantities of land through their own farm bailiffs. They had become possessed of the land because the borrowers could not pay the interest. He mentioned that as a caution to those who would rush too quickly into the mortgage of land in the Australian Colonies, which had no doubt, for some years past, been at a very fictitious value. He thought it was possible to educate to a higher extent than at present some of those who were coming forward in life offices, with regard to the selection of securities. The selection of lives was an everyday

matter: the selection of securities should be the same. The internal machinery of offices was not quite adapted to many modern requirements. Great delays were not helpful to offices who wished to take advantage of surplus funds—at such times as, for instance, the week following last Lord Mayor's day, when, from Monday to Saturday, high rates were paid on undoubted securities for three, six or 12 months to those who could give a ready reply as to whether the means could be had. Although it was true that capital or stored-up labour had increased rapidly in recent years, the field for the employment of capital was not used up yet—for example, in countries such as North-West Canada, Africa, and Australia. The managers of offices had difficulties to contend with, no doubt, and the wants and expectations of the assuring public were more and more exacting.

Mr. M. N. ADLER remarked that one point they had pretty clearly arrived at was, that the decline in the rate of interest earned by British offices in the last 20 years had not been a material one: it had fallen from £4. 10*s*. to £4. 2*s*. 6*d*. per-cent. He had before him a statement from the *Economist* of August last, which spoke of the decline in the rate of interest on the investments of insurance companies in America. Mr. Wright, the actuary of the New England Mutual Life Assurance Association, had made an inquiry into the average rates realized on the investments of 20 leading insurance companies in the States during the last 21 years, and those results were summarized in a table which gave the highest, the lowest, and the average rate of interest for every year from 1869. In the five years ending 1873, the average rate of interest was 6·1; it went down the following five years to 5·9. From 1879 to 1883 the rate of interest was 5 per-cent: in the following five years it was 4·7; and, in the year 1889, it came down to 4·6 per-cent. That was a very marked decline, and the *Economist* added: "It is worth noting that the average rate of interest realized "by the British life offices in 1889 was just about 4 per-cent: so that "the American offices now retain virtually little of the advantage they "previously enjoyed over British companies in the much higher "returns realized by them." It was only fair to say that the *Economist* brought into that comparison the years during which the civil war raged in America, which tended very much to depreciate the securities; but, in any case, the contrast between the fall in the rate of interest of English offices, as compared with that of American offices, was very marked. With regard to German offices, he believed the fall in the rate of interest was very considerable. Many offices made advances to counties and boroughs years ago at 5 per-cent, and one corporation after another found they could place loans in the open market, and wanted to repay those advances. How should these applications be dealt with? With regard to interest on loans on policies, the author seemed to think that the reduction in the rate did not appreciably increase the investments in that branch. So far as his (Mr. Adler's) own office was concerned, it charged 4½ per-cent to its policyholders, and they found that during the last 20 years they had nearly trebled their loans. With regard to Indian and Colonial securities, the author had grouped

those together, but he thought that Indian securities deserved a special place. There were, first of all, Government loans, the interest on which, by means of conversions, had considerably dropped, but there were also the Indian Railway stocks, guaranteed to the extent of 5 per-cent by the Government. Those guarantees had still a fair number of years to run, and offices who were lucky enough to invest in them in years gone by, had reaped greater benefit than by investing even in those debenture stocks which Mr. Coles was so kind as to advise many companies to do. The Bombay and Baroda stock, to the extent of many millions, stood now at 187, and the Great Indian Peninsula at over 170. In 1870 the Indian Government gave notice to purchase the East Indian Railway. Its ordinary stock represented a capital of about 30 millions. When it was found out that the Government were going to purchase it by means of annuities spread over 73 years, as they had the right to do, great was the terror of trustees who had invested in the stock, and it fell from 150 to 120. At that juncture, he (Mr. Adler) suggested that a part of the annuities should be applied as a sinking fund, and be administered by public trustees. That scheme was adopted, and other lines which the Government had since purchased followed the same plan. In the B annuities 1*s.* 4*d.* for each £1 was set apart each year to form that sinking fund. The A annuities, which had not a sinking fund attached to them, were quoted at $24\frac{1}{2}$ per £1 annuity. The B annuities, which had a sinking fund, which, during the 11 years had, with compound interest, accumulated to about 18*s.* or 19*s.*, stood above 29, a difference of very nearly £5,—or £4, allowing for the sinking fund. That showed what a difference it made if securities were rendered available for trustees. Assurance offices could form their own sinking funds, and to them the A annuities were cheap at the price.

Mr. T. Y. STRACHAN said that the author called attention to the loan insurance companies, and had remarked that the most important point was whether those companies could re-insure their risks. The nature of the risk was a most important element in considering what was to be held, and whether it was to be re-insured or not. It must be remembered that in taking a security guaranteed by a loan insurance company, it was not proposed to take from the investing company its security, and to substitute something else: the proposal was to add to the security. What the loan insurance company had to look to was the loan and the margin. If they insured a life, the time came when it died, and they had to pay the whole amount. If they insured against fire, they ran the risk in many cases of total loss; if they insured loans guaranteed upon property they looked to the property first, and the margin between the realized value of the property and the loan at the time of realization was the measure of the probable loss. If the risks were of the nature of those which fire insurance companies called one-and-sixpenny risks, then the amount they would be prepared to hold would be much larger than if they were extraordinary risks. The most important question was, what were the nature of the risks they took, and how much they were to take upon a risk. Securities were offered to insurance companies which, although good, required strengthening. Many such loans bore a good rate of interest, and it was thought that if they had some

guarantee the defect would be cured. It did not necessarily follow that the effect of the loan insurance was to reduce the rate of interest. There were many forms of debentures in English undertakings, and the advantage of a loan insurance company was, that before the company guaranteed the issue of debentures, it dictated the terms upon which they were to be issued. It dictated the form of debenture, it went into the question of title, it made the debenture as good as possible for the advantage of the debenture-holders. American securities had been alluded to, but he was not quite sure whether American municipal securities were a thing that they would care much about. New York was borrowing at $2\frac{1}{2}$ per-cent, and they were not very anxious to go there. He was advised that there was nothing more full of pitfalls for the investor than the issues of small municipalities, but if these were guaranteed by a company that first saw that everything was right, it would give the lender something in addition to what he was originally offered. Then, as to Australia, a loan insurance company might present to offices that were not large enough to send out men of their own the means of introducing securities guaranteed after investigation on the spot, and thereby enable them to share in the high rate of interest so long as it obtained in the Colonies.

Mr. T. G. C. Browne said he could not take the same sanguine view as that of Mr. Adler as to the comparative unimportance of the decline of 7*s.* 6*d.* in the rate of interest, which amounted to about $\frac{1}{4}$ of the margin of profits from interest where companies valued at 3 per-cent. It seemed to him that they could scarcely comfort themselves with the greater misfortunes of their neighbours and rivals. The decline of 7*s.* 6*d.* in their rate was a most serious thing. It naturally made them think what they could do to rectify the matter, but he would submit that it must not on any account be done at the expense of security. The returns of life assurance companies showed that, apart from rare cases of dishonesty, the great mass of the insurance investments had been made with very great ability and discretion. It might be very difficult to define what were first-class securities fit for insurance companies, but he ventured to quote the words of the late Mr. Bagehot, in his book called "Lombard Street", which he applied to what he called first-class mercantile paper. He said, "What constitutes first-class mercantile paper in the City is an ever-varying tradition." The meaning of that was, that if on a certain day they took half a dozen experienced bankers and placed a parcel of bills before them they would come to a pretty unanimous decision as to what bills were first-class paper in that bundle, and what were not. In the same way, if half a dozen experienced life assurance managers were got together and certain securities put before them they would come to a fairly unanimous decision as to what were first-class securities, and what were not. Therefore, though he did not attempt to define first-class securities, most of them who had practical experience had a pretty fair notion of what they consisted, and the question to be considered was, what addition was to be made to that group? It might occur that a security which had hitherto been considered first class might have to be excluded, and, as Mr. Ryan had pointed out, Consols had arrived at that point.

He thought that Consols, as a permanent investment, might not unfairly be described as a security yielding a very low rate for the next 12 years, with a fair prospect of losing a considerable percentage of the capital at the end of that period. Another security whose existence in the group of first-class securities had been questioned of late was that of loans on broad acres. The recent depreciation in the value of land had brought out the weak points of many securities: but he should be very sorry to see the day when a considerable proportion of the funds of life assurance companies was not invested in English agricultural land. They must exercise the greatest care both with regard to margin of income and margin of value, but it must not be forgotten that it was much safer to lend upon a depreciated security than upon an inflated one. There was nothing that more distinctly emphasized the fall in the rate of interest than the fact that although many business men had come to the conclusion that loans on broad acres were not desirable securities, the rate of interest that could be obtained on such securities of the best class had diminished since the depreciation in the value of land commenced. There was another fact to be considered: most of them had lived through a period during which this country had not been involved in war to any great extent, and they should consider what would be the effect of a prolonged naval war upon many of the securities they held. In the event of such a war where the Government of the country would have to borrow very largely, the limit of the rate that securities would yield would be governed by the rate that the Government of this country could borrow at, and it was quite conceivable that this might be 4 or 5 per-cent. It was enough to consider what would become of the premiums upon railway debenture stock if such an event occurred. On the other hand, the probable effect of a prolonged naval war upon English land would be to increase its value materially, because anything that interfered with the supply of food from abroad would have that effect. Those considerations emphasized the necessity for their investments having the characteristic of variety. They should not hesitate, after careful consideration, to admit new kinds of investments into the great groups of first-class securities for insurance companies, but they should exercise caution not to put too much in any one class.

Mr. A. H. Bailey thought they must go back to first principles in defining what was meant by the term rate of interest. Interest was made up of two parts: one, the remuneration for the use of capital; the other, a premium for the risk of its loss. For life assurance purposes it was the first of those they had to consider—at what rate did they improve their capital? To follow that out properly there must be added to what was brought into account as interest, the profits arising from the sale of securities, and any loss on realization must be deducted, the balance being the rate of interest realized. It was quite true, as Mr. Humphreys had said, that, in strictness, income tax ought to be treated like other taxes, but if that were done it must be added to the expenses. Most offices would object to this, and it therefore seemed unavoidable that in calculating the rate of interest income tax should be deducted. He was inclined to adhere to the canons laid down in the paper to which Mr. Mackenzie

had referred; but it must be remembered that they could not control the market. Circumstances had altered very much since 1862. In the early part of the century the investments open to trustees were practically two only, the funds and mortgages on land. Assurance societies were then limited to those investments, and he recollected at an early meeting of the Institute, a director of the Equitable Society stated that it had been their policy to have their investments divided as nearly as might be in equal portions between those two forms, and in nothing else. It was afterwards found out that there were a number of investments well suited for insurance societies and not for private individuals. Several companies sprang up in consequence, which were loan societies first and insurance societies secondly. One class of securities which they sought was life interests, which commanded a high rate. Loans on life interests in those days could not be obtained at less than 6 per-cent, and he had known such transactions carried out at 8 per-cent. Another class was reversions. It was found out that they could afford to lock up capital in that way, but the early assurance societies never heard of such a thing, and, in consequence, reversionary societies came into existence. Then loans repayable by instalments were made to towns on the security of their rates. Now the great towns issued stocks of their own. From his point of view railway debentures were more suitable for insurance companies than debenture stocks, but railway debentures were not now to be had; and, therefore, it being now hardly possible as in former times to obtain investments in which the principal was secure with a reasonable rate of interest, insurance companies must act as private investors would, namely, they must go on the Stock Exchange, and not have all their eggs in one basket, but they must be prepared to run risks of fluctuation in value.

Mr. Augustus Hendriks said he would confine himself to one or two statements made by previous speakers in which he did not concur. With regard to Mr. Humphreys' views as to not deducting income tax from the rate of interest, he agreed with what had been said by Mr. Bailey. It seemed to him that when they were receiving interest as individuals it made a very great difference to them in their income whether they had a dividend declared from which income tax was deducted, or whether they got their dividend in full. Mr. Humphreys illustrated his argument by the house tax, but that was rather an unhappy illustration, because supposing they possessed a house and the house tax was deducted, they would certainly look upon it as a deduction from personal income, and, therefore, why not a deduction from the interest they were earning by similar investments when they represented the interest-earning powers of an insurance company? Mr. Ryan said that the fall in the rate of interest was of greater consequence to companies carrying on annuity business than as applied to life assurance. From that view he utterly dissented. It would be found that, looking at the accounts of insurance companies dealing largely in annuities, the reserve applicable to that class of business was only about eight times the annuity granted. Annuities were generally granted at advanced ages, and their average duration would only be a limited number of years; therefore, as any fall in the rate of interest would take place gradually, it was easy for companies carrying on annuity business to gradually increase their

reserve, if necessary, for existing annuities, and alter their terms for new transactions, the process being much easier than for life insurances. Mr. Coles gave illustrations as to what investments they should cultivate, and what they should avoid. He did not agree with him in the strong objection he expressed to loans upon debentures of "Trust and Loan Companies." Certain of those companies had almost unlimited borrowing powers, as compared with their capital, but there were others that had been most careful, both by their deeds and Acts of Parliament, to limit their borrowing power on debentures, and these offered a field for investment which, if properly cultivated, was second to none. As to Colonial investments, that was a matter which would right itself in the course of a few years: it all turned upon the question of supply and demand. In the old days there used to be only a few companies and individuals investing abroad: there were now a great many. The companies were all gradually getting into the same fields for investment, the tendency thus being towards the equalization of the rate of interest throughout the world.

Mr. George King thought the fall in the rate of interest was really greater than the author had shown, because, in the paper, account was taken of the average rate earned on all the accumulated funds, and but little reference was made to the rates at which investments could be made at the present day. The average rate realized conformed only slowly to the market rate. Between 1830 and 1850 insurance companies were realizing considerably lower rates of interest than they were now making. That being so, he did not think there was any great ground for despondency. The author had mentioned five composite companies whose returns figured in all the three series of accounts. Looking at those five companies, he found, taking the life department, the average rate of interest realized in the author's last period was £4. 2*s*. 8*d*. per-cent, whereas the other departments of the same companies, possibly not under the same managers, but certainly under the same boards of directors, only yielded £3. 14*s*. per-cent. That was a very instructive fact, and showed how, by seeking investments suitable for life companies, they might do a great deal better than others who were compelled to invest their money in more readily convertible securities. Another fact was that the youngest of the five companies made in both departments the highest rate of interest—that showed how by being on the alert, even at the present day, they might get good securities with good interest. He thought that insurance companies should be left unfettered in their powers, and that directors should have unlimited discretion, so as to be able at once to take advantage of new opportunities.

The President was in accord with those who thought that the real fall in the effective rate of interest had not been wholly disclosed by the paper. The contrast presented between what they knew of the movements of the rates of interest in many quarters, especially in those places which in days gone by produced high rates of interest, and the comparatively small fall which had been shown by Mr. Mackenzie, seemed to point to that conclusion. A statement prepared by Mr. Wright with regard to the interest earned by 20 American companies had been referred to. The figures appeared to show that the rate of decline in the interest earned by those companies was from 6·1 to 4·6. That in itself was sufficiently serious, but it by no means

stated the whole case. The rate earned (6·1) was from 1869 to 1872, but about 1870 there was a very interesting publication brought out by one of the great American life offices, on the occasion of what it called its silver wedding, in which it was shown that during the previous 25 years it had earned interest at the average rate of 7 per-cent. Therefore, to go back to a period well within living memory, the reduction in the rate of interest earned by American companies had been from 7 per-cent to 4·6. If, as the author stated, the decline in the rate of interest earned by English assurance companies amounted to only 7*s.* 6*d.* per-cent, in what was that small decline to find its explanation? He thought it had its explanation in this: that a large proportion of the funds of the companies were old investments, some being fluctuating securities the marketable value of which had risen, but the cost price of which had not gone up in the companies' books; and some being reversionary interests or loans by way of annuities, or even ordinary mortgages, comprising a considerable proportion of the securities held by the insurance offices, which had not been subject to the modern decline in the rate. If they could have a return as to the rates at which they could now invest money the contrast would be far more serious than the author had brought out. It was an exceedingly unwise saying, that a high rate of interest necessarily meant bad security. It meant very possibly a kind of security which, because it was unknown, was doubted, or was not suitable to many people. Loans on rates within comparatively recent periods commanded a high rate of interest—5 per-cent. Loans of this kind, where the form of repayment did not suit ordinary investors, were suitable only to a very limited number of investors, and there being a large demand and a small supply there was naturally a high rate. The general decline in the rate of interest had arisen from a complete change in the economic conditions of the world. That which caused the high rate of interest in America and in Australia was a cause which was passing away. There was a large country with a sparse population, with unsettled government, and a want of credit. Those were conditions which were vanishing, and they were approaching the time when the rate of interest would be equalized. But that was not all. The days when high rates of interest were obtainable in those countries were days when they were practically isolated in the world. He believed that the permanent rate of interest had gone down for all time, and that was the broad moral which he drew from the paper. Then came the question, what was the way in which managers of life assurance companies ought to endeavour to meet this? Ought they not to reconsider the rate at which, in their calculations, they assumed they could during long periods of years invest their funds? In addition to endeavouring to maintain the rate of interest in the manner pointed out in the paper, they should look to the other side of the question, and if they had not the large interest income of former days, let them cut their coat according to their cloth, and reduce the rate at which they valued their liabilities.

Mr. Mackenzie, in reply, said that with regard to income tax being deducted from interest, Mr. Humphreys had stated his reasons very clearly from his point of view, and the reasons on the other side had been plainly stated also. What he wished to emphasize was, that

whichever rule was adopted it should be adopted uniformly, and an authoritative direction from the Board of Trade would be very useful. Mr. Coles had spoken as to the securities which were dealt with on the Stock Exchange, and he agreed that many even of the debenture companies required most careful consideration. In saying that companies might turn their attention to these securities, he meant, of course, that they must be very carefully investigated. He was bound to take notice of Mr. Strachan's criticisms of his remarks upon companies that insured debentures and other securities. He was sorry to say that Mr. Strachan's remarks did not convince him in the least. He still thought that what he had stated was perfectly sound doctrine—that if they were to ask another company to relieve them of certain risk they must consider what the liabilities of that company were, and what was the prospect of having a really sound security to fall back upon in the insurance. Mr. King had stated that in 1840 the offices were making lower rates of interest than at the present moment. That was perfectly correct, and Mr. Bailey had shown the reason why. They were then devoting their attention to a very limited number of securities, and if they had made better use of their opportunities they might have earned a good deal more interest. He hoped they would be able, by increased diligence in search, to prevent a serious fall in the rate of interest in the future. He quite agreed with the President that the fall was really greater than was shown in his tables.

Final Remarks on Mr. Woolhouse's Method of Graduation, with a Numerical Example showing how far it distorts the Law of a perfectly graduated Series of Numbers. By T. B. SPRAGUE, M.A., F.I.A., &c.

THOSE who have studied my paper on the Graphic Method of Graduation will remember that one of my objections to Mr. Woolhouse's method of graduation was that it must necessarily disturb the law of any numbers to which it is applied, unless these have throughout constant third differences. I gave a mathematical demonstration of this proposition, which establisht it beyond all question, but did not show the extent of the disturbance. Mr. Woolhouse nevertheless says that a proper application of his method can never have a tendency to distort the law which the facts may follow. In this state of matters I think it will be useful to apply to the question the conclusive test of numerical calculation; and in this way I shall be able, not only to prove the correctness of my statement that there is a disturbance of the law of the numbers, but also to show the exact extent of such disturbance.

When Mr. Woolhouse's method is applied to a series of ungraduated figures, it is scarcely possible to show how it affects the law of the series, because that law is itself unknown until all

the irregularities are got rid of. But if there are no irregularities in the series of numbers to which the method is applied, the effect of the graduation on the series becomes at once apparent. I have accordingly applied Mr. Woolhouse's method to regraduate a mortality table which is perfectly free from irregularities, namely, the table given in the *Institute Text Book*, Part II, page 88, which is a graduation of the H^{M} Table by Mr. Makeham's formula. The values of log l_x are given in that work to 7 decimal places; but the value of l_x is calculated to a radix of 102,906 at age 10, so that throughout the greater part of the table l_x contains only five figures. This I found was not sufficient for my purpose, and I have therefore had the values of l_x computed to an additional place, as shown in column (2) of the following table. The values of d_x are given in column (3), and their differences in column (4). The values of d_x, as regraduated by Mr. Woolhouse's method, are given in column (5), and their differences in column (6). The regraduated values of l_x (which I denote by l'_x) are given in column (7), and the corresponding probability of dying in a year, q'_x, to five decimal places, in column (8). Column (9) contains the same probabilities according to the original table, and column (10) contains the difference $q_x - q'_x$. Lastly, column (11) contains the difference $l_x - l'_x$.

We see that these differences follow a regular law. Up to age 54 the values of q_x and q'_x are practically identical, then up to age 72 the regraduated value, q'_x, is greater than the original, q_x, by a small amount, never exceeding ·00003; then up to age 88 q_x is the smaller, the difference gradually increasing from ·00001 to ·00084, and then diminishing rapidly. From age 89 to the end of the table, q'_x is again the larger of the two, the difference constantly increasing from ·00094 to ·18280 at age 97. A similar law pervades these differences. The numbers living, l_x and l'_x, are exactly (or approximately) equal at the ages 10, 54, 77, and 91, and in the intervals between these ages the values in the one table are alternately greater and less than those in the other; that is to say, in the intervals from 10 to 54, and from 77 to 91, l'_x is the larger of the two; and in the interval from 54 to 77, and from 91 to the end of the table, l_x is the larger. It will be observed also that, whereas the original table extends to the age of 103, the regraduated table stops at the age of 98; at which age the process of calculation brings out 77 deaths, although the number living is only 76. If the calculation is carried further we get 26 deaths at age 99, and 2 at 100: in

other words, at these high ages Mr. Woolhouse's method produces absurd results. At still higher ages the process gives negative values of d'_x, namely, -7, -8, -6, -4, -2, -1, -1.

Taking a general view of the figures, however, we infer that, although Mr. Woolhouse's method slightly disturbs the law of the series (l_x) to which it is applied, the practical effect of this disturbance on the probability of dying is so slight as to be of no practical importance except at extreme old ages. I feel, therefore, bound to admit that my objection to the method, on the ground of its tendency to disturb the law of the series, although theoretically well founded, may be disregarded in practice.

x	TEXT BOOK TABLE			REGRADUATION BY WOOLHOUSE'S METHOD				TEXT BOOK	DIFFERENCES	
	l_x	d_x	Δd_x	d'_x	$\Delta d'_x$	l'_x	q'_x	q_x	$q_x - q'_x$ (9)−(8)	$l_x - l'_x$ (2)−(7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
10	1,029,060	6,609	−18	6,609	−19	1,029,060	·00642	·00642	0	0
1	1,022,451	6,591	−16	6,590	−16	1,022,451	·00645	·00645	0	0
2	1,015,860	6,575	−14	6,574	−13	1,015,861	·00647	·00647	0	−1
3	1,009,285	6,561	−11	6,561	−12	1,009,287	·00650	·00650	0	−2
4	1,002,724	6,550	− 9	6,549	− 8	1,002,726	·00653	·00653	0	−2
5	996,174	6,541	− 5	6,541	− 5	996,177	·00657	·00657	0	−3
6	989,633	6,536	− 3	6,536	− 3	989,636	·00660	·00660	0	−3
7	983,097	6,533	0	6,533	0	983,100	·00665	·00665	0	−3
8	976,564	6,533	5	6,533	5	976,567	·00669	·00669	0	−3
9	970,031	6,538	8	6,538	8	970,034	·00674	·00674	0	−3
20	963,493	6,546	12	6,546	12	963,496	·00679	·00679	0	−3
1	956,947	6,558	17	6,558	17	956,950	·00685	·00685	0	−3
2	950,389	6,575	21	6,575	21	950,392	·00692	·00692	0	−3
3	943,814	6,596	27	6,596	27	943,817	·00699	·00699	0	−3
4	937,218	6,623	34	6,623	33	937,221	·00707	·00707	0	−3
5	930,595	6,657	38	6,656	39	930,598	·00715	·00715	0	−3
6	923,938	6,695	45	6,695	45	923,942	·00725	·00725	0	−4
7	917,243	6,740	53	6,740	53	917,247	·00735	·00735	0	−4
8	910,503	6,793	61	6,793	60	910,507	·00746	·00746	0	−4
9	903,710	6,854	69	6,853	69	903,714	·00758	·00758	0	−4
30	896,856	6,923	77	6,922	78	896,861	·00772	·00772	0	−5
1	889,933	7,000	88	7,000	88	889,939	·00787	·00787	0	−6
2	882,933	7,088	99	7,088	98	882,939	·00803	·00803	0	−6
3	875,845	7,187	109	7,186	110	875,851	·00820	·00821	1	−6
4	868,658	7,296	122	7,296	122	868,665	·00840	·00840	0	−7
5	861,362	7,418	135	7,418	135	861,369	·00861	·00861	0	−7
6	853,944	7,553	149	7,553	149	853,951	·00884	·00884	0	−7
7	846,391	7,702	164	7,702	164	846,398	·00910	·00910	0	−7
8	838,689	7,866	181	7,866	181	838,696	·00938	·00938	0	−7
9	830,823	8,047	198	8,047	197	830,830	·00969	·00969	0	−7
40	822,776	8,245	216	8,244	217	822,783	·01002	·01002	0	−7
1	814,531	8,461	236	8,461	236	814,539	·01039	·01039	0	−8
2	806,070	8,697	257	8,697	257	806,078	·01079	·01079	0	−8
3	797,373	8,954	281	8,954	280	797,381	·01123	·01123	0	−8
4	788,419	9,235	302	9,234	303	788,427	·01171	·01171	0	−8
5	779,184	9,537	328	9,537	329	779,193	·01224	·01224	0	−9
6	769,647	9,865	355	9,866	354	769,656	·01282	·01282	0	−9
7	759,782	10,220	383	10,220	383	759,790	·01345	·01345	0	−8
8	749,562	10,603	409	10,603	411	749,570	·01415	·01415	0	−8
9	738,959	11,012	442	11,014	441	738,967	·01491	·01490	− 1	−8

x	Text Book Table			Regraduation by Woolhouse's Method				Text Book	Differences	
	l_x	d_x	Δd_x	d'_x	$\Delta d'_x$	l'_x	q'_x	q_x	$q_x - q'_x$ (9)−(8)	$l_x - l'_x$ (2)−(7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
50	727,947	11,451	471	11,455	471	727,953	·01574	·01574	0	−6
1	716,493	11,925	502	11,926	503	716,498	·01665	·01664	−1	−5
2	704,568	12,427	536	12,429	536	704,572	·01764	·01764	0	−4
3	692,141	12,963	566	12,965	567	692,143	·01873	·01873	0	−2
4	679,178	13,529	599	13,532	599	679,178	·01992	·01992	0	0
5	665,649	14,128	629	14,131	631	665,646	·02123	·02122	−1	3
6	651,521	14,757	661	14,762	660	651,515	·02266	·02265	−1	6
7	636,764	15,418	687	15,422	688	636,753	·02422	·02421	−1	11
8	621,346	16,105	714	16,110	714	621,331	·02593	·02592	−1	15
9	605,241	16,819	733	16,824	735	605,221	·02780	·02779	−1	20
60	588,422	17,552	751	17,559	751	588,397	·02984	·02983	−1	25
1	570,870	18,303	761	18,310	762	570,838	·03208	·03206	−2	32
2	552,567	19,064	764	19,072	765	552,528	·03452	·03450	−2	39
3	533,503	19,828	760	19,837	759	533,456	·03719	·03717	−2	47
4	513,675	20,588	742	20,596	743	513,619	·04010	·04008	−2	56
5	493,087	21,330	717	21,339	715	493,023	·04328	·04326	−2	64
6	471,757	22,047	673	22,054	674	471,684	·04676	·04673	−3	73
7	449,710	22,720	619	22,728	616	449,630	·05055	·05052	−3	80
8	426,990	23,339	544	23,344	543	426,902	·05468	·05466	−2	88
9	403,651	23,883	454	23,887	451	403,558	·05919	·05917	−2	93
70	379,768	24,337	344	24,338	341	379,671	·06410	·06408	−2	97
1	355,431	24,681	216	24,679	211	355,333	·06945	·06944	−1	98
2	330,750	24,897	68	24,890	64	330,654	·07528	·07527	−1	96
3	305,853	24,965	−97	24,954	−104	305,764	·08161	·08162	1	89
4	280,888	24,868	−280	24,850	−285	280,810	·08849	·08853	4	78
5	256,020	24,588	−475	24,565	−480	255,960	·09597	·09604	7	60
6	231,432	24,113	−677	24,085	−683	231,395	·10409	·10419	10	37
7	207,319	23,436	−885	23,402	−889	207,310	·11288	·11304	16	9
8	183,883	22,551	−1,088	22,513	−1,091	183,908	·12241	·12264	23	−25
9	161,332	21,463	−1,283	21,422	−1,282	161,395	·13273	·13304	31	−63
80	139,869	20,180	−1,457	20,140	−1,454	139,973	·14389	·14428	39	−104
1	119,689	18,723	−1,604	18,686	−1,598	119,833	·15591	·15643	49	−144
2	100,966	17,119	−1,717	17,088	−1,708	101,147	·16894	·16955	61	−181
3	83,847	15,402	−1,788	15,380	−1,775	84,059	·18297	·18369	72	−212
4	68,445	13,614	−1,812	13,605	−1,797	68,679	·19810	·19890	80	−234
5	54,831	11,802	−1,786	11,808	−1,771	55,074	·21440	·21524	84	−243
6	43,029	10,016	−1,713	10,037	−1,698	43,266	·23198	·23277	79	−237
7	33,013	8,303	−1,593	8,339	−1,582	33,229	·25096	·25151	55	−216
8	24,710	6,710	−1,439	6,757	−1,430	24,890	·27147	·27156	9	−180
9	18,000	5,271	−1,255	5,327	−1,254	18,133	·29377	·29283	−94	−133
90	12,729	4,016	−1,058	4,073	−1,061	12,806	·31805	·31550	−255	−77
1	8,713	2,958	−859	3,012	−867	8,733	·34490	·33949	−541	−20
2	5,755	2,099	−668	2,145	−682	5,721	·37493	·36473	−1,020	34
3	3,656	1,431	−498	1,463	−513	3,576	·40912	·39141	−1,771	80
4	2,225	933	−354	950	−367	2,113	·44960	·41933	−3,027	112
5	1,292	579	−238	583	−251	1,163	·50129	·44814	−5,315	129
6	713	341	−151	332	−160	580	·57241	·47826	−9,415	133
7	372	190	−92	172	−95	248	·69355	·51075	−18,280	124
8	182	98	−50	77	−51	76	1·01316	·53846	−47,470	106
9	84	48	−26	...	...	...	...	·57045	...	...
100	36	22	−13	...	...	...	...	·61111	...	...
1	14	9	−6	...	...	...	...	·64286	...	...
2	5	3	−1	...	...	...	...	·60000	...	...
3	2	2	−2	...	...	...	...	...	...	...
4	0	...	...	...	...	...	...	...	...	...

Having had an opportunity of reading Mr. Woolhouse's *Further Observations* (see below), I have very little to add to what I have already said; for while those observations contain, as usual, much strong assertion, which I think it better not to notice further, they contain very little argument. There are only two points which really call for notice. *First,* Mr. Woolhouse says that a properly graduated mortality table will not be altered by the application of his system of adjustment; and although he claims to have demonstrated this, it is clear that his argument is inconclusive, because the figures given above show by a numerical example, what I had previously proved by mathematical demonstration, that a perfectly graduated table *is* altered by the application of Mr. Woolhouse's method. *Secondly,* Mr. Woolhouse frankly tells us that science, as represented by the mathematical theory of probable errors, is powerless to get rid altogether of the irregularities. For this reason I think myself justified in putting that theory on one side. It is, no doubt, useful in its proper place, but it is out of place in the graduation of mortality tables; and I appeal to science, as represented by the graphic method, and find that it is capable of getting rid of the irregularities. Of course I repudiate altogether Mr. Woolhouse's assumption, that his method is scientific, and mine empirical.

The readers of the *Journal* have now the whole of the facts before them. Mr. Woolhouse prefers his adjustment of the $H^{M(5)}$ Table because it is based on the theory of errors. I prefer mine because it is free from the irregularities which disfigure Mr. Woolhouse's, and yet keeps closer to the original observations during fully half the extent of the table. There the matter must rest, so far as I am concerned.

Some Further Observations on the Method of Adjusting Mortality Tables. By W. S. B. Woolhouse, F.I.A., &c.

I WAS under the impression that enough had been said by me in elucidation of the above subject in my former observations (*J.I.A.*, xxvi, 420); and, independently of this, I was assured that the principles of the method were well understood by the generality of readers interested in such matters. But after a prolonged silence of upwards of three years, Mr. Sprague has returned to the subject (*J.I.A.*, xxix, 59). Mr. Sprague, in his criticisms on this particular subject has been, to my mind,

unfortunately, too much disposed to sit in judgment upon a question without fully ascertaining or duly recognizing its actual conditions from a thoroughly practical point of view; and thus all his statements, drawn from considerations of curves of the fourth and higher orders, and of interpolations with fourth and higher orders of differences, are wholly inapplicable and untrue.

In opposition to my statement of the fact that there is nothing of an arbitrary nature in my process of adjustment, Mr. Sprague asserts that the selection of quinquennial values in the series to be adjusted is entirely arbitrary. It is somewhat difficult to comprehend that so meagre an assertion can be meant seriously. The interval to be adopted is of course governed exclusively by practical considerations, according to the nature of the enquiry, and is in other respects quite immaterial. I was originally induced to take quinquennial values for two prominent reasons: The first and principal consideration was that the column of the numbers living, as shown by a well-adjusted mortality table, with intervals of five years, will be practically susceptible of interpolation for intermediate ages, by the employment of first and second central differences. The other reason was in regard to facility and accuracy of calculation in consequence of the coefficients used in the interpolations being all of them limited to two places of decimals. The result of my practical experience fully confirms me in the opinion that, for mortality tables, for which the method was especially designed, I could not have chosen a better interval. With a less interval than five years the adjustment would be less effective, and with a greater interval a higher order of differences would be required in the interpolations.

To my statement that if the original points, taken in groups, were to range in curves of the third order, they would in such case not be subject to alteration, Mr. Sprague objects by saying that no mortality table proceeds by constant third differences *throughout the whole* extent of the table. Here Mr. Sprague entirely overlooks the gist of my argument, which is this:—That if a group of fifteen consecutive points of the adjusted curve of mortality practically range in a curve of the third order, the central point of that group will not be altered by the application of my formula of adjustment. The same consideration of course applies to each of the other groups; but each of these curves of the third order may be slightly different from all the others, so that it is not necessary to have constant third differences throughout the whole of the table, as Mr. Sprague supposes.

Thus, in the vicinity of every point the curve is closely fitted with a different curve of the third order, and no restraint is put upon the series of values.

Mr. G. King specially called Mr. Sprague's attention to this point in the discussion which followed the reading of his paper on the graphic method, but Mr. Sprague did not take the matter up.

To bring my system of adjustment briefly before the reader, it may be convenient here to re-state the fundamental process:

1. Begin with the first age in the table, and extract the numbers living for quinquennial intervals of age; then, by the formula for interpolation, with central differences, determine all the values for the intervening ages, and so obtain a complete series of values, constituting a distinct Table A.

2. Begin with the second age in the table and, as before, extract the numbers living at quinquennial intervals, and by interpolation obtain another complete series of values, constituting Table B.

3. Begin with the third age, and, in like manner, complete a Table C.

4. Begin with the fourth age, and, similarly, complete a Table D.

5. Begin with the fifth age, and complete a fifth and last Table E.

6. All the original numbers living are equally employed in the production of these five fundamental tables, A, B, C, D, and E. For every given age x these tables will show, for the number living, five values, l_x^A, l_x^B, l_x^C, l_x^D, and l_x^E. The adjusted value, $(l)_x$, is found by taking the average value of these; and is, therefore,

$$(l)_x = \tfrac{1}{5}(l_x^A + l_x^B + l_x^C + l_x^D + l_x^E).$$

The six operations here detailed constitute the radical basis of the entire system of adjustment, and clearly exhibit its true and logical character. It was precisely thus that I at first manipulated the adjustment of the H^M Table before the operations were moulded into the known algebraic formula. The whole of the results were afterwards thoroughly checked by being re-worked from the formula.

By way of hypothesis, let us conceive a properly-graduated and correct mortality table, the progression of the terms of which is so regular that no adjustment is needed. If we operate upon such a table, as it will be susceptible of interpolation, it will follow that the interpolations will only reproduce the original numbers, and that, in each of the five fundamental tables, A, B, C, D, and E, the numbers living must practically coincide at every age with those of the original table. Therefore, the average of the five tables, which gives the adjusted value, will be just the same.

Thus, it is clearly shown to a demonstration that, if the table operated upon be correct, it will not be altered by the application of my system of adjustment. Hence, in any actual case of adjustment the process, as I have before explained, operates upon the errors alone, and thereby brings the true law of the series into greater prominence.

Mr. Sprague cannot admit that when we ask for an adjusted value of l, the values of l_{-7}, l_{-6}, l_{+6}, l_{+7} necessarily have always a legitimate influence on the result. However, as these values necessarily enter into the formation of the differences upon which the interpolations are founded, and as the interpolations are perfectly legitimate operations, it is self-evident that the said values thus properly exercise their legitimate influence, and no more than their legitimate influence.

Mr. Sprague (p. 62) remarks as follows:

"Mr. Woolhouse admits that his method regards smoothness of progression only as an adjunct; and that, when his method has been applied, there may still remain traces of irregularity which are to be amended by inspection of the differences. I believe that, if Mr. Woolhouse had so plainly stated all this in the first instance, his method would not have been so greatly esteemed, and so popular, as it was for a time."

I beg to say that this very small matter was most distinctly pointed out when my method was first introduced, and was explained in all my papers on adjustment, to the following effect:

"It is possible, however, that at one or two exceptional places there may yet remain some slight traces of irregularity, but they are sure to be quite isolated, and so trivial as to be readily amended by mere inspection."

I have only to add here that these "slight traces", if any, very rarely occur, and are, when compared with the errors of observation, so very small and insignificant, that whether they

are amended or not is a matter of perfect indifference as regards any considerations of accuracy. The only motive is that suggested by the very satisfactory progression of the differences of the series of adjusted results. Referring, as examples, to the Mortality Tables of the Institute, the regularity observable in the progression of the differences of the adjusted numbers living is indeed very remarkable when we consider that these adjusted values are carried out to figures one place further than the unadjusted values which constitute the original data from which they are drawn.

The following conclusion of the paragraph is vastly more important, and cannot be too strongly impressed upon the mind of the operator:

"It may be further observed that the final table, as a correct representation of the average mortality of the class of lives under consideration, will be entitled to the fullest reliance when the corrections are small in proportion to the respective numbers. This will invariably be the case throughout the table when the experience is accurately recorded, and the materials are sufficiently ample. It is almost needless to add that if, on the contrary, the computed corrections come out proportionately large, the fact will surely imply that the probable errors of the data are considerable, that the original observations are essentially defective, and that no results deduced from them alone can be trustworthy."

If the resulting table comes out as in the latter case here mentioned, it should be at once abandoned as practically useless. In my paper "On the Philosophy of Statistics" (see *J.I.A.*, xvii, 37), I have specially pointed out some prominent faults occasionally to be found in statistical methods of enquiry. It has often surprised me how many investigators of repute are habitually committing the fourth of these faults, namely, reasoning upon insufficient numbers and drawing precise, formal, and confident deductions from a too-limited number of observed facts which are quite insufficient to convey any reliable statistical information.

Mr. Sprague continues thus:

"I maintain, on the contrary, that the true law of mortality can show no irregularities as we pass from age to age; and, accordingly, in my method of graduation I lay it down as a fundamental principle, that the irregularities must be got rid of."

And further on he says:

"The difference of opinion between Mr. Woolhouse and myself on this point arises from the fact that he is satisfied with reducing the irregularities, or rather distributing them over other parts of the curve; whereas I say that a method of graduation, in order to be satisfactory, must altogether get rid of the irregularities."

The opinion, or fundamental principle, here so confidently laid down by Mr. Sprague, is in direct opposition to the mathematical theory of probable errors. From a series of values, derived from observation, which are severally affected by unknown errors, science is powerless to get rid altogether of the irregularities. All that can be done is to reduce the probable errors. If the irregularities are got rid of by artificial or arbitrary means, the desired effect on the probable errors will not be attained. The table, instead of being adjusted, will merely be graduated, and it will not so approximately represent the experience. On these considerations, I am bound to accept my adjusted $H^{M.5}$ Table as more reliable and trustworthy than Mr. Sprague's empirical table. Its superiority is, indeed, plainly indicated by Mr. Sprague's own figures, given in the last columns of his Tables A and C. According to my adjustment, the progressive "totals of the excess of the expected over the actual deaths" show only six cases over 20·0 ($\mp$). By Mr. Sprague's graduation, there are 22 cases exceeding 20·0 ($\mp$); and a most objectionable persistency of negative values continues from age 74 to age 85.

In conclusion, I emphatically repeat that, with practicable data, the application of my system of adjustment to a mortality table will always give the best possible results. It is satisfactory to know that every advantage and superiority I have claimed for the system of adjustment has been fully recognized and approved by several mathematicians of repute, amongst whom I may refer to the highly favourable and unqualified opinion so freely and spontaneously given by Mr. Makeham in his able and valuable paper "On the Further Development of Gompertz's Law" (*J.I.A.*, xxviii, 152). On page 325, Mr. Makeham expresses himself thus:

"By supposing that a law of mortality does exist, but without making any *a priori* assumption whatever as to its nature, Woolhouse has deduced a beautiful and exceedingly valuable formula of adjustment, the principle of which is not likely, I think, soon to be superseded by anything better adapted to the purpose required. The beauty and value of the formula, to my mind, consists chiefly in the fact that each individual observation is treated exactly like all the rest—a very great advantage indeed over the old method of selecting, arbitrarily, isolated terms of the mortality table, and deducing therefrom the intervening values by the ordinary processes of interpolation, as all who have been engaged in investigating the normal law of mortality will very well know. By his ingenious contrivance, Woolhouse enables us to take, at the outset, a most important preliminary step in approximating to the true probabilities,

and thus to facilitate greatly the subsequent investigation of the *nature* of the law—respecting which, as I have said, he makes no assumption whatever, but the knowledge of which must certainly prove of the greatest service in the determination of the best possible substitute for the true probabilities of life at successive ages."

It is impossible to go further than the unreserved opinion here so clearly and comprehensively stated.

I cannot hold myself responsible for the exceptional opinion formed by Mr. Sprague, but in my earnest pursuit of truth, I am compelled to assert and maintain what I well know to be correct.

On the Theory of Inverse Probabilities. *By* William Matthew Makeham.

Another investigation into the mortality experience of life assurance companies by the Institute of Actuaries will, in all probability, take place at no very distant period; and I think it is extremely desirable that this subject should previously be ventilated in the pages of the *Journal.* A very elementary acquaintance of that branch of the doctrine of probabilities which treats of the law of "Errors of Observation" will suffice to enable anyone to follow my reasoning in the following pages, and to form his own opinion as to whether the conclusions which I have arrived at are right or wrong.

Section 1.

In paragraph 5 of the "Useful Knowledge" Society's Treatise on Probability (by Lubbock and Bethune), the following observation occurs: "Simpson has defined the probability of an event to "be the ratio of the chances by which the event in question may "*happen* to all the chances by which it may *happen or fail.* In "this definition, by the word 'chance' must be understood "*a way of happening.*" The use of the word "chance" to signify "a way of happening" appears to me, in certain cases, to be very convenient, and I purpose availing myself of it in the course of the present article. For instance, in this sense of the word, we may (quite properly) be said to be ignorant of, or imperfectly acquainted with, the "chances" for and against any particular event; and may be altogether unable to determine accurately their *true* ratio, although their *probable* ratio it may be quite within our power to estimate. The term "probability", on the

other hand, cannot properly be used in the same way. We cannot be said to be ignorant of the *probability* of a given event, for the term "probability" has no reference to the chances (for and against) actually *existing*, but only to our *knowledge* of them. The *probability*, therefore, can always be determined by calculation, provided, of course, that we possess the skill necessary for the purpose.

Let us take, by way of illustration, the familiar instance of tossing a coin. If the coin were perfectly symmetrical, or evenly balanced, the relative chances of "head" and "tail" would be equal; and whether this fact (of perfect symmetry) were known or not to the observer, he would, *in either case*, correctly infer that the probability of head in a single throw is one-half. But the probability in question would likewise be one-half if the coin were known to be *not* symmetrical, *provided* that the observer did not know which side (head or tail) had the preponderance of chances in its favour. But if the observer knew on which side the balance lay, although the relative *chances* for and against head or tail would, of course, still be the same, the *probability* of head or tail would not then be one-half, but something different.

We must not, however, infer from this that if a coin, not perfectly symmetrical, be tossed *twice* (the knowledge of the observer being limited as in the above proviso) the probability of its falling head (or tail) *twice in succession* will be $\left(\frac{1}{2}\right)^2$ or $\frac{1}{4}$. Laplace has supposed the case where the chances for and against head in one throw are as $1+w$ and $1-w$ respectively, the magnitude of w being supposed to be known but not its sign. If positive (w denoting its numerical value), the probability of head twice in succession is $\left(\frac{1+w}{2}\right)^2$, if negative, it is $\left(\frac{1-w}{2}\right)^2$, and these two suppositions being equally likely the probability of head twice in succession would be the mean value $\frac{1}{2}\left\{\frac{(1+w)^2}{4}+\frac{(1-w)^2}{4}\right\}=\frac{1}{4}+\frac{w^2}{4}$. And the same expression will evidently denote also the probability of *tail* twice in succession. Hence, the *probability* of head twice in succession, and also of tail twice in succession, are *both* increased by the defective symmetry of the coin, *on whichever side the preponderance of chances may happen actually to lie.* Laplace speaks of this "aberration of the ordinary theory" (as he terms it) as very important in the practical application of the theory of probabilities. Of its

importance there can be no doubt, but I think it can hardly be called an "aberration" of that theory: for a little consideration will, I think, suffice to show that the result is, after all, not so very paradoxical as it might appear at first sight. What it really shows, I think, is the necessity of distinguishing between the actual "ratio of chances" on the one hand, and on the other the extent of our *knowledge of that ratio*, which alone determines the "probability."

Laplace's well-known formula in inverse probabilities, $\frac{m+1}{m+n+2}$, which expresses the probability that a coin, having fallen head m times and tail n times in $m+n$ trials, will again fall head in the next trial, is deduced upon the supposition that nothing whatever is known beforehand of the actual relative chances of head and tail; and therefore that the antecedent probabilities of these events are each equal to $\frac{1}{2}$. If, in Laplace's formula, we put $n=0$, we have $\frac{m+1}{m+2}$ for the probability that a coin which has fallen head m times in m trials will again fall head in the $(m+1)$th trial. And putting m in $\frac{m+1}{m+2}$ successively $=0, 1, 2,$ &c., we have

(*Laplace's Formula*)

Number of times Head has successively turned up (m)	Probability of Head in the next Trial $\left(\frac{m+1}{m+2}\right)$	
0	1 : 2	·5
1	2 : 3	·667
2	3 : 4	·75
3	4 : 5	·8
4	5 : 6	·833
⋮	⋮ ⋮	⋮
998	999 : 1,000	·999

The first value in the preceding table, namely, $\frac{1}{2}$, which corresponds to $m=0$, represents the antecedent probability of the event in question. The next, $\frac{2}{3}$, corresponding to $m=1$, represents the altered probability of the *second* trial, by taking into account the known fact that head has actually turned up in the

first—the *knowledge* of which fact, we see, has had the effect of increasing the probability of head in the next trial from $\frac{1}{2}$ to $\frac{2}{3}$. And so on with the subsequent values, each succeeding addition to the number of times that head has turned up (that is, the knowledge of it) having the effect of increasing still further the probability of head in another trial; until we arrive at the value ·999, which represents the probability of head when m has reached 998. And if the table were continued indefinitely, we should continue to approximate still more and more nearly to unity, or certainty, but without ever actually reaching it. These results are evidently precisely what we should expect from the fundamental assumptions of Laplace's investigation, which imply that the antecedent probability of head turning up in a single trial is $\frac{1}{2}$.

These fundamental assumptions of Laplace's formula are—first, that the ratio of chances, for and against, may have any value from 0 to 1; and, secondly, that all values within those limits are *a priori* equally probable. It must, however, be evident that, in many applications of the theory of inverse probabilities, these assumptions may differ very widely from the facts; and therefore must have the effect of circumscribing materially the availability of the very valuable results of Laplace's investigation. In a very interesting discussion of this subject, which took place not long ago in the columns of the *Insurance Record,* Mr. G. F. Hardy very reasonably objected, upon this ground, to the proposed adoption of Laplace's formula for the purpose of deducing the probabilities of life from the experience of assurance offices. I propose, therefore, in the next section, to endeavour to discover a practicable remedy to the defects pointed out by Mr. Hardy, by *generalizing* the formula obtained by Laplace for the one particular case he has dealt with, and in which the antecedent probability of the event is $\frac{1}{2}$.

Section 2.

Suppose we have a mixed mass, of unlimited magnitude, consisting exclusively of white and black balls in definite and known proportions. Let the ratio of the number of white balls to the whole be denoted by p, and consequently the ratio of black balls to the whole by $1-p$, or q, $(p+q=1)$. The ratio of the

chances, for and against, of drawing a white ball will therefore be as p to q.

Next, let a number of urns be filled from this hypothetical mass of white and black balls, and, in order to avoid the possibility of unconscious selection, let the operation be supposed to be performed blindfold. It is evident that the ratio of chances, for and against, of drawing a white ball (from any given urn) will not now necessarily be as p to q; although in each urn it will tend to approximate more and more to that ratio as the number of balls placed therein is increased. Let the actual ratio in any particular case be as p' to q', $(p'+q'=1)$.

The relation between p and p', above indicated, constitutes, in fact, the fundamental and essential characteristic of the quantity p. Hence we have the following

Definition.—The quantity denoted by p is the limit towards which the unknown ratio p' (in any particular urn) necessarily tends more and more to approximate as the number of balls contained in the urn is increased.

The ratio p being *known*, it will evidently denote what is termed the antecedent, or *a priori*, probability of drawing a white ball from any one of the urns. It will also bear that relation to all the *possible* values of p', in any given urn, which Laplace has designated "le milieu de probabilité"; that is to say, p is a quantity such that the true value of p' in any particular urn is just as likely to be *above* as *below* it. Further, if a large number of urns have been filled from the supposed mass of white and black balls, the expected number in which the ratio p' is of greater magnitude than p, is equal to the expected number of those in which p' is of less magnitude than p. Therefore p is also the *milieu de probabilité* of the several values of p' actually existing in the different urns.

Suppose, now, that a succession of $m+n$ drawings have been made from one particular urn (each ball drawn being replaced) with the result that m white and n black balls have been drawn; and that it is required to determine the probability $(p_{m.n})$ of drawing a white ball in the next trial.

First, we may observe that p and $p_{m.n}$ both denote the probability of drawing a white ball in the next trial, but at different epochs, and under different circumstances. For p represents the *a priori* probability (before any drawings have yet been made); while $p_{m.n}$ represents the *a posteriori* probability (after the fact that m white and n black balls have been drawn *has become known*

to the observer). Although the actual *ratio of chances* necessarily remains the same in both cases, the relative *probabilities* differ, because the information respecting those chances is greater after the $m+n$ drawings have been made than it was before.

If, however, m, the number of white balls drawn, should happen to be equal to $p(m+n)$, the expected number, it is evident that $p_{m.n}$ will necessarily be equal to p; for in that case the result of the $m+n$ trials will have afforded no reason for altering the estimate of the probability, either by an increase or a diminution of p. The first of the two following postulates is, therefore, sufficiently evident; and as regards the second, very little consideration is, I think, required to ensure its admission.

Postulate 1.—If $p=\dfrac{m}{m+n}$, then $p_{m.n}$ is also equal to $\dfrac{m}{m+n}$, and to p.

Postulate 2.—In all other cases $p_{m.n}$ will necessarily lie between $\dfrac{m}{m+n}$ and p.

Because p is the *milieu de probabilité* of the possible values of p' in the particular urn in question, if $\dfrac{m}{m+n}<p$, it is *probably* less than p', $\therefore$ $\dfrac{m}{m+n}<p_{m.n}$.[1]. And because p is the *milieu de probabilité* of the several values of p' in the different urns, if $\dfrac{m}{m+n}<p$, then, in the particular urn in question, p' is *probably* less than p and $\therefore$ $p_{m.n}<p$.[2]. Thus, it appears that if $\dfrac{m}{m+n}<p$, it follows that $\dfrac{m}{m+n}<p_{m.n}<p$. And in the same way it may be shown that if $\dfrac{m}{m+n}>p$, it follows that $\dfrac{m}{m+n}>p_{m.n}>p$. Whence the truth of the second postulate is evident.

From the relations thus established, it appears that $p_{m.n}$ must be a function of m, n, and p of such a form that, upon giving to each of these three variables all the values which they can possibly take, the value of the function in question shall always lie within the limits $\dfrac{m}{m+n}$ and p. Hence, $p_{m.n}$ may be supposed to represent a function of the form $\dfrac{m+rp}{m+n+r}$ (where r is some positive quantity, as yet undetermined), for, by a well-known property, the value of the fraction $\dfrac{m+rp}{m+n+r}$ necessarily lies in all cases

within the limits $\frac{m}{m+n}$ and $\frac{rp}{r}$ or p. And it is evident that by giving a suitable (positive) value to r, all possible values which are comprised within the above limits (and no others) will be expressible by the function in question. Let $p_{m.n}$, then,

$$= \frac{m+rp}{m+n+r} = \frac{\frac{m}{m+n} + \frac{r}{m+n} \cdot p}{1 + \frac{r}{m+n}} \quad \text{(A)}.$$

Now, an examination of the last form (A) of the function shows that $p_{m.n}$ is to be found by adding together the observed quantity $\frac{m}{m+n}$ and the antecedent probability p; not, however, in equal proportions, but in the respective proportions of 1 and $\frac{r}{m+n}$. But we know that the combination-weight (w) of the observed quantity $\frac{m}{m+n}$ (for different values of $m+n$) varies directly as $(m+n)$, so that we may put $w = c(m+n)$; and it is evident that the combination-weight (w') of the antecedent probability p must be constant for all values of $m+n$. Let w' be denoted by c'. Putting

$$p_{m.n} = \frac{w \cdot \frac{m}{m+n} + w'.p}{w+w'} = \frac{\frac{m}{m+n} + \frac{w'}{w} \cdot p}{1 + \frac{w'}{w}} \quad \text{[B]},$$

and comparing the equivalent expressions, A and B, we conclude that $\frac{r}{m+n} = \frac{w'}{w} = \frac{c'}{c(m+n)}$; or $r = \frac{c'}{c}$, *i.e.*, = some undetermined constant independent of $(m+n)$.

But although r $\left(\text{in } p_{m.n} = \frac{m+rp}{m+n+r}\right)$ has been shown to be independent of $m+n$, the number of trials, it is, of course, quite possible that it may be a function of p. Suppose, then, that $p_{m.n} = \frac{m+\phi(p).p}{m+n+\phi(p)}$. Now by interchanging m and n, and substituting q for p, in the last formula, we get $\frac{n+\phi(q).q}{m+n+\phi(q)}$ for $q_{n.m}$, the probability of drawing a *black* ball in the next trial (n black and m white balls having previously been drawn); and we must evidently also have $q_{n.m} = 1 - \frac{m+\phi(p).p}{m+n+\phi(p)} = \frac{n+\phi(p).q}{m+n+\phi(p)}$. Comparing this result with the first value of $q_{n.m}$, we have

$\frac{n+\phi(p).q}{m+n+\phi(p)} = \frac{n+\phi(q).q}{m+n+\phi(q)}$, a relation which shows that if r varies with p, it must be such a function of p that its value is not affected by the substitution of q or $1-p$ for p. Therefore, r must be either a constant or what is termed a symmetrical function of p and q.

"Pour fixer les idées", as the French mathematicians say, I have supposed the hypothesis of an unlimited mass of white and black balls in definite and known proportions, such proportions, to the whole, being represented respectively by p and q; and I have supposed the contents of the urn, from which the drawings are made, to be supplied indiscriminately from this mass. By means of this assumed hypothesis we have seen that although the unknown ratios p' and q' of this particular urn will probably differ, respectively, more or less from p and q, they will nevertheless tend to approximate more and more to the limits p and q as the number of balls contained in the urn is increased. This property, as I have already observed, constitutes the essential characteristic of the functions p and q; for which reason we may henceforth consider these functions, not merely as the antecedent probabilities (a property which may be considered as accidental and subordinate), but as representing the limits towards which the ratios p' and q' necessarily tend if the number of balls in the urn is indefinitely increased. And the conclusion to be drawn at this stage of the enquiry is that the true values of the probabilities denoted respectively by $p_{m.n}$ and $q_{n.m}$ cannot (for any useful purpose) be determined until the values of the limits in question shall have first been ascertained.

Section 3.

Comparing Laplace's formula $\frac{m+1}{m+n+2}$ with the general expression obtained in the last section, namely, $\frac{m+rp}{m+n+r}$, it is evident that the two become identical by assuming $r=2$, inasmuch as the supposed value of p, or the antecedent probability, is $\frac{1}{2}$ in Laplace's investigation. For convenience of calculation I propose, for the present, to assume a mean value in all cases for the function r, whatever may be the value of p, for which purpose it is evident that the mean value in question must be taken $=2$.

Accordingly, putting $r=2$ in $\frac{m+rp}{m+n+r}$, the (approximate) *generalized* formula for different values of p becomes $\frac{m+2p}{m+n+2}$.

In order to illustrate the results obtained by giving different values to p in the formula $\frac{m+2p}{m+n+2}$, I have computed the table appended hereto. As before, I have assumed $n=0$, whereby the formula becomes $\frac{m+2p}{m+2}$, denoting the probability $(p_{m.0})$ of drawing a white ball after m white balls have been already drawn in succession in m trials. This probability is given for successive values of m and for two widely-different values of p, namely, $p=\cdot01$ and $p=\cdot99$. For comparison, I have also given the corresponding values computed by the ordinary formula, $\frac{m}{m+n}$, and by Laplace's formula, $\frac{m+1}{m+n+2}$.

Although the results embodied in this table are based upon an assumed mean value of r (and therefore have no claim to perfect accuracy), they suffice, nevertheless, to show how important is the quantity denoted by p when the number of trials is limited; and how little we can depend, in such cases, upon the results derived (by supposing $n=0$) either from the ordinary formula or from the improved formula of Laplace.

The ordinary formula, $p_{m.n}=\frac{m}{m+n}$, was, I believe, first given (as an approximation only) by the celebrated James Bernouilli. It follows directly (by my first postulate) from the assumption of $p=\frac{m}{m+n}$; and of course it may, consequently, be obtained by substituting $\frac{m}{m+n}$ for p in the general formula $\frac{m+rp}{m+n+r}$. Laplace's formula, $\frac{m+1}{m+n+2}$, on the other hand, is based upon the equally unsatisfactory assumption that p may be anything between 0 and 1, all values between those limits being supposed also to be equally probable. The indispensable key to the general solution of the problem is p, the limit of the ratio of the actual chances, p', when the number of balls in the urn is indefinitely increased. We may frequently find it difficult to determine this limit as accurately as might be desired; but I do not think (for

practical purposes) we shall often have to resort to the assumptions involved in Laplace's formula, the effect of which is to restrict so greatly the general utility of that formula.

Probability of drawing a White Ball in the $(m+1)$th Trial (m White Balls having been drawn in m Trials).

m	By Bernouilli's Formula	By Laplace's Formula	By New Formula	
			$p = \cdot01$	$p = \cdot99$
0	1	·5	·01	·99
1	1	·667	·340	·993
2	1	·75	·505	·995
3	1	·8	·604	·996
4	1	·833	·670	·997
5	1	·857	·717	·997
6	1	·875	·753	·997
7	1	·889	·780	·998
8	1	·9	·802	·998
9	1	·909	·820	·998
10	1	·917	·835	·998
100	1	·990	·986	·9998
1,000	1	·999	·998	·99998

(*To be concluded.*)

THE INSTITUTE OF ACTUARIES.

EXAMINATIONS IN THE COLONIES, &c.

Notice is hereby given that, by a recent decision of the Council, Examinations for admission to the Classes of Associates and Fellows will, in future, be held in the Colonies and Dependencies of the Crown. The Examinations will take place annually in April, commencing in 1892, and, until further advised, the places where they will be held will be Melbourne, Sydney, Wellington, Cape Town, and Montreal.

Associates and Students of the Institute intending to present themselves for Examination at any of the above-mentioned places must give notice by letter, accompanied by the Examination Fee of One Guinea, addressed to the Honorary Secretaries, reaching London not later than the 31st December previous to the dates of the Examinations.

By Order,

T. G. C. BROWNE,
THOS. H. COOKE, } *Hon. Secs.*

Staple Inn Hall,
May 1891.

EXAMINATIONS OF THE INSTITUTE, April 1891.

Examination for Admission to the Class of Associate (Part I).

Examiners—F. T. M. Byers, Esq.; F. E. Colenso, Esq., M.A.; G. F. Hardy, Esq.; A. G. Hemming, Esq.

First Paper.

1. A man borrows £854. 7*s.* 1*d.* at 5 per-cent compound interest with half-yearly rests. He pays off £500 at end of 3 years, and wishes to pay the balance of the debt at the end of 5 years. What must he then pay?

2. Give the rules for the multiplication and division of decimals, and investigate the conditions under which a vulgar fraction can be expressed by a finite decimal.

3. An Agent abroad draws, at 90 days' date, on his Company for £200 in settlement of a Life Claim. What entries will you require to make in the several Ledger Accounts—"Bills Payable", "Agents", and "Claims"?

4. Divide 17 into three integers such that when the first is multiplied by 5, the second by 4, and the third by 7, the sum of the three products will be 80.

5. How soon after half-past 1 o'clock will the second-hand of a clock

(*a*) overtake the hour-hand?

(*b*) bisect the angle between the other two hands?

(*c*) overtake the minute-hand?

6. Solve the following equations:

$$(a)\quad \begin{cases} y+\sqrt{\dfrac{y}{x}}=\dfrac{42}{x} \\[2ex] \dfrac{x^2}{3}+\dfrac{x}{2\sqrt{y}}=\dfrac{54}{y} \end{cases}$$

$$(b)\quad \begin{cases} \dfrac{1}{x+y}+\dfrac{1}{x-y}=\dfrac{3}{4} \\[2ex] 2x^3+6xy^2=\dfrac{9}{64}(x^2-y^2)^3 \end{cases}$$

$$(c)\quad \begin{cases} x^2+y^2-x-y=78 \\ xy+x+y=39 \end{cases}$$

7. Obtain the cube root of 26 to 5 places of decimals by means of the Binomial Theorem.

8. A and B throw 2 dice once for a pool of £1,000. If the throw is 7, A takes the whole; if 8, B takes the whole; in any other case, each takes one-half of the pool. What should A pay B to equalize their chances?

9. A cylindrical saucepan, 8 inches in diameter, has a circular hole in the bottom, centrally placed, 1 inch in diameter. A coin of 2 inches diameter is thrown in. Show that the chance of its falling clear of the hole is 3 to 1.

10. Find a convenient expression for the sum of the series

$$u_1x+u_2x^2+u_3x^3+\&c., \textit{ ad inf.},$$

when the differences of u are rapidly convergent.

11. Having given

$$u_0=\ \cdot9545$$
$$u_1=1\cdot4018$$
$$u_2=1\cdot8322$$
$$u_3=2\cdot2474$$

find x when $u_x=1\cdot5000$.

12. In an equilateral triangle, a perpendicular is drawn from one angular point to the opposite side. What proportion does the length of the perpendicular bear to a side of the triangle?

13. What is the *locus* of the following equation:

$$(m-1)x^2+(m-1)y^2-2amx+ma^2=0.$$

Second Paper.

14. A person invests

£10,935 : 0 : 0	in Consols	at	$94\frac{3}{16}$
7,342 : 6 : 0	,, India $3\frac{1}{2}$ per-cent	,,	$107\frac{7}{8}$
852 : 17 : 0	,, India 3 per-cent	,,	$100\frac{5}{8}$
79 : 15 : 6	,, Canada 4 per-cent	,,	$105\frac{3}{4}$

What will be the amount of his income during the next 5 years?

(Note.—*Candidates will be supplied with Logarithm Books in working out this Question.*)

15. Show that every equation of an odd degree has at least one real root.

16. Two casks containing, the one a, and the other a' gallons, are filled with a mixture of wine and water such that the first contains β, and the second β' gallons of wine. By means of 2 smaller casks a gallon is drawn from each of the larger casks simultaneously, and transferred to the other. This operation having been repeated n times, how much wine will there be in each of the 2 casks?

17. During the time that the shadow on a sun-dial, which shows true time, moves from 1 o'clock to 5, a clock, which is too fast by m hours and n minutes, strikes a total number of strokes equal to $m+n$, and it is observed that the number of minutes is less by 41 than the square of the number which the clock strikes at the last time of striking. The clock does not strike 12 o'clock during the time. How much is it too fast?

18. Of 9 men, all can row, but only 2 can steer: in how many different ways can you arrange them in an eight-oar?

19. Sum to n terms, the series

$$\frac{1}{1+\sqrt{2}}+\frac{1}{3+2\sqrt{2}}+\frac{1}{7+5\sqrt{2}}+\&c.$$

20. Assuming that $\left(\frac{a+x}{a-x}\right)^{\frac{1}{2}}$ is expanded in ascending powers of x, write down the coefficient of x^{2r-1} and x^{2r}.

21. Expand $\log_e(1+x)$ in a series of ascending powers of x, and deduce therefrom a more rapidly convergent series for the calculation of the logarithms of consecutive numbers.

Find by any method $\log_e 3$ to 4 decimal places given $e=2{\cdot}7183$.

22. A die has 2 sides marked with the number 3, and 4 sides marked with the number 5: it is thrown 100 times. What is the probability that the sum of the numbers thrown will be exactly 400?

23. In the series u_0, u_1, u_2, &c., in which the successive differences Δu_x, $\Delta^2 u_x$, &c., converge slowly, it is observed that the ratio $\frac{\Delta^2 u_x}{\Delta u_x}$ is nearly $=\frac{\Delta u_x}{u_x}$. What expedient would you adopt to obtain the best results upon interpolation.

24. If a straight line be divided into any two parts, show that the squares on the whole line and on one of the parts are equal to twice the rectangle contained by the whole and that part, together with the square on the other part.

25. Three straight lines intersect to form an equilateral triangle, of which the side $=m$. Find the rectangular co-ordinates of the points of intersection when the origin of the axes is at the centre of the triangle, and the axis of y passes through the apex.

Examination for Admission to the Class of Associate, or to the Class of Fellow (Part II).

Examiners—T. G. Ackland, Esq.; J. Blakey, Esq.; L. M. Simon, Esq.; E. Woods, Esq.

First Paper.

1. A Lands Improvement Loan of £s is repayable, with interest, by an uniform annual charge on the estate, payable half-yearly for n years. Immediately after payment of the tth half-yearly instalment the owner wishes to redeem the loan, and the lender agrees to discount the remaining payments at a lower rate of interest. How would you proceed to ascertain the profit realized by the lender in excess of the capital outstanding?

2. Show the relation existing between the values of annuities certain of the $(t-1)$th and tth orders of the figurate numbers, and give a general formula for the value of an annuity for n years, whose successive payments are $u_1, u_2, \ldots, u_n$.

3. (α) Find the probability that (x) will be alive m years after the death of (y).

(β) Show that the compound probability ${}_np_x \times {}_{n-1}p_y$ may be expressed by either of the formulas

$$\frac{{}_np_{x \cdot y-1}}{p_{y-1}} \quad \text{or} \quad p_x \times {}_{n-1}p_{x+1 \cdot y}.$$

4. If the chance that a man who reaches the age of 35 will eventually die of cancer is 1 in 21, and the chance that a woman who reaches the same age will eventually die of cancer is 1 in 12; find in how many of 100 families, each consisting of three males and three females, all of whom have attained the age of 35,

(α) no death,
(β) one death,
(γ) two or more deaths,

may be expected to occur from cancer.

5. Deduce a formula for the net annual premium for a pure endowment, the office premiums paid to be returned in the event of death before attaining the age fixed. Assume that the office premium P_x is deduced from the net premium π_x by the formula—

$$P_x = \pi_x(1+\phi)+c.$$

6. Give a verbal explanation of the formula $\frac{1}{P_x+d}-1$ for the value of an annuity of 1 per annum upon the life of (x). State what sum assured is required to protect the purchaser, and show that n years hence the value of the annuity and the policy together will still be $=\frac{1}{P_x+d}-1$.

7. Find the single premium to assure a perpetuity of £100 in the event of A, aged 30, dying within 10 years, the first payment of the perpetuity to be due at the end of the year in which A dies. Interest to be taken at the rate of 4 per-cent per annum, and the law of mortality assumed to be that of De Moivre's hypothesis. Given v^{10} at 4 per-cent $=\cdot 675564$.

8. Find an expression for the value of an annuity of 1, payable during the joint lives of (x) and (y), and for t years longer if the survivor (whichever he may be) live so long.

9. State the various expressions which have been suggested for the value of a reversionary annuity to (x) after (y), payable m times a year, and explain under what circumstances each is applicable.

10. Given a complete table of temporary life annuities for all ages and terms, how would you proceed to construct, by a continued method, a table of policy-values for endowment assurances?

11. Describe fully the method which you would adopt to calculate approximately, from the column l_x of a mortality table, one isolated value of an annuity on three joint lives, at a given rate of interest.

12. Define a differential coefficient, and prove that, if $u=yz$, then

$$\frac{du}{dx}=y\frac{dz}{dx}+z\frac{dy}{dx}.$$

Second Paper.

13. Give a verbal explanation of the formula $a_{\overline{n|}}=\dfrac{1}{P_{\overline{n|}}+i}$, and show how the formula will be modified in the case where the sinking fund $P_{\overline{n|}}$ is to be re-invested at a rate of interest i' to replace the capital.

14. A perpetuity of £x per annum, the first payment of which is due at the end of n years, is to be purchased by annual instalments, commencing at P, and diminishing by $\dfrac{P}{n}$ each year, so that the last instalment, $\dfrac{P}{n}$, will be payable at the beginning of the nth year. Find the value of P.

15. State the probabilities respectively indicated by the following formulas, and find their values in terms of the numbers living at each age:

$$(\alpha)\quad {}_{n-1}q_x \times {}_{n-1|}q_y$$

$$(\beta)\quad {}_{n-1|}q_{\overline{xy}}$$

16. Find the probability that of three lives (x), (y), (z),

(α) (x) will die first,

(β) the survivor of (x) and (y) will die before (z).

17. The sum of £s is deposited by each of l_x persons in a fund, and accumulated at compound interest. £a is paid on the death of each member, at the end of the year in which he dies, and at the end of n years the amount remaining in the fund is applied to the purchase of an annuity upon the life of each of the surviving members. Find the amount of the annuity.

18. A, at the age of x, effected an ordinary whole-life assurance at an annual premium; m years having elapsed, he wishes to convert it into an endowment assurance, payable at age $(x+m+t)$, or at previous death. Find an expression for the annual premium on the new policy.

19. Deduce a formula for the reduced or paid-up policy which may be granted under a whole-life assurance effected at age x, after a duration of n years, the formula to be expressed in terms of the net annual premiums at entry, and at present age. How will this formula be modified to meet the case of (i) an endowment assurance; (ii) a whole-life assurance with limited payments; (iii) a temporary assurance; (iv) an endowment; (v) a contingent assurance; and (vi) a last survivor assurance, (a) provided that both lives are in existence, (b) provided that one of the lives is dead?

20. Investigate an expression for the annual premium for a whole-life assurance on (x), entitled to a reversionary bonus every five years, at the rate of £1. 10s. per-cent per annum on the sum assured and previous bonus additions; an interim bonus being allowed after the first five years, at the rate of £1 per-cent for each annual premium paid since the preceding division.

21. State a formula for the value of a life annuity on (x), payable m times a year, and thence deduce the value of a reversionary annuity, payable quarterly to a life (x) in the event only of his surviving two other lives, (y) and (z). How would this latter formula be modified to give the value of a complete annuity on the same status?

22. How would you proceed to approximate to the value of an annuity payable to (x) after the death of (y) and (z), provided that (z) dies first?

23. Given a table of values of l_x at all ages, how would you proceed to construct most expeditiously a table of values of annual premiums for whole-life assurances (P_x)?

24. Show that the limit of $\dfrac{\Delta^n y}{\Delta x^n}$ is equal to $\dfrac{d^n y}{dx^n}$.

25. Integrate, with regard to x, the following expressions:

$$\frac{x}{x^2+a^2},$$

$$\sqrt{x^2+a^2},$$

$$(x+a)^{-1}.(x^2+a^2)^{-1}.$$

Examination for Admission to the Class of Fellow (Part III).

Examiners—G. King, Esq.; A. W. Sunderland, Esq., M.A.; G. Todd, Esq., M.A.

First Paper.

1. What do you consider to be the object of graduating a Mortality Table, and what are the conditions of a successful graduation?

Give instances in which an assumed Law of Mortality has been employed for purposes of graduation.

2. How was the Carlisle Table constructed and graduated? Can you suggest any improvements on the method of procedure?

3. Enumerate the principal sources of surplus in a Life Office, and say how, in your opinion, the surplus should be distributed so as to deal fairly by the different policyholders.

4. A Reversionary Interest Society possesses among its assets a large number of life interests, some of them wholly or partially protected by policies, and also a large number of policies not held in connection with life interests. By what methods would you value these portions of the assets at the periodical stock-takings?

5. State shortly the objects of the Settled Land Act. In the event of a sale of land by the tenant for life, how would his mortgagees be affected?

6. A assigns a policy effected by himself on his own life to B, by way of mortgage, the deed giving full powers of sale or of surrender; but B does not serve notice on the company of the assignment. A becomes bankrupt, and the officials of the company, seeing the fact mentioned in the papers, make a note of it in the books. Next day B gives formal notice of his deed, and subsequently the Official Receiver gives notice of the bankruptcy. What are the respective rights of the parties, and how must the company act:

1st. If B's deed prove to be regular in every respect?
2nd. If B's deed prove to be insufficiently stamped?

7. Discuss the various methods which have been adopted in answering question 9 of the 6th Schedule of "The Life Assurance Companies Act, 1870", namely, "The average rate of interest at which the life assurance fund of the company was invested at the close of each year during the period since the last investigation." State what, in your opinion, are the respective advantages of each.

8. What alterations in the constitution of the Bank of England, or the currency system, suggest themselves to you as a means of averting monetary crises, or mitigating their effects?

9. Express in the form of integrals $\bar{A}^{3}_{xyz}$ and $\bar{a}_{\overline{xy}|z}$, and prove the formula

$$A^{1}_{xy}=\mu_x\bar{a}_{xy}+\tfrac{1}{2}(\bar{a}_{x-1:y}-\bar{a}_{x+1:y})$$

assuming
$$-\frac{d}{dx}\bar{a}_{xy}=\tfrac{1}{2}(\bar{a}_{x-1:y}-\bar{a}_{x+1:y})$$

10. Apply Lubbock's Summation formula to determine the value of an annuity payable m times a year ($a_x^{(m)}$), and hence deduce the value of $\bar{a}_x$.

11. A, aged 30, is insurable for the whole term of life, only at an addition of 10 years to his age. B, aged 60, is a good life. How would you determine the correct extra for

(*a*) A joint-life assurance on A and B.
(*b*) A last-survivor assurance on A and B.
(*c*) A contingent assurance on A against B.
(*d*) An endowment assurance on A, payable at 60.

12. A testator left property of the value of £10,000 in trust, out of the income to pay an annuity of £100 to his sister, A (now aged 78), during the joint lives of herself and of his wife, B (now aged 75), and the residue of the income to his said wife for life, and after her death to sell the securities and pay one-fourth of the proceeds to his said sister, if she be then living, and to divide the remainder, or, if the said sister be then dead, the whole of the proceeds between such of the children of his deceased brother, C, as shall be then living, and the issue then living of any child of C who may be then dead, *per stirpes* and not *per capita*. There were in all five children of C, who are now all living at various ages from 35 up to 50.

Explain exactly the methods and data by which you would estimate the value for sale of the prospective one-fifth share of D, the eldest child, now aged 50; and say what description and what amount of policies you think ought to be effected by a purchaser to protect his investment.

13. Explain the method of obtaining the "Exposed to Risk" employed:

(*a*) By Mr. Galloway in the Amicable Experience; and
(*b*) In the preparation of the Institute of Actuaries' H^M Table, and point out the relative advantages of each.

Second Paper.

14. How far, and in what directions (if at all), does Mr. Woolhouse's method of graduating Mortality Tables depart from the conditions of successful graduation, and what are the reasons of such departure?

15. The surplus shown at each valuation of a Life Office is distributed among the policyholders as follows:

To each policy in existence at the beginning of the valuation period is allotted the interest profit on its then reserve value, and the balance of the surplus is distributed among all the policies in proportion to the premiums received upon them during the valuation period.

A valuation having been made so far as to ascertain the surplus, by what operations would you carry out the distribution?

16. Explain exactly the principles and procedure adopted by Mr. Woolhouse in his "Easy Method of getting out a rough Estimate Valuation of a Whole-Life Assurance Business."

17. What method of procedure would you advise were you consulted by the Court of Chancery as to the reduction of contracts in the case of a company to be reconstructed under "The Life Assurance Companies Act of 1870"? If the company be a limited proprietary one, what rights, if any, have the shareholders after reconstruction?

18. Write a short account of the legal incidents and difficulties connected with trust policies issued under "The Married Women's Property Act, 1882."

19. Explain the meaning of the term "Base Fee." In making an advance to a tenant-in-tail what legal procedure is usually adopted by Life Offices for obtaining a charge upon the Fee?

20. What would be the advantages and probable effects of the issue of £1 notes by the Bank of England, and what arrangements as regards reserve would be necessary were such issue to take place?

21. Give a short summary of the main considerations which should guide a Life Office in the investment of its funds.

What have you to say with regard to the following investments:

(*a*) Consols.
(*b*) Railway Debenture Stocks.
(*c*) Debentures of trading companies.
(*d*) Colonial Mortgages.

22. Solve the following equation for ϕ:

$$\frac{1}{\phi}\cdot\frac{d\phi}{dx}=a+bc^x$$

where a, b, and c are constants.

Deduce Makeham's formula,

$$l_x=ks^x\,.\,g^{c^x}.$$

23. Sketch a form of "annual report", to show concisely the chief operations of a life assurance society during any year of its existence.

Before commencing to answer these questions, the Candidate must give up the rest of his paper. He will then be supplied with such books of tables as he may desire.

24. A, aged 25, takes a life interest in estates after his father, B, aged 60, the present life tenant. An assurance company agrees to advance to A £10,000 in cash. It is arranged that A may, at any time within the first five years, terminate the transaction by repaying to the company the advance, and premiums, &c., out of pocket, the whole accumulated at 5 per-cent compound interest. If, however, redemption be not effected, then the company is to take an annual

reversionary charge of amount to be now fixed, and which is not to be redeemable until it falls into possession.

Determine the amount of the annual reversionary charge, and say what should be the redemption-money when it falls into possession. What amount of policies must be effected on the life of A, the transaction being so planned that at the outset policies are effected of amount only just sufficient to secure the company during the first five years, the balance of the amount which will be ultimately required being now arranged for by means of assurances to be issued, without medical examination, five years hence, at the rate for A's then age.

25. A, aged 29, an officer in the British Army, is entitled on the death of the survivor of B, aged 65, and C, aged 75, provided he survives B, to £10,000 Consols. Estimate the value of A's interest for sale in the open market.

PROCEEDINGS OF THE INSTITUTE.—Session 1890–91.

First Ordinary Meeting, 24 *November* 1890.

The President (Mr. B. Newbatt) in the Chair.

The President delivered an inaugural address.

Second Ordinary Meeting, 22 *December* 1890.

The President (Mr. B. Newbatt) in the Chair.

Mr. James Chatham read his Prize Essay on "The Rate of Mortality among recently-selected Lives."

The following gentlemen took part in the discussion:—The President, Messrs. Sprague, G. King, Newman, Ryan, Ackland, G. F. Hardy, and Tarn.

Third Ordinary Meeting, 26 *January* 1891.

The President (Mr. B. Newbatt) in the Chair.

A paper on "The Practice and Powers of Assurance Companies in regard to the Investment of their Life Assurance Funds", was read by the author, Mr. A. G. Mackenzie.

The following gentlemen took part in the discussion:—Messrs. Humphreys, Ryan, Coles, Adler, Strachan, Browne, Bailey, A. Hendriks, George King, and the President.

Fourth Ordinary Meeting, 23 *February* 1891.

The President (Mr. B. Newbatt) in the Chair.

A paper entitled "Some Notes on Laws affecting Policies of Life Assurance" was read by the author, Mr. W. K. Lemon.

The following gentlemen took part in the discussion:—Messrs. Bailey, A. Hendriks, J. McKee (non-member), W. Trower (non-member), C. D. Higham, A. G. Mackenzie, and F. Bell.

Fifth Ordinary Meeting, 23 *March* 1891.

The President (Mr. B. Newbatt) in the Chair.

Mr. Sunderland read his paper "On the Method of treating Under-"average Lives for Assurance Purposes, by making Temporary Reductions "from the Sums Assured, with special reference to the Mode of Calculating "those Deductions commonly adopted in Practice."

The following gentlemen took part in the discussion:—Messrs. Rothery, Nash, Frankland, Pulley, Ryan, Bailey, Nightingale, Wyatt, A. G. Mackenzie, Schooling, Searle, Tarn, Chisholm, Warner, and Humphreys.

Sixth Ordinary Meeting, 27 *April* 1891.

The President (Mr. B. Newbatt) in the Chair.

Mr. Young read a paper entitled "The German Law of Insurance against Invalidity and Old Age."

Messrs. Sutton, R. P. Hardy, Whittall, Adler, Bailey, and Geo. King; also Canon Blackley, Sir Julius Vogel, Messrs. Lloyd and Hunter (non-members), took part in the discussion which followed.

The Forty-Fourth Annual General Meeting, 1 *June* 1891.

The President (Mr. B. Newbatt) in the Chair.

The proceedings at the Annual General Meeting will be found on page 267.

REPORT, 1890-91.

"The Council have pleasure in reporting to the members upon the progress of the Institute during the session of 1890-91, the forty-third year of its existence.

"The appended schedule shows the additions, changes, and losses in the membership, which have occurred during the year ending 31 March last.

"The increase in the number of members was 19, as compared with 38 in the preceding year. Since the date of the Charter the number of members has been as follows:

1884-85 . .	434	1888-89 . .	563
1885-86 . .	441	1889-90 . .	601
1886-87 . .	484	1890-91 . .	620
1887-88 . .	521		

Schedule of Membership, 31 *March* 1891.

	Honorary Members	Fellows	Associates	Students	Corresponding Members	Total
i. Number of Members in each class on 31 March 1890 .	2	173	207	208	11	601
ii. Withdrawals by						
(1) Death . . .	...	7	3	1	...	
(2) Resignation . .	...	2	3	31	...	58
(3) Default in payment of Subscriptions . .	...	1	2	8	...	
	2	163	199	168	11	543
iii. Additions to Membership	...	...	...	77	...	77
	2	163	199	245	11	620
iv. Transfers						
(1) By Examination: *from* Associates	...	...	5	...	...	...
to Fellows .	...	5	...	...	...	...
	2	168	194	245	11	620
(2) By Examination: *from* Students	...	...	...	8	...	...
to Associates .	...	...	8	...	...	...
	2	168	202	237	11	620
(3) By Ballot: *from* Associates	...	...	...	...	...	...
to Fellows .	...	...	...	...	...	...
v. Number of Members in each Class on 31 March 1891 .	2	168	202	237	11	620

"The accounts for the year are very satisfactory, the total amount of funds now being £4,114. 1*s*. 5*d*., showing an increase of £570. 19*s*. 1*d*.

"The income and expenditure account and balance sheet are given herewith.

Dr. Income and Expenditure for Year ending 31 March 1891. Cr.

		£ s. d.	£ s. d.
1890-91.			
Amount of Funds, 31 March 1890—			
Messenger Legacy Fund		280 8 9	
Brown Prize Fund		272 8 9	
Library Fund		291 19 0	
General Fund		2,698 5 10	3,543 2 4
Composition of Subscriptions			67 4 0
Annual Subscriptions—			
Town Fellows	£239 8 0		
Country ,,	71 8 0		
Fellows admitted since 1884:			
(1) By Examination	99 15 0		
(2) By Ballot	28 7 0	438 18 0	
Town Associates	214 4 0		
Country ,,	77 14 0		
Associates admitted since 1884:			
(1) By Examination	26 5 0		
(2) By Ballot	25 4 0	313 7 0	
Students		246 15 0	1,029 0 0
Entrance Fees—			
Students			85 1 0
Examination Fees			117 12 0
Sales of Publications—			
Text-Book, Part I.		33 5 3	
,, ,, II.	130 6 0		
Less Charges for Binding	10 8 4	119 17 8	
Log. Cards		1 17 0	
Mortality Experience		2 18 8	
Life Tables		18 18 0	176 16 7
Dividends and Interest			134 18 0
Donation to Library Fund			1 1 0
			£5,154 14 11

	£ s. d.	£ s. d.
1890-91.		
Journal—		
Cost of Nos. 155-6-7-8	251 13 2	
Less Sales	181 3 10	70 9 4
Library—		
Binding and Purchases		58 9 6
General Expenditure—		
Rent	250 0 0	
Salaries, including Income Tax	271 18 4	
Lecturers	52 10 0	
Examination Charges	34 1 9	
Charter and Bye-Law Expenses	68 3 0	
Meetings	51 16 10	
House Expenses	37 11 6	
Corporation Duty	3 18 2	
Fire Insurance	11 8 0	
Postage and Stationery	121 18 11	
Sundries	8 8 2	911 14 8
		1,040 13 6
Funds 31 March 1891—		
Messenger Legacy Fund	286 2 1	
Brown Prize Fund	280 4 9	
Library Fund	293 0 0	
General Fund	3,254 14 7	4,114 1 5
		£5,154 14 11

Examined and found correct, 29 *April* 1891.

(Signed) A. B. Woods, E. B. Trew, A. F. M. Gamble, } *Auditors.*

Balance Sheet, 31 March 1891.

LIABILITIES.	£ s. d.	£ s. d.
Messenger Legacy Fund (£211. 1s. 10*d*. Consols), cost	203 17 8	
Accumulated Dividends, at 2¾ per-cent	82 4 5	286 2 1
Brown Prize Fund (£200 Metropolitan Railway 4 per-cent Debenture Stock), cost	200 0 0	
Accumulated Dividends	80 4 9	280 4 9
Library Fund		293 0 0
General Fund		3,254 14 7
		£4,114 1 5

ASSETS.	£ s. d.
Consols, 2¾ per-cent (£2,000) cost	1,886 10 0
Metropolitan Railway 4 per-cent Debenture Stock (£700), cost	715 1 9
Great Eastern Railway 4 per-cent Debenture Stock (£800), cost	1,031 10 3
On Deposit at the London and Westminster Bank	100 0 0
On Current Account at London and Westminster Bank	358 18 5
Arrears of Subscriptions	22 1 0
	£4,114 1 5

Examined and found correct, 29 *April* 1891.

(Signed) A. B. Woods, E. B. Trew, A. F. M. Gamble, } *Auditors.*

"Four Fellows have taken advantage of the new rule enabling them to compound for their subscriptions.

"The annual subscriptions, together with compositions, admission and other fees, amount to £1,298. 17*s*., being £81 in excess of those of the previous year.

"The total expenditure for the year is £1,040. 13*s*. 6*d*., which shows some increase, owing chiefly to expenses occasioned by the change in the Bye-Laws.

"The stock in hand of the Institute publications at date is as follows:

No. of Copies	Description of Work
525	*Text-Book*, Part I.
734	,, ,, II.
163	Mortality Experience Tables.
27	Mortality Experience.
216	Logarithm Cards.
430	Messenger Prize Essay (Friendly Societies).
512	Index to 10 Vols.
120	,, 20 ,,
9,516	Parts of *Journal*.

"The Adjudicators appointed by the Council to consider the three Essays on "THE EFFECT OF SELECTION AMONG ASSURED LIVES", sent in to compete for Mr. Sprague's prizes, awarded the first prize of £30 to Mr. James Chatham, and second prizes of £20 each to Mr. Philip Lewin Newman and to Mr. Emory McClintock.

"At the first meeting of the session, held on the 24th day of November 1890, the President (Mr. B. Newbatt) delivered an address; and the following papers have since been submitted:

"22 *December* 1890—Mr. Chatham's Prize Essay.

"26 *January* 1891—'The Practice and Powers of Assurance Companies in regard to the Investment of their Life Assurance Funds'—by Mr. A. G. Mackenzie.

"23 *February* 1891—'Some Notes on Laws affecting Policies of Life Assurance'—by Mr. W. K. Lemon.

"23 *March* 1891—'On the Method of treating Under-average Lives for Assurance Purposes, by making Temporary Deductions from the Sums Assured'—by Mr. A. W. Sunderland, M.A.

"27 *April* 1891—'The German Law of Insurance against Invalidity and Old Age'—by Mr. T. E. Young, B.A.

"An Intermediate Examination in Part I was held on 25 October 1890, when twenty-six candidates presented themselves, of whom twelve passed, namely:

Boyd, H. N.	Moodie, P. B.
Brown, E. H.	Moran, J. F.
Covington, O. H.	Taylor, J.
Mackinney, F. W.	Trenerry, C. F.
May, G. E.	Wilson, G.
Milton, H.	Wright, R. Y. M.

"For the Examinations held on 24 and 25 April last, eighty-five candidates entered their names—of whom two did not present themselves—namely:

37	for Part	I,	of whom	11	were successful.
36	,,	II,	,,	11	,,
12	,,	III,	,,	5	,,

"By order of Council, successful candidates are now to be divided into three Classes, the names in each Class being arranged alphabetically, as below:

Part I.

Class I:

Kitchin, F. H.
Todhunter, R.

Class II:

Hooker, R. H.

Class III:

Cheshire, H. F.
Claridge, W.
Gardiner, R. E.
Morgan, B. C.
Rintoul, P.
Spencer, J.
Thomson, H. A.
Whigham, C. F.

Part II.

Class I:

Milton, H.

Class II:

Cook, W. P.

Class III:

Clarke, A. H.
Dawson, C. P.
Fulford, F. W.
Gayford, H. S.
Harris, A. S.
Hutcheson, W. A.
Taylor, James.
Thomas, E. C.
Turnbull, A. D. L.

Part III.

Class I:

Tilt, R. R.

Class II:

Gamble, A. F. M.
Phelps, W. P.

Class III:

Anderson, W. S.
Pulley, W. P.

"The Lectures in the subjects of Parts I and II of the Examinations have been satisfactorily attended.

"The Council have made arrangements to examine candidates in future in Melbourne, Sydney, Wellington, Cape Town, and Montreal, simultaneously with the Examinations held in the United Kingdom annually in April, and it is hoped that the increased facilities thus offered will tend to further the objects of the Institute.

"The Council congratulate the members on the fact that the session just

closed has been remarkable for having produced several papers by new contributors, and for having stimulated many of the younger members to take part in the discussions; and in this manifestation of zeal and healthy ambition the Council are glad to see a worthy response to the appeal for assistance of this kind which they made last year.

"B. NEWBATT,

"12 *May* 1891." "*President.*"

PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The Annual General Meeting of the members was held at Staple Inn Hall, on Monday, 1 June, the President, Mr. B. Newbatt, in the chair.

The Report of the Council (given on p. 262) having been read,

The PRESIDENT, in moving the adoption of the Report, referred to the statistical records which it presented, which showed in the first place, that the increase of numbers had resulted solely from the influx of students, there not having been any net addition to either the class of Associates or the class of Fellows. While they had to lament the loss of 10 Fellows, 7 of them by death, and of 8 Associates, 3 of them by death, no fewer than 40 Students had gone, 1 only from death, the other 39 by resignation or default. The 40 students so lost this year compared with 11 only in 1889, and with 14 only in 1890—a comparison which seemed to show that they were on the eve of attaining the maximum number. The broad result of the whole enquiry was—that the class of Fellows and Associates had declined, and showed a tendency to decline, and that it was to the Student class alone that the increase in the aggregate membership was due. The latter would constitute the future source of supply, and upon the degree in which they could attract, train, and keep them would depend the future strength of the Institute. Upon the general question of examinations he remarked that while it was the aim of the Institute, and the object of its training, to pass as many as possible of its Students into the class of Fellows, it recognized the certainty that only a comparatively limited number would be so passed. With regard to several questions relating to the examinations the Examination Committee, and, in its turn the Council also, would very seriously concern themselves. But assuming that in the end, whatever pains might be taken to bring about better results, a large proportion of the Students would fail to pass into the class of Fellows, and even into the class of Associates, that would not imply a failure of the system. The universities were not held to fail because the vast majority of their *alumni* took only a poll degree. The Associate degree was something more than a poll degree, while the Fellowship degree corresponded to the higher honour degrees of the universities. In order that the question "Are we making the best of the position?" might be answered in the affirmative, he thought that they needed a poll degree for admission into the class of Associates, of the nature that he had indicated on a previous occasion. He cordially congratulated those five gentlemen who had now become entitled to the fellowship, and hoped that they would, in the larger arena of practical life, gain as much distinction as they had won within the walls of the Institute. In conclusion, he expressed his satisfaction that so many new men had come forward as contributors of papers or as debaters, and his thanks that in more than one case they had done so in ready response to his personal solicitation. The force and the variety of the papers read, and the keenness and vivacity of the discussions, had given distinction to the session, and proved that the Institute had lost none of its vitality, nor its members any of their devotion.

Mr. AUGUSTUS HENDRIKS, in seconding the adoption of the Report, said that the success of the past session was largely owing to the President, his eloquent address of 24 November, and to the good advice he had given. As one of the adjudicators to decide upon the prizes offered by Mr. Sprague,

he might mention their great satisfaction when, upon comparing the motto with the name contained in the other envelope, they found that one of the several able assistants of Mr. Sprague became the recipient of the first prize.

The resolution was unanimously adopted.

Election of Officers.

A ballot having been taken, the Scrutineers, Mr. T. J. Searle and Mr. A. F. M. Gamble, reported that the following list had been adopted:

President.

Benjamin Newbatt.

Vice-Presidents.

Thomas G. C. Browne.	Augustus Hendriks.
George Stephen Crisford.	Thomas Emley Young, B.A.

Council.

Marcus Nathan Adler, M.A.	Charles Daniel Higham.
Arthur Hutcheson Bailey.	George Humphreys, M.A.
Geo. Wm. Berridge.	George King.
Thomas G. C. Browne.	Henry William Manly.
Arthur Francis Burridge.	Benjamin Newbatt.
James Chisholm.	Arthur Pearson.
*Henry Cockburn.	Howard James Rothery.
Francis Ernest Colenso, M.A.	Thomas Bond Sprague, M.A.
Ernest Colquhoun.	William Sutton, M.A.
Thos. Homans Cooke.	*George Todd, M.A.
George Stephen Crisford.	Andrew Hugh Turnbull.
Archibald Day.	William Wallis.
*David Deuchar.	*William J. H. Whittall.
Thomas Charles Dewey.	*Frank Bertrand Wyatt.
Augustus Hendriks.	Thomas Emley Young, B.A.

* New Members of Council.

Treasurer.

Henry William Manly.

Honorary Secretaries.

Thomas Homans Cooke.	George King.

Mr. A. B. Woods, Mr. E. B. Trew, and Mr. W. R. Makeham, were elected Auditors for the ensuing year.

The President announced that the Council, desirous of extending the range of studies of its members, had determined upon offering two "Samuel Brown" Prizes, of the respective values of 50 guineas and 25 guineas, for the best two essays, to be written by members of the Institute, on the following subject:—"The Enfranchisement of Leaseholds and the Taxation of Ground Rents, Chief Rents and Kindred Charges on Land in England and Wales." A syllabus, and the conditions of the competition, would be shortly issued to all the members.

Mr. H. Cockburn proposed, and Mr. E. Justican seconded, a vote of thanks to the President, Vice-Presidents, Council, and Officers of the Institute for their services during the past year.

The resolution was cordially adopted, and Mr. G. S. Crisford responded.

A vote of thanks was accorded to the Auditors, on the motion of Mr. W. J. H. Whittall.

The proceedings then terminated.

CONTENTS OF NO. CLXII.

ANALYTICAL INDEX.

NOTICE TO CORRESPONDENTS.

Communications for this *Journal* must be sent in at least one month prior to the day of publication, or their insertion will in all probability be deferred.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

The German Law of Insurance against Invalidity and Old Age: a History, Analysis, and Criticism. By* T. E. Young, B.A., *a Vice-President of the Institute of Actuaries.*

[Read before the Institute, 27 April 1891.]

I.—Introduction.

THE law adopted by the German Reichstag on the 23rd of May 1889 for insurance against invalidity and old age must demand careful attention as one of the most daring social and economic experiments in modern history.

Its many varied aspects solicit and repay the sedulous thought of the philosopher, as a contribution to one of the deepest problems in the evolution of man: of the statesman, as a signal tentative experiment in the compacting of a nation into an intimate and interdependent unity: of the economist, as a bold departure from principles of social condition which had generally prevailed, and as a practical criticism upon the doctrine of *laissez faire*: of the historian, as a problem in the mode of development of nations: of the moralist, as a contribution to practical social ethics: and of the actuary, as affording him a question which peculiarly is fitted to

* Die Alters- und Invaliditätsversicherung.

engage the disciplined thought and experience in which he has been trained.

I venture to enlarge upon the latter aspect, as I feel great confidence that if the actuary does not permit himself to be restricted within the more narrow technical boundaries of his science or generalised experience, but follows in their natural and legitimate direction the studies with which his specific training is competent to deal, he is especially equipped to form a useful critic to his fellow-citizens, to the economist and philosopher, upon the many-sided relations and consequences of this intricate scheme and upon the principles of which it forms an exhibition in a particular concrete shape. It is clear, I think, from the nature and scope of his work, that he need in no way be necessarily confined within, what I may term, the instrumental range of his craft, but may justly and rightfully expatiate in this large economic and financial field, where the instruments he has mastered are capable of a wide and fruitful use.

May I diffidently hasten, however, by anticipation, to disclaim for the present paper any example of the service I have thus described? I cannot aspire, with my brief study of the question, to contribute anything of original value upon the nature of the scheme and the possible consequences it involves. The time, too, is not yet ripe for a connected and valid survey, and experience of the working of the law can alone afford an adequate and stable basis for deduction. My ambition is rather limited to the hope that the materials I have collected may be so displayed that hereafter, without occupying the time and difficult labour necessary to discover them, some future actuarial student of the question may, with added experience and reflection, contribute a paper upon the basis of these facts which shall justify the competency of actuarial knowledge to examine the subject successfully, and which shall at the same time prove of practical value as an economic elucidation, based upon scientific study, of so deep and pervading a problem of civilisation. Though I purpose, for the sake of partial completeness, to submit a few considerations upon these points at a later stage, I feel regretfully that they must largely partake of a random and discursive character—rough predictions rather than organised criticism and well-based conclusions.

At the present stage I defer any discussion of the origin of this project: whether it has been wrung from the Government by the oppressive burden of Militarism and Protection and their effect

upon industrial life; or whether it is a legitimate and evolutionary result of the progress of national development in what Mr. Herbert Spencer has termed its altruistic direction.

I will at present content myself with stating my conviction that it affords a signal illustration of that dual organic process under which St. Simon and his school endeavoured, many years ago, to range all social phenomena. They were men who possessed the rare faculty of combining isolated and apparently disconnected social facts, and gathering them into the forcible and impressive unity of a serviceable and hopeful generalisation. Thus, they sought to introduce into the region of human social life, which had hitherto been deemed to be largely chaotic, the uniform relations or laws which the study of physical nature had copiously revealed, so that in every sphere of nature the reign of law might be observed to prevail. They startled the thoughtful world, at that date, by their induction that social phenomena invariably exhibited an Organic Period when the adjustment between social needs and surrounding institutions was approximately intimate and complete: followed, necessarily, by a Critical Period, when those needs had advanced beyond the scope and capacity of existing social environment, so that the relations involved became the subject of examination and dissatisfaction, until the disturbing and adapting process had emerged from its merely critical phase, and had resulted in a wider and closer adjustment which heralded the approach and continuance of another Organic Stage and era of social rest. I have always, myself, entertained a deep belief in this rhythmical law. Mr. Spencer, in his First Principles, long ago pointed out the rhythm existent in organic and inorganic nature, and a study of social history will confirm the St. Simonian doctrine that a similar rhythm pervades the human series. Whatever, then, be the special circumstances which have culminated in the adoption of this German law, its existence furnishes evidence, I think, that a social Critical Period has now arrived throughout the world, and will continue and develop until it vanishes into its inevitable sequence of an Organic Stage.

Bold experiments will not in themselves alarm us: all passages into the unknown—intellectual and social—must, from the necessity of the case, largely assume the character of pure experiment:

> "We were the first
> That ever burst
> Into that silent sea."

For the experience of a limited and defective Past—and all reform is based upon this postulate—must obviously fail to furnish a sure and capable pioneer to that Future which, besides being comparatively indefinite and unknown, is intended to be widened and deepened by the advent of the reform itself, and to be divergent accordingly from the conditions by which our experience was bounded. In these great questions, however, we must be chiefly guided, not so much by the mere *facts* of experience as by the *indications* of those facts—essentially, therefore, must we be directed by such an *interpretation* of the meaning and tendency of the past, which a study of man generally, and of the particular class of man with which we are dealing; of the consecutive history of existing social relations; and of the character of the nation and the progress of its institutions, as may, after careful and prolonged enquiry, be revealed.

A condition of Faith, therefore, grounded definitely upon the interpreted assemblage of experiences of the past, must accompany the projects of all reformers, so that, as Coleridge points out, our experience may not simply resemble the stern lights of a ship, which illumine only the wastes over which we have voyaged, but may, with the inspiration of disciplined faith, provide a helpful, though feeble and flickering, light across the unnavigated spaces yet to be traversed.

Whether the present vigorous scheme be conceived really in comparative darkness: the result simply of the pressure of irritation from political and social burdens: an agitated concession to misery and possible national danger: a blind extemporaneous effort, involving consequences of permanent and incalculable importance, and adventured only by precarious energy, and cheered and aided by the vaguest hope: or whether it be a serious and validly-grounded attempt, constituting a definite era in national evolution, though necessarily attended by imperfect and arbitrary accompaniments: on these points I trust that the ensuing exposition will afford some approximate means for a tentative estimate, besides furnishing the basis of special deductions by the economist and actuary.

In preparing—I regret to add, hastily and imperfectly—the paper now submitted, I have employed various documents issued by our own Government, but especially, I have enjoyed the great advantage, in respect of the statistical and mathematical foundation of the law, of the possession of an official document which I obtained through the kindness and courtesy of our Ambassador

at Berlin, and which, I believe, has never yet been presented to an English reader.

I ventured also, in elucidation of the measure, and in explanation of difficult portions, to submit certain additional enquiries, to which, by his instruction, I received the most courteous and helpful response.

The memorandum from which I have compiled the statistical and mathematical elements of the basis of the scheme was originally submitted to the Reichstag as an appendix to the Bill on its first introduction in November 1888.

I propose, as a necessary introduction to the apprehension of the law, to furnish a brief history of fiscal and social legislation in Germany; and then, in as lucid and compact a form as I have been able to achieve, to arrange together in appropriate sections the several provisions of the enactment, with such explanatory notes, not contained in the law itself, which the supplementary official information, collated and digested, will enable me to supply. And after explaining the details of the law and the basis on which they are founded, I shall finally venture to submit a few observations upon the scheme and its possible results.

It is not unfitting—and, indeed, it is requisite as aiding a comprehension of the object and orderly sequence of the measure—to refer parenthetically to the unofficial pioneers who have rendered aid to social justice by the formulation of schemes for its concrete expression, and whose labours undoubtedly have largely contributed to the origination and successive development of State Socialism in Germany.

The German nation, from whatever cause, has always been fertile in the production of social and political reformers; and, in historic sequence, have appeared Rodbertus, Engels, Marx, Lassalle, and Liebknecht. Whatever may be thought of the shapes into which these pioneers cast the dominating principles of their social creeds, many of them were men not only noted for original acumen, large knowledge of economics, wide organising and systematising powers, and enlightened social views in essence—however distorted in particular forms—but the majority, too, were inspired by a genuine feeling for humanity and the possibilities of human character and progress through the instrumentality of human aid. Many, again, have been men of social rank, and in some instances of high position: who deserve respectful recognition for self-negation and the unselfish merging of their individual views and hopes in the welfare and struggling

efforts of the "dim common populations." The impulse which their schemes have given to popular feeling and thought, and the direction of social aims; the deep and pervasive influence which they produced in all sections of society—an influence, of course, far from uniform in motive and exhibited in various modes—the redemptive formulas they announced for progress and national contentment, and the systems in which these were organised—all these causes have wielded a powerful force in the more recent course of German legislative thought. The pioneers rest from their toils—defective and impracticable in manifold ways, —but whenever did Sibyl, under the energising passion of deep feeling, reach the perfect gift of shaping her oracular utterances into fully intelligible and helpful forms?—and the legislation of Germany has furnished an enduring epitaph for their tombs! It demands the deep, irregular feelings and unofficial inspirations of a Peter the Hermit to form the materials and the stimulating impetus out of which the practical social and political Crusader is to construct and elaborate his work. I need not, however briefly, discuss these schemes—notwithstanding the useful help they furnish to the understanding of German, and, indeed, of universal legislation upon these subjects—irregular and often lawless precursors, as they were, of the fitful and glimmering social dawn—for the serious student of the present phase of German legislation must personally become familiar with their efforts, as a valuable key and guide, and cease to be content with a simple enumeration of their names.

II.—A Brief History of Fiscal Legislation and State Socialism in Germany.

It will assist, as I have stated, in the study and interpretation of the law, and help to assign its place in the legislative development of the nation, and thus to throw light upon its character and its order in social history, if I attempt in a compact shape to trace the fiscal legislation of the country and the record of its attempts at State socialism.

The student will find it a mistaken assumption to suppose that State socialism is a modern innovation in German politics. Legislation there has shown a distinct continuity in this direction; and Prince Bismarck has only extended and developed the social system which was initiated by the Great Elector and by Frederick the Great of Prussia.

State socialism first assumed a definite form in the legislation of the latter Sovereign; and under his reign, too, native industries and trades were protected and artificially stimulated by the imposition of import duties, by premiums or bounties on exports, and by direct subsidies to nascent manufactures.

The mercantile theory was dominant. But in 1818 appeared the first impetus to a free trade movement in Prussia, and this was largely produced and aided by the division of the country into a number of States, each with its separate fiscal laws and customs, and each dominated by the notion, that its prosperity was involved in the repression, by stringent protective measures, of its competing neighbours. The results of this internecine commercial warfare were disastrous. In 1819 a German Commercial and Industrial Association was established, based upon the policy of freedom of trade at home, combined with reciprocity in respect of foreign competitors. Still, with fluctuations, up to 1877, the economic policy of Germany was, as a rule, founded upon free trade.

In viewing the influence of Prince Bismarck upon the economic history of his country, it should be borne in mind that in his earliest utterances he proclaimed the doctrine that the State exists for the benefit of all: that social duties were obligatory upon the citizens, and that the State, in its executive functions, existed to ensure that these duties should be scrupulously fulfilled.

His economic legislation has been founded on two facts: the distressed condition into which industry generally had fallen, and the miserable circumstances of the working classes, stimulated and emphasised as they were in their expression by the development of socialism.

The financial and social disorder resulting from the Franco-Prussian War—*victa victoris victrix*—the havoc which had thus been created in industrial life, terminating in the disorganisation of trade in general; the speculative mania which succeeded the distribution of the French indemnity; the prevailing unrest, and difficulty of national settlement into routine channels, after so deep and abrupt a diversion; the artificial stimulation of production, based upon the new supply of money, with consequent financial failures, and disturbance of a regular commercial life; the temporary increase of wages which ensued, and the elevation of the labourer to a novel and more fortunate sphere, for which his previous history had not adapted him; the necessarily resulting collapse and disenchantment; all conspired to produce and deepen that wide-spread social disorganisation and distress which

formed the prelude, in Bismarck's estimation, to the adoption of protectionist ideas, and the supposed panacea of State socialism, as nationally imperative. In 1877, then, the Chancellor felt that definite and stringent measures were demanded to confer comfort upon the struggling labouring classes, and to stem, by forestalling, the rapid influence of socialism.

In 1879 the Chancellor announced his departure from the general fiscal policy of the time and his reintroduction of the traditional protectionist policy of the country. He truly stated that his revival of this system was simply a reversion to the ancient fiscal practice of Prussia. Industry was to be unconditionally protected: the policy of practical free trade had inundated Germany with foreign goods to the depression of home values and the impoverishment of native trade; and on the 7th of July 1879, the passing of a revised tariff proclaimed a farewell to the doctrine of free trade which had existed during many fitful years. In justice to Bismarck's views, it is well to point out that neither free trade nor protection appeared to him an absolute and essential doctrine, independent of time and place: he regarded them, as far as I can gather, as functions, so to speak, of a nation's condition, useful then, useless now: and the position of Germany at that epoch seemed to him to demand the alleviations supposed to be involved in the protectionist scheme.

Tariffs in ascending scale of severity followed one another in necessary sequence.

The State, under his inspiration, subsequently attempted a further step in the direction of universal commercial nationalisation by endeavouring to obtain a monopoly in the trades of tobacco and brandy, though, in each case, without success. This effort at monopoly was stated to be simply a means to an end—the alleviation of general taxation—by obtaining revenue from two prime articles which could justly bear a heavy burden of taxation. The assumption, too, of monopoly would confer upon the State the position and power of an employer of labour in the interests of the working classes, and this doctrine was distinctly recognised.

Bismarck has also frequently expressed the hope that the State should exclusively possess a monopoly of the insurance system in its entirety, so that, as he phrased it, capitalists might not, in opposition to reason, conduct insurance companies for their own benefit at the expense of the community. And the subsequent compulsory insurance laws which he devised and carried into execution were, no doubt, deliberately expected to constitute

successive steps towards the ultimate realisation of this idea, by habituating the citizens progressively to the conception of a general State control in every social form.

In the Prussian railway law of 1838, the principle was expressed that the construction of railways should be assigned to private enterprise alone, though the State ought to possess a right of control in various directions with an ultimate power of purchase. Prince Bismarck endeavoured to familiarise the people with a national absorption of the railway system: and in 1876, under his advice, a Bill was introduced into the Prussian Diet for the transference of railways to the German Empire. The Bill passed; but its powers remained dormant, since the railways failed to pass under State control. Prussia having thus indicated the way, the road was opened for railway nationalisation.

The post and telegraph services are imperial institutions, except that Bavaria and Wurtemberg are constitutionally allowed a wide latitude of local administration. The surpluses derived from the State working of these undertakings are applied to the liquidation of the ordinary State expenditure.

In 1854 compulsory membership in Sick Relief Societies was adopted by the legislation of Prussia, and the other Northern States, generally, imitated the example. It was estimated in 1875 that the number of relief societies in Germany was 12,000, with a total membership of 2,000,000, and a capital of nearly £4,000,000.

In 1871 the owners of railways, mines, and factories were rendered liable by law for injuries or deaths caused to the employed by accidents, where the sufferers were not themselves to blame: and contracting out of the provisions of the law was prohibited.

Attention has also been directed to the regulation of factories: the hours of labour: female and juvenile labour: and Sunday labour.

The Sickness Insurance Law of 1883, the Accident Insurance Laws of 1884 and 1885, and the Law which is the subject of this paper, are founded on the principle of compulsion which was originated by the sick insurance legislation of Prussia of 1854.

In 1882 a combined Accident Insurance Bill and Sickness Insurance Bill was introduced by the Government, proposing that the organisations to which sick insurance was assigned should provide for the support of workpeople during the first thirteen weeks of incapacity to work, with a transfer of the burden of their maintenance after that period to the Accident Insurance Funds. In connexion with an accident insurance measure previously

proposed, the radical party had become alarmed at the development of State insurance; and hence, I infer, it was due to this feeling that the Government in the present measure proceeded cautiously, and abandoned the notion of insurance by the State direct. Bismarck, indeed, stated that the Imperial Insurance Office (contemplated as the central authority in the former measure, which, as abortive, I have omitted) was too "bureaucratic." Its intended place was to be occupied in the amended Bill by Trade Organisations acting on the principle of mutual liability.

The sickness insurance portion of the conjoint Bill alone passed on the 31st of May 1883, and came into operation on the 1st of December 1884.

Extensions of its scope were introduced in 1885 and 1886, so that the whole of the working classes and of the smaller official classes of the country were ultimately included in the sickness insurance scheme.

The accident insurance part of the original Bill was resumed in 1884. Prince Bismarck stated in the Reichstag his conviction that the State should monopolise the entire scheme of insurance and divert it from private enterprise, which he contended merely signified private exploitation in the misfortunes of the labouring population.

The measure became law on the 6th of July 1884, and came into operation on the 1st of October 1885. Extensions have subsequently been introduced, by which successively larger and larger classes of the population—the postal, railway, naval, and military services: agricultural labourers: certain civil service officials: soldiers and sailors—have been embraced within its range.

These two measures—though in their passage into law disjoined for the moment—retain their original unity, for they have been made co-dependent, the accident insurance supplementing that of sickness, and the period of inability to work constituting the point where the one Act ceases and the other commences.

These two laws apply to the whole of the wage-earning classes, and extend, it is calculated, to upwards of 12,000,000 work-people.

No contracting out of either Act is permitted, and a renunciation of the claims of any of the insured is stringently prohibited.

In respect of sickness, a labourer must compulsorily become a member of a local sick fund, a factory sick fund, a building sick fund, a mining sick fund, or a voluntary relief fund. The

employer, as a rule, is required to pay one-third, and the member two-thirds, of the premium: and the premium is assessed at $1\frac{1}{2}$ to 2 per-cent of the average daily local wages current in the trade to which the member belongs, a maximum contribution of 3 per-cent being fixed. The grant of relief is restricted to a period of 13 weeks, and at the expiration of that time, if the illness continues, the charge of maintenance is transferred to the local relief funds. The minimum relief includes free medical assistance, medicine (and even spectacles), and if the labourer is incapacitated he receives sick allowance to the extent of one-half the wages earned by an ordinary day-labourer in the locality where he resides. A distinguishing principle of this law is the fact that no formality is imposed before the workman can claim insurance. The simple incapacity to work is *ipso facto* the sole condition of inclusion in the scheme, and constitutes membership. Upon the employer is entailed the obligation to see that his workpeople are insured, and upon him falls the whole of the premium, with the right of deduction of his employees' share from their wages prior to payment.

The accident insurance scheme commences after 13 weeks of inability to labour produced by casualties. Every accident involves compensation, with the sole exception of wilful injury by the insured themselves.

During the continuance of illness due to accident, support is furnished, with compensation at death to the insured's relatives or those dependent upon him. The entire obligation to insure the work-people is imposed upon the employer, and at his sole cost, since neither the insured nor the State provide any aid towards the allowances and compensation. The Government had originally proposed that both the State and the insured should contribute their shares to these payments, but the idea was abandoned. It will subsequently be observed that this cancelled provision constitutes an essential feature of the law of 1889. The employers, for the due execution of the insurance provisions, are formed into Trade Organisations with mutual liability: the premiums to be discharged by the employers are assessed annually upon the basis of (1) the amount of wages and salaries paid by the particular employer during the preceding year, and (2) the nature and extent of the risk involved in the individual trade or industry. The tariffs thus settled by the Organisations require the sanction of the Imperial Insurance Office. Where complete inability to work exists, the injured claimant is entitled during absence from

labour (commencing with the 14th week of his illness, since the charge of his maintenance is entailed upon the sickness insurance fund during that previous period) to an allowance of two-thirds of his ordinary wages earned in the past year: and in the case of partial inability, to a certain proportion of those wages, based upon the extent of his remaining capacity to work. If death occur, the relations or dependents are entitled, as a funeral contribution, to a sum equivalent to 20 times the daily wages of the deceased—the minimum being fixed at about £1. 10*s.*—as well as an annuity, payable monthly, and assessed as follows: to the widow, during widowhood or until death, one-fifth of the husband's earnings, and for every child $\frac{15}{100}$ths of the earnings until the attainment of the age of 15: but so that the combined annuities to any family shall not exceed three-fifths of the earnings of the deceased.

The serious burden entailed upon employers is assumed to contain compensation in the care thus produced in adopting effective precautions against the occurrence of accidents.

With this preamble—important in its bearing upon the origin, interpretation and orderly sequence of the subject of this paper—I now arrive at the epoch of the law of insurance against invalidity and old age.

III.—Analysis of the Law of 1889.

I purpose, for convenience and clearness, to remodel entirely the provisions of the law, and, disregarding its actual form, to divide it into its essential sections.

I adopt the divisions of:

A.—The Range of the Measure.

B.—The Contributions payable, and the Rules affecting them.

C.—The Allowances provided, and the Conditions relating to them.

D.—The Nature of the Official Machinery devised.

E.—Points of Procedure not already described.

I shall then subjoin the more interesting and important portions of the official memorandum which I have already mentioned, with any concluding observations which the scheme and its details may suggest.

I have felt great perplexity in ensuring uniformity of expression in respect of the principal terms to be employed. The correlative terms "Healthy" and "Unhealthy", "Active" and

"Infirm", "Actives" and "Invalids", "Activity" and "Infirmity", "Activity" and "Invalidity", have at various times been adopted: and though I am far from satisfied with the nomenclature I have selected, I have decided, as being most in consonance with the language of the law, to use the expressions "Active", "Actives", "Invalid", "Invalids", "Activity", and "Invalidity"—an "Active" being a person capable of work, and "Activity" denoting this condition of capacity: while 'Invalid" and "Invalidity" signify the opposed conditions.

A.—*The Range of the Measure.*

i. The following classes of persons are to be insured on completing the age of 16, namely:

(i) Those who are engaged as workmen, assistants, apprentices, and servants, and who receive in return a payment or wage:

(ii) Assistants in shops, and apprentices (except assistants and apprentices of apothecaries) who receive a payment or wage, but whose regular annual earnings do not exceed £100:

(iii) Members of the crews of German ships, both for sea and inland navigation, who receive a payment or wage. [An exception is specified in respect of certain ships affected by the law of the 15th of March 1888.]

If the Federal Council*—which forms the principal machinery in the execution of the provisions of the law—so decides, the scheme may be extended to:

(iv) Persons who carry on an independent business but who do not regularly employ at least one paid workman: and

(v) Masters, who would be excluded by the preceding limitation No. iv, may be admitted, whatever may be the number of paid workmen in their service, provided they are employed in the course of their independent industry at the orders and on the account of other masters in the manufacture of industrial products

* It will be remembered that the Federal Council—the Bundesrath—is the assembly which represents the confederated individual States of Germany: and that its members are appointed by the Governments of those States for each session. The Reichstag, or Diet of the Realm, represents the German nation: and its members are elected by universal suffrage. In these bodies vest conjointly the legislative functions of the Empire.

(*i.e.*, "small" masters): and the inclusion of such masters within the scope of the law is also permitted where they purchase the raw materials of their manufactures at home, as well as while they are temporarily employed in working on their own account.

Further powers in special cases are accorded to the council in connexion with the obligation entailed upon masters, employing such "small" masters, of carrying out, as regards "small" masters and their assistants, the duties imposed by the law on employers.

And certain minor regulations exist which need not be specified.

To the Federal Council also is left the decision as to the extent to which temporary services are not to be regarded as coming within the scope of the law.

(vi) Officials in the service of the Empire or of the Federal States, and of the Communal Unions, and soldiers who are also engaged as servants, are *not* liable to compulsory insurance.

(vii) The scheme, too, is *not* compulsory upon persons who, by reason of their physical or mental defects, are unable to earn regularly by their labour at least one-third of the daily wages of an ordinary labourer, as fixed for their place of employment according to the Sick Insurance Law of the 15th of June 1883.

(viii) An employment where free maintenance alone is furnished to the labourer is not subject to the provisions of this law.

(ix) Persons entitled by the existing law to receive an allowance for invalidity are also excluded from compulsion under the Act.

(x) If a person receive a pension or half-pay, to the extent of at least the lowest allowance granted for invalidity, from the Empire, or from one of the Federal States, or from a Communal Union; or if he is entitled by the existing laws of insurance against accidents to receive a yearly allowance of at least the same amount, he is permitted exemption, upon petition, from the obligation of insurance.

Provisions as to the decision on such petitions are specified with a power of appeal to a higher court.

(xi) A person, excluded from the range of section vi, page 282, who is employed in the service of the Empire, or of any Federal State or a Communal Union, is deemed to fulfil the obligation of insurance by belonging to special insurance or benefit funds, either existing or hereafter established for his department of service, where a provision equal to that obtainable by the law of the land is secured, provided the following requirements are met by such funds, namely:

1. That the contributions payable to these funds may not exceed one-half of those required to be paid by section vii, page 287.

   This provision does not apply to those special insurance funds where the system of collecting contributions does not agree with that expressed in section vii, page 287, in consequence of which higher contributions are required in order to cover charges due to the payment of allowances for invalidity and old age at the rate which the present law prescribes.

2. Whenever higher contributions are to be collected, those of the insured shall not exceed those of the employers.

(xii) The Federal Council is constituted the final authority as to the descriptions of insurance and benefit funds which satisfy the preceding requirements. Participation in insurance and benefit funds thus scheduled is considered equivalent to insurance in one of the institutes established under this law: and the State subsidy (section vi, page 292) will be assigned to the funds so recognised in respect of the allowances for invalidity and old age which they discharge, so far as the claims to such pensions accord with the provisions of the present law.

A rule is also laid down with reference to the calculation of the "period of waiting" (wartezeit) before an allowance vests, and the amount of such allowance.

[*Note.*—The "period of waiting" is a fixed period during the entire term of which the insured must pay contributions before they can claim the right to an allowance or pension.]

(xiii) The Federal Council will decide whether and to what extent the provisions of section vi, page 282, may be extended to officials in the service of other public corporations or bodies, and who possess a future claim to pension: and whether also the provisions of sections xi and xii, page 283, shall apply to members of other insurance funds which afford allowance during invalidity and old age.

It may be of interest if I briefly summarise these regulations, and, for the sake of clearness and simplicity, anticipate a few of the provisions which I shall later on describe:

i. The law provides a *right* to an allowance in invalidity and old age.

ii. Every person is entitled to an allowance for invalidity—without regard to age—who has become confirmedly unfit for work. If this incapacity is due to an accident, the individual only possesses a right to an allowance for invalidity when no allowance can be paid under the provisions of the Imperial Statute for insurance against accidents.

iii. A person may be regarded as unfit for work who, by reason of bodily or mental condition, is unable to earn, by means of labour paid in a manner corresponding to his capacities and powers, a sum of at least one-sixth of the average standard of wages (section xii, page 288), according to which contributions have been paid for him during the previous five contributory years, and of one-sixth of 300 times the amount determined by the Sickness Law of the 15th of June 1883, as the customary local daily wages of an ordinary labourer of the last place of employment where the claimant was engaged in a manner not merely temporary.

iv. Each insured member is entitled to an allowance for old age, without necessarily proving incapability of work, who has completed his 70th year.

v. A person insured who, without being a confirmed invalid, is unfit for work during an entire year, is also entitled to an allowance for invalidity during the subsequent continuance of his inability to labour.

vi. Where the insured has brought upon himself, either purposely or by criminal act—which may be so declared by a Court of Law—a state of unfitness for work, no claim for allowance in invalidity is recognised.

vii. The insured who has become unfit for work in consequence of illness, but who has not observed the provisions of section viii,

forfeits all right to an allowance for invalidity if the incapacity to labour has been produced by his own conduct.

viii. Each institute is authorised to require that the sick fund to which an insured belongs, or has belonged, shall provide such treatment for him as the institute deems appropriate: and the latter must refund the expense incurred for the treatment which it has prescribed. One-half of the lowest amount granted for sickness under the sick insurance law is to be paid as compensation for these charges. And provision is made for the settlement of disputes between the two bodies on these points.

ix. Provisions also apply to the treatment of workmen paid in kind, either entirely or partially, and to the amount of claimable allowance.

x. If the person entitled to an allowance is a foreigner, he may, if he cease residing within the German Empire, be paid off with a "lump" sum equivalent to three times his annual allowance.

xi. As the scope of the law is practically to secure to every labouring person or servant throughout the Empire a sufficient annual provision to relieve him from entire dependence on others (1) in case of the occurrence of confirmed invalidity, and (2) in case of the attainment of age 70 (provided he is not already receiving an allowance for invalidity), it is interesting to note that the scheme is expected to include 12,000,000 persons.

B.—*The Contributions payable, and the Rules affecting them.*

i. To secure the right to an allowance for invalidity and old age the claimant, besides proving incapacity to work, or the attainment of age 70, must

1. have completed the prescribed period of "waiting"—(wartezeit): and
2. have paid the required contributions.

ii. As to condition No. 1, the period of waiting consists of five contributory years for an allowance for invalidity, and 30 contributory years for an allowance for old age. (In respect of the period for invalidity where the insured become unfit for work in the course of the first five years subsequent to the date of the law, a special provision applies to the reduction of the term in question.)

iii. Forty-seven contributory weeks are reckoned as completing one contributory year: (this diminution in the calendar year is adopted in favour of persons who may be unable to continue in regular employment throughout the year): and a period of illness

is not counted as a contributory period when the insured has brought the illness upon himself by methods before described (section vi, page 284), or by guilty participation in brawls, by drunkenness, or evil and dissipated habits.

Where contributory weeks fall within different calendar years, they are added together to complete the contributory year, saving the provisions specified in section xx, page 293.

iv. Where persons, after having been employed not merely provisionally in some service entailing compulsory insurance, are prevented for a period of seven or more consecutive days from continuing their insurance on account of a certified illness involving inability to work, or where, in fulfilment of their military service, they have been recruited for the army or navy in time of peace, mobilisation, or war, or have rendered voluntary military service in time of mobilisation or war, they are permitted to reckon these periods of time as contributory periods. [No contribution requires to be paid during a certified illness (vouched by the proper authorities) nor during engagement in military service, though, as already stated, the time is nevertheless reckoned as though contributions had been received: and in the latter contingency—military service—the State discharges the obligation of contribution.]

v. Where an illness continues for upwards of a year consecutively the period subsequent to the lapse of that year is not to be counted as contributory. And provisions apply for proof of an illness by a certificate of the directing board of the sick or benefit fund to which the insured belongs, or by a certificate of the parish authorities.

vi. The means required for the payment of allowances for both invalidity and old age are to be supplied by the Empire, the employers, and the insured. As regards the Empire, the charge consists of subsidies to the allowances actually to be paid in each year, and as regards the employers and insured, of continuous contributions.

The contributions of the latter classes are to fall in equal shares upon employers and insured, and require to be paid for every calendar contributory week during which the insured has been in an employment which entails obligation to the law, *i.e.*, compulsory insurance.

[If any fractions of pfennigs appear in the settlement of accounts between employers and insured, the share due from the former is to be "rounded off" to five pfennigs (about $\frac{1}{2}d.$) above the fraction, and that of the latter to five pfennigs below.]

vii. The contributions payable for the contributory week must be settled in advance for each insurance institute for the first 10 years after the passing of the law, and afterwards for periods of five years each.

The contributions are to be so assessed as to rate as to prove sufficient—while taking into consideration cases of non-payment due to illness—to cover the costs of administration; to provide a sum for a reserve fund; to meet the charges likely to be entailed by the repayment of contributions; and to raise a sufficient capital to ensure the payment by the insurance institutes of their share of the allowances which will, in all probability, require to be granted during the period in question. The sums necessary to provide the reserve fund are to be so fixed for the first contributory period as to prove equivalent at its close to one-fifth of the estimated capital needful to supply the payment of the probable allowances that will fall due from the institutes during that term. Any deficiency in the reserve at the end of the first period must be met in the ensuing contributory periods.

viii. The insurance institute has power by bye-laws to determine that the reserve fund shall be increased to twice the amount thus prescribed.

ix. The reserve fund and the interest obtained on its investment may only be touched—until it has attained its appointed amount—in cases of extreme necessity under the sanction of the Imperial Insurance Office.

x. In order to form a basis and measure for the contributions (and allowances), the following classification of the insured has been devised on the foundation of the yearly rates of wages they receive, namely:

Class	Annual Wages earned
I	Up to £17. 10*s*. inclusive
II	From between £17. 10*s*. and £27. 10*s*.
III	" " £27. 10*s*. " £42. 10*s*.
IV	Upwards of £42. 10*s*.

Class	Average Wages' Standard
I	£15
II	25
III	36
IV	48

xi. Provisions apply to the ascertainment by the authorities of average annual wages in special cases, such as agriculturists and railway officials, and of those of seamen insured under the Maritime Insurance Law of the 13th of July 1887; and also for members of colliers' benefit clubs, with sick funds of every kind.

And, as a general rule, the annual wages are reckoned as 300 times the local daily wage of ordinary day-labourers in the particular locality of employment: and this forms the standard of the average yearly wage.

xii. The annual standard of wages already furnished, is finally thus settled:

For Wage-Class	I:	a standard of	£15
,,	II:	,,	25
,,	III:	,,	36
,,	IV:	,,	48

The contributions are so to be measured according to the wage-classes that those payable by each class shall cover the claims to allowances, based upon the contributions paid by members of that class, which it may be anticipated that the institute will have to satisfy.

xiii. For the first contributory period (10 years) weekly contributions (payable by employer and insured together) are to be levied by each insurance institute, at the following rates:

For Wage-Class	I:	about	$1\frac{1}{2}d.$	Half from the Employer, and half from the Insured.
,,	II:	,,	2	
,,	III:	,,	$2\frac{1}{2}$	
,,	IV:	,,	3	

For subsequent contributory periods, the committee of each institute must determine the rate of contribution in connexion with sections vii and xii, pages 287 and 288.

[It is estimated that the highest rate of contribution after the lapse of 80 years, when financial equilibrium is expected to be reached, will not exceed

For Wage-Class	I:	about	$2\frac{1}{2}d.$
,,	II:	,,	4
,,	III:	,,	6
,,	IV:	,,	$7\frac{3}{4}$

The contributions levied during the early years will, it is anticipated, greatly exceed the payments for allowances, so that a sinking fund can be formed, which may not be touched until an adequate capital has been created, by means of which it is hoped that the necessity of making the contributions unduly burdensome, as soon as claims for allowances begin to accrue more heavily, will be avoided.]

At the same time deficits or surpluses which result from previous collections of contributions and can be accounted for, must be so treated as to effect an exact balance of accounts when the new contributions are levied.

The decision of the committee requires the sanction of the Imperial Insurance Office.

xiv. If any contributory period should have been allowed to proceed without any such authorised decision having been arrived at, the insurance office itself must assess the contributions to be collected from the insured at the particular Institute in question, according to section xii, page 288.

xv. The rates of contribution and the dates at which their collection commences require to be published in the newspapers at least two months prior to the time when the contributions are to be levied on the determined scale.

xvi. The Insurance Institute may settle other rates of contribution in substitution for those prescribed by section xiii, page 288, even for the first contributory period: and these decisions require the authorisation of the insurance office.

xvii. In order to levy the contributions, Stamps bearing the mark of their respective value are to be issued by each insurance institute for the several wage-classes in its district: and these are supplied at all the post offices situated within the particular district, and at other places which the institute may appoint for their sale, on payment of the amount of their nominal value.

Distinguishing marks (settled by the Imperial Insurance Office) are to be affixed to the stamps, and the duration of their validity recorded: within two years after the expiration of such duration stamps which have lost their validity may be exchanged for fresh ones.

xviii. An ingenious device* is employed for effecting the payment of contributions by employers and insured.

This is done by pasting stamps of a corresponding value on the Receipt Card of the insured.

This receipt card bears the day and year in which it is issued, the conditions affecting it (according to section ix, § 3, page 299), and the penal provisions of the same section.

The cards are to be numbered in regular succession for each insured, and provision is made for the exchange of new cards for

* It will be remembered that in our own system of Post Office Savings' Banks this plan was introduced in November 1880, which the German officials have evidently copied.

old ones and for cards that have been lost or have become defaced.

xix. Each receipt card possesses space for stamps for 47 contributory weeks: each bears also the name of the insurance institute of the district in which the insured was at the time employed.

If the insured is not provided with a card, the employer procures one for him, and deducts the cost from the next wages.

xx. The insured possesses a right of objection on certain points connected with these cards and the calculations exhibited on them.

xxi. The employer pastes upon the receipt card, on the day on which wages are paid, stamps (purchased at his own cost) for an amount to be reckoned according to section xxii, page 290.

xxii. The payment of contributions by employers and insured is arranged by that employer in whose service the insured was engaged during the previous calendar year: and if an insured has not been employed in the same service for an entire calendar week, the master who first employed him must discharge the payment of the contributions.

Employers are empowered, when paying wages to their servants, to deduct one-half of the required contributions.

xxiii. Persons who cease to be in a condition entailing compulsory insurance may, nevertheless, continue their insurance voluntarily, or may renew it by payment of the contributions assessed for Wage-Class II, in the stamps of the institute of their district: and, as a kind of fine, an extra stamp is to be added for every week of voluntary insurance. Various provisions on different points of the scheme apply to this permission.

C.—*The Allowances provided, and the Conditions relating to them.*

i. The grounds on which the allowances are granted have already been expressed.

ii. The allowances are fixed by the calendar year, and are furnished by a sum to be supplied by the insurance institute—saving the proviso of section vii, page 292,—and by a fixed subsidy from the State.

iii. In settling the share of allowances for invalidity, payable by the insurance institute, the sum of £3 is taken as the basis,

and this increases with every contributory week completed, according to the following scale:

For Wage-Class	I:	by the amount of about	$\frac{1}{4}d.$
"	II:	"	$\frac{3}{4}$
"	III:	"	1
"	IV:	"	$1\frac{1}{2}$

[Thus, the basis of *each* allowance for invalidity is formed by £2. 10*s.* contributed by the State, and by £3 which is the *fixed* annual contribution of the institute, or, together, £5. 10*s.* In addition to this, the allowances for invalidity are increased at the following ascending rates:

For Wage-Class	I:	by about	$\frac{1}{4}d.$	per week.
"	II:	"	$\frac{3}{4}$	"
"	III:	"	1	"
"	IV:	"	$1\frac{1}{2}$	"

The lowest allowance for invalidity therefore would amount to

After Years	Class I			Class II			Class III			Class IV		
	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
5	5	14	$8\frac{1}{2}$	6	4	$1\frac{1}{4}$	6	11	$1\frac{3}{4}$	7	0	3
10	5	19	$4\frac{3}{4}$	6	18	$2\frac{1}{2}$	7	12	$3\frac{1}{2}$	8	11	$1\frac{1}{4}$
20	6	8	$9\frac{1}{2}$	8	6	$4\frac{3}{4}$	9	14	$7\frac{1}{4}$	11	12	$2\frac{1}{2}$
30	6	18	$2\frac{1}{2}$	9	14	$7\frac{1}{4}$	11	16	$10\frac{3}{4}$	14	13	$3\frac{1}{2}$
40	7	7	$7\frac{1}{4}$	11	2	$9\frac{1}{2}$	13	19	$2\frac{1}{2}$	17	14	$4\frac{3}{4}$
50	7	17	0	12	11	0	16	1	6	20	15	6

].

iv. The share of the allowances for old age to be supplied by the insurance institutes is to amount for every contributory week:

For Wage-Class	I:	to about	$\frac{1}{2}d.$
"	II:	"	$\frac{3}{4}$
"	III:	"	1
"	IV:	"	$1\frac{1}{4}$

[These grants, combined with the State subsidy, bring up the total allowance obtainable during old age to

£5. 6*s.* 5*d.*	in respect of Wage-Class	I.
£6. 14*s.* 7*d.*	"	II.
£8. 2*s.* 10*d.*	"	III.
£9. 11*s.* 0*d.*	"	IV.]

v. In this scale, 1,410 contributory weeks are reckoned to the account of each person. If contributions have been paid for any of the insured for more than 1,410 contributory weeks in different wage-classes, those 1,410 weeks are taken into account during which the highest contributions were received. (The period

of 30 contributory years is, of course, equivalent to 1,410 contributory weeks.)

vi. The subsidy provided by the State amounts to £2. 10*s*. for each allowance in invalidity and old age.

vii. Wage-Class II is adopted as the basis of calculation in reckoning allowances for periods of certified illness or military service, which, in accordance with sections iii and iv, page 284, are to be treated as contributory periods. And the State will assume the obligation of supplying that portion of an allowance which is payable in consequence of a term of military service.

[The State charge consisting of £2. 10*s*. for every allowance recognised as valid, it was finally estimated that this subsidy will amount to £320,000 in the first year and gradually attain to £3,450,000 in the 80th year of insurance, when it is expected that, approximately, a balance will for the first time be established between the number of allowances payable and the capital accumulated for liabilities, in consequence of which the subsidy will no longer be required and will gradually be extinguished. It is calculated, also, that the allowances alone will ultimately amount to £12,520,000 per annum, and the costs of administration (at 1*s*. per head of the insured) to £600,000 per annum immediately the law is put into execution.]

viii. The allowances are payable in advance in monthly instalments, and may be rounded off by an addition of upwards of $\frac{1}{2}d$.

ix. Provisions in respect of the assignment of a wage-class (both as regards allowances for invalidity and old age) apply in the case of persons belonging to any insurance or benefit club, colliers' benefit club, or any local sick fund.

x. Allowances for invalidity are due from the date of the day on which the insured was rendered unfit to work, and the day is defined as that on which the petition for "invalidity" allowance was forwarded to the authorities.

xi. The allowance for old age begins on the first day of the 71st year of life, and is forfeited whenever the applicant holds an allowance for invalidity.

xii. Women who marry before entering upon the receipt of an allowance may claim a refund of one-half of their contributions, provided payments have been made for at least five contributory years, and their reversionary right (based upon their former contributions as insured) thereupon lapses. The claim must be established within three months after marriage.

xiii. If a man whose contributions have been paid for at least five contributory years should die before entering on the receipt of an allowance, his widow, or, if no widow be left, his legitimate children under 15 years of age, are entitled to obtain a refund of one-half of the contributions paid.

xix. If a woman for whom contributions have been paid for five contributory years should die before receiving an allowance, the children she leaves—if also fatherless—under 15 years of age receive a refund of one-half the contributions.

xx. The reversionary right arising from a condition of insurance is extinguished if contributions have only been paid for less than a total of 47 contributory weeks during the period of four consecutive calendar years: but the right is revived whenever the condition of insurance is renewed by re-employment in a business entailing compulsory insurance, or by voluntary payment of contributions, and provided a term of five contributory years shall have been thereafter completed.

xxi. If the condition of a recipient of an allowance for invalidity should change by which he ceases to be confirmedly unfit for work, the allowance thereupon may be terminated, and if the allowance is restored the former period during which he received payment is to be counted in the same way as a term of certified illness. (See sections iii and v, pages 285 and 286.)

xxii. The right to an allowance under this law ceases (i) where the person receives an annuity under the law for insurance against accidents as soon as, and by so much as, the "accident" allowance, together with the allowance which would (under ordinary circumstances) be granted under the present law, exceed the sum of £20. 15*s.*: (ii) in the case of officials and soldiers (see sections vi, page 282, and [xiii], page 284) as long as, and by so much as, the pension or half-pay obtained by them, together with the allowance they should receive under this law, exceed the sum of £20. 15*s.*: (iii) when and so long as the individual entitled is undergoing a sentence of more than one month's imprisonment, or as long as he is an inmate of a workhouse or reformatory: and (iv) while the insured is not domiciled within the Empire. (The Federal Council, however, in the latter case may decide otherwise in connexion with certain frontier districts.)

xxiii. Obligations imposed by the law of the land upon parishes or charitable unions, with the object of supporting necessitous persons, are not affected by this law; and regulations

apply in respect of the right of allowance to needy persons receiving support from a parish or union during a period in which they would be entitled to an allowance for old age or invalidity under the present law.

xxiv. Factory benefit funds, colliers' funds, and other similar institutions which allow their members an annuity for old age or invalidity, whilst insuring them under the requirements of this law, are empowered to reduce the pecuniary support they grant to such persons as are also entitled to the receipt of an allowance for old age or invalidity under such law, by the amount of the latter, or by a less amount, provided the contributions of employers and members of the fund are diminished in a corresponding ratio.

[I may mention that soldiers disabled in time of war already receive insurance allowances on a special scale, which are considerably higher than those granted under the new law.]

xxv. Allowances under this law cannot legally be mortgaged or transferred to third parties, nor can they be seized for other claims than those of a wife and legitimate children, and those of parishes or unions entitled to repayment.

xxvi. Persons claiming allowance for old age or invalidity must give notice to the proper Lower Administrative Authorities for their place of residence. The receipt card and documents, serving as vouchers for the validity of the claim, must accompany the notice. If the claim is made for an allowance for invalidity, the local officials examine and report to these authorities.

xxvii. The administrative authorities then transmit the petition (with the documents and other relative information), together with their report, to the last insurance institute to which the applicant contributed, as shown by his receipt card.

xxviii. The directing board of the institute then examines the petition, and unless it is summarily rejected (with a written notice containing the reasons for refusal), the Board requires production of the previous receipt cards. The costs attendant on these proceedings and investigation fall upon the institute.

xxix. Assuming the claim to be considered valid, the rate of allowance is at once fixed, with a notice in writing to the person entitled to the allowance, from which also he may understand the mode in which the grant is reckoned.

xxx. The fact that inability to earn wages has been caused by an accident, for which compensation is granted by the law of insurance against accidents, is not a valid reason for rejection of

the claim to "invalidity" allowance, and the insurance institute will then claim compensation from the trades' associations concerned for payments so allowed. Disputes regarding such compensation in the case of accidents are to be settled as prescribed by the law of insurance against accidents of the 6th of July 1884, or otherwise by an ordinary judge.

xxxi. An appeal lies with the petitioner to the arbitration court in respect of rejected claims or the settlement of the rate of allowance: and a legal remedy of revision is available to both parties against the court's decision.

The Imperial Insurance Office is the final arbiter upon such revision.

xxxii. A claim to allowance for invalidity which has been finally rejected can only again be submitted before the expiration of a year from the date of final decision if it can be proved that circumstances have in the meantime arisen through which the applicant has been rendered unable to earn a living for a prolonged period.

xxxiii. Assuming an allowance has been finally settled and assessed, the directing board of the institute prepares a certificate of title for the annuitant in respect of the annuity he is to receive, in which are indicated the post office by which the allowance will be paid, and the periods of payment, while, at the same time, the lower administrative authorities of the place of the claimant's residence are made acquainted with the rate of allowance granted. When the rate of allowance has been finally decided, a copy of the notification thereof, with the certificate of validity and the receipt card, must be forwarded by the board to the reckoning department of the Imperial Insurance Office.

xxxiv. The payment of allowances is effected through the particular post office in whose district the recipient resided at the time of forwarding his petition: and on due notice of change of residence the post office for the new district becomes the source of payment. The post office is authorised to make payments to holders of certificates of title.

D.—*The Nature of the Official Machinery devised.*

i. The chief machinery consists of Insurance Institutes, which are to be established by order of the State Governments for the larger Communal Unions of their territory, or for the whole territory of the Federal State.

ii. On each institute devolves the insurance of all persons whose places of employment are situated in its district.

The establishment of institutes requires the sanction of the Federal Council, and the seat of each institute is determined by the State Government.

iii. The insurance institute may bring a suit before a court of justice, or itself be proceeded against in the course of its administration. If bankruptcy occur, the Communal Union for which the institute has been established, or the Federal State for which the institute is founded, is responsible for its liabilities to creditors to the extent of the deficiency of its capital. Where several Communal Unions or Federal States possess the same institute, the liability of each, in respect of any deficiency in the funds, is to be measured by their respective populations, as ascertained at the preceding census.

iv. The property of the institute (whose business is to be limited to the operations prescribed by this law) is applicable solely to the objects of the law.

Accounts of income and expenditure are to be strictly kept, and the assets vested in trust.

v. The expenses of the first establishment of the insurance institutes are borne either by the Communal Union or by the Federal State for which they are founded: but the subvention thus advanced to the institutes must be repaid from the first contributions received.

vi. Each institute is governed by a Directing Board, representing it both judicially and extrajudicially, and the board possesses the powers of a public authoritative body.

vii. Provisions regulate the appointments, duties, and salaries of officers connected with the board. These officers are to be selected either by the Communal Union or the State Government, and their salaries and the pensions to their families are charged to the institute.

viii. Other officers on a par with these officials may be appointed, who shall be paid, or not, as bye-laws may prescribe.

ix. With each insurance institute is connected a Committee of at least five members, representing the employers and the insured, and these representatives are elected by the governing boards of the local benefit and insurance funds existing in the district of the institute.

x. Since the governing boards of such funds and associations are composed of representatives of employers and representatives of the employed, the members of the boards belonging to the class of employers shall take part only in the election of employers' representatives, and those members of the board who are of the class of the insured may only elect representatives of the insured. The elections are valid for five years.

xi. Those employers only are eligible as representatives of employers who have in their service persons insured under the law, and persons only who are insured under the present law can act as representatives of the class of the insured.

xii. A Council of Supervision may be formed (to watch over the proceedings of the board), and the numbers of the representatives of the employers and of the insured must be equal; and officers chosen from the employers and insured are to act as the local administrative instruments of each institute. But neither members of the council of supervision nor such officers can sit on the directing board.

xiii. Provisions regulate the voting power and the creation and scope of bye-laws of the institutes, and the latter must be sanctioned by the Imperial Insurance Office.

xiv. Bye-laws must be prepared for each institute, and their range is defined in detail.

xv. The posts of the unsalaried members of the directing boards, of the members of the committee, of the council of supervision, and those of the officers and assessors of the arbitration court (section xviii) are honorary positions, and these officials are paid only for expenses incurred in the discharge of their duties.

xvi. The representatives of the insured must inform their employers whenever they are required to perform the duties of their office, and receive, in addition to expenses incurred in the fulfilment of their functions, compensation for loss of wages during their attendance.

xvii. A State Commissioner, with a due array of officials, is appointed by the Government of the State and the Imperial Chancellor for the district of every institute, in order to protect the interests of the other institutes and of the State.

xviii. One Arbitration Court at least, with assessors, is established for the district of every institute, and provisions regulate its constitution and duties. The costs attendant on it,

together with the expenses of its proceedings, are chargeable to the institute. The number of assessors must include at least two taken from the class of employers, and two from that of the insured.

xix. Several insurance institutes may combine for the purpose of bearing in common the burdens imposed by insurance against invalidity and old age.

xx. The insurance institutes are subject to the supervision and inspection of the Imperial Insurance Office, which is empowered to examine their affairs at any time.

E.—*Points of Procedure not already described.*

i. The reckoning department of the Imperial Insurance Office is responsible for the calculations and statistical work included in the duties of that office. This work comprises (i) the apportioning of allowances, and (ii) the preparation of statistics relating to the working of the law.

ii. The department apportions the allowances to the State and to the interested insurance institutes. This allotment is effected—after deduction of the subsidy to be paid by the State—according to the relative contributions which have been paid into the separate institutes for the insured, or which were payable by the State in accordance with section vii, page 292. The several directing boards are empowered to raise objections within a limited time to any apportionment.

iii. The department must inform the central postal authorities, at the end of each financial year, of the amount to be refunded by the State and by the separate institutes. The institutes must recoup the advance payments made by the post offices within a specified time, and the Imperial Insurance Office can adopt compulsory measures for the recovery of arrears.

iv. Officers appointed for the purpose by the central State authorities effect the issue and exchange of receipt cards: reckon the stamps pasted upon the cards by the insured, in order to ascertain the number of contributory weeks for every separate wage-class, which are to be placed to the credit of the holders of the cards. The duration of certified illness and of military service are also recorded on the card: and a certificate is furnished to the holder showing the result of these calculations.

v. The insured can protest against the calculations, and he

possesses a right of appeal against the decision that may be pronounced on his objection.

vi. The moneys of the insurance institutes may be invested at interest according to the provisions of the law for insurance against accidents, but a wider range of investment is granted if permitted by the Communal Union or the central authorities of the Federal State: bonds and mortgages are to be placed in the custody of some public authority or office empowered to keep moneys or valuable papers.

vii. The financial year is the same as the calendar year.

viii. Each institute is to bear the expenses connected with the issue of receipt cards, whenever they are not chargeable to the insured.

ix. A large number of pains and penalties and legal remedies are prescribed, such as:

1. Arrears due to the institutes are to be collected in the same way as parish rates.
2. Employers may not detrimentally affect the benefit of the law to the insured by efforts which would restrain or restrict the latter in exercising the functions of any honorary posts which the law permits them to fill, and any agreement to the contrary effect is invalid.
3. Employers making other entries upon the receipt cards than those permitted by the law are liable to a penalty not exceeding £100, or to imprisonment. (This provision arose from the fact that the labouring population strongly expressed the fear that the cards might prove a revival of the ancient "labourers' books" formerly kept by employers, in which they recorded remarks upon the character and acts of their servants: hence information would have been possible relating to the private working life of the insured, and to obviate this danger the stringent penalty was inserted.)
4. Penalties also apply to
   (*a*) unjust deductions by employers from the wages of the insured;
   (*b*) the use by employers of other than the prescribed stamps;

(*c*) the publication by directing boards and other administrative bodies of the institutes and their officials of the secrets of their office, and for making public, with a view to damaging the owner of a business, trade secrets which may have come to their knowledge in the prosecution of their duties, or for copying private business processes or methods which they have similarly learnt.

x. Provisions apply, among other matters, to the reduction of the "period of waiting" before receiving an allowance. In respect of (i) Invalidity—where the insured has become unfit for work during the first five years subsequent to the date of the law—provided the legal contributions have been paid for one contributory year—the period may be reduced by that number of weeks during which he had been in some employment (dealt with by this law) before the date of the law but within the five years previous to his becoming incapable of work, and Wage-Class I is to be the basis for reckoning the average rate of wages for the period by which the "waiting time" is shortened. In respect of (ii) Old Age—where the insured had completed the 40th year of age at the date of the law, and had been engaged in some employment (subject to the law) during the three years immediately preceding the passing of the law for the space of 141 weeks, the period may be curtailed by so many contributory years as the years of life at the date of the law exceed the number of 40. The minimum basis for the benefit in this case is that of the rates assigned to Wage-Class I.

The law is thus retrospective.*

I have, thus, at last emerged from this toilsome task, and emerged with the registered vow that never again will I attempt to analyse a complicated German Act of Legislation, or endeavour to fashion its articles into shapeliness and order.

I have necessarily excluded a large number of its provisions: several of the elaborate checks and counter-checks in balanced equipoise which have been provided, especially with respect to the legal rights and powers of appeal of the various bodies and the insured; and many details of the official machinery prescribed. The omitted portions, however, deserve a careful study in order to

* For a complete explanation and illustration of these "transitory" provisions, *vide* the Addendum to this Paper on page 362, *et seq.*

perceive completely the intricacies of the measure, and the implicit sources of difficulty and discontent, tending, by reason of their minuteness and comprehensiveness, I fear, to entail constant popular and official friction in execution.

In order to render the lengthy exposition more intelligible and compact, I had also intended to adopt a concrete example of a labourer entering the scheme at age 35; to show the annual amount of his contribution, that of his employer, and of the State; to include the estimated amount of deduction for management; to exhibit the annuity for invalidity and old age to which he would be entitled and its conditions: the financial pressure entailed upon the population generally: the minute and inquisitive supervision to which throughout life the citizen is exposed: the possibilities of dissatisfaction sure to be engendered: and then, finally, to attempt to ascertain the ultimate approximate relation between the accumulated contributions, and the estimated value of the obligations of the Institutes. But the brief time at my disposal has prevented the pursuance of this intention. It is true that in such a survey, average results must alone be regarded: but still an individual illustration in these aspects would have facilitated a comprehension of the details of the law. The student of the subject however can construct such a conspectus for himself from the materials which I have furnished. For the same reason I have been compelled to abandon an interesting and helpful comparison between the statistics employed (to be detailed hereafter), and such similar data—rates of sickness, death and wages—as I might have been able to select (from materials I have collected), relating to classes of British workmen, corresponding as far as possible in social condition to those of Germany who are embraced within the scope of the law.

It will be observed that in the essential difference between the present law and its legislative precursors, the Government may be said to have adopted for the first time a genuinely socialistic principle, since, under the preceding insurance laws, the burden of contributions fell solely on employers, as in the insurance against accidents—an outline of which I have sketched—or upon employers and the insured, as in the sickness insurance which I have described, while in this latest scheme a third part of the charge is contributed by the State.

During the progress of the Bill objections were raised to the low rate of allowances, but the scale was affirmed on the ground

that at the inception of the scheme it was inexpedient to impose too serious a burden upon the people. It is to be observed on this point that the lowest scale—£5. 6*s*. 5*d*.—as compared with an English standard, is equivalent to upwards of one-third of the average annual earnings of that class: and, moreover, it was never the intention of the Act to provide aged and invalid persons with a sufficient income for independent livelihood, but simply to aid them by an augmentation to any other provision or assistance they might possess or be able to secure.

Referring to the fact that the cost of administration was assessed at one Mark per head of the insured, or £600,000 a year, this charge would have been almost indefinitely increased had not existing official machinery (*e.g.*, the postal administration) been attached to the scheme: it is estimated that the employment of the post offices for the sale of insurance stamps has avoided the necessity of establishing 83,000 additional centres for that purpose.

IV.—Statistical Basis of the Scheme.

I now proceed to explain, as briefly as is possible consistent with an accurate survey (though I fear tediously), the data and methods on which this superstructure has been raised, as they are contained in the voluminous statistical and mathematical memorandum which I have had the good fortune to secure. I shall necessarily omit a large amount of interesting materials, and even with regard to those which I have compiled I feel my task cannot escape the charge, in some parts, of needless minuteness of comparative worthlessness. It was of interest, however, I thought, in the exposition of this daring project, to describe as completely as appeared necessary the searches, and even the gropings, after some substantial experiential basis which the authors had carefully and laboriously attempted. The effort too to secure some approximately valid foundation for so elaborate and far-reaching a system, and the endeavour to construct it as far as feasible upon scientific methods will, I trust, involve a sufficient apology for the extent of an exposition so original and, indeed, unique. The interest is further enhanced by the fact that no previous attempt has been made to exhibit the contents and reasonings of this memorandum to English economists and actuaries, so that in itself it furnishes an original

contribution—however imperfect and limited by the indefinite character of the data—to our actuarial knowledge. Moreover, the digest, I believe, forms the first serious and methodised effort to establish an imposing and extensive social scheme upon a definite statistical and scientific basis.

The authors of the memorandum, as I have stated, seek as far as previous detached investigations afford a guide, to establish a valid relation upon scientific grounds between the contributions or premiums required and the allowances to be granted, and they appear to be definitely directed by the fact that—admitting the existence of varieties of risk in the different occupations involved in the scheme—the premiums and annuities calculated must apply to the "totality"—excess in one direction balancing deficiency in another.

The total value of the annuities or allowances is rightly made dependent upon the number and value of the annuities, which again depend upon the date of commencement of invalidity or old age, and the subsequent duration of life of those entitled to the benefits. The total value of the contributions or premiums is sought to be made dependent upon the number of contributors, and the duration of the contributions (*i.e.*, the period of activity), terminating by death or the advent of invalidity.

The authors endeavour to determine, as the basis of the value of allowances and contributions:

i. A table of invalidity.

ii. A table of mortality

(*a*) Of persons who become invalids.

(*b*) Of active persons who die without the occurrence of invalidity.

The value of the element of age is admitted, and they accordingly also endeavour to construct—in order to obtain the total value of both contributions and allowances for all the insured—

iii. A table expressing the proportion in which the persons, as above, are distributed over the different years of life.

It is considered that these data exist in sufficient precision for males: while in respect of females the statistics are deficient, both with regard to the probabilities of invalidity and death.

It is deemed to be sufficient, however—though the detailed

grounds of this opinion are not stated—that the relative values employed for males should be applied to females.

The calculation of independent bases of values for each institute is not at present attempted, but at a later stage it is intended to investigate the question of discrimination in respect of special bases of calculation for the several districts, the importance of which is admitted, since the rates of invalidity and death will obviously show different results, according as one institute contains a larger proportion of employments which involve greater hazards than those which constitute the chief area of occupation in another.

It is considered that division of ages would not produce sufficient deviations in the separate districts to affect the calculated results.

In accordance with the preceding obvious consideration, it is admitted that the contributions in the various institutes will hereafter require modification, if it should not be proved that the higher charge entailed by a heavier liability to invalidity is compensated by an augmented mortality, *i.e.*, a diminished duration of life among the invalids, so that the increased claims for "invalidity" allowances are not balanced by fewer numbers (in consequence of a heavier death-rate) who survive to claim in invalidity, and, further, claim that allowance for a shorter period. This necessary and tentative adjustment, however, is impracticable at present. Consequently the same rate of contribution applies universally, notwithstanding the accepted probability of different rates of risk for the several districts.

(I).—*Table of Invalidity.*

The authors then pass in review the various investigations which have been made, with a view to the construction of a table of the probabilities of invalidity. They recite:

i. Dr. Heym, of Leipzig, as the first investigator into the subject, who arbitrarily assumed that at age 20 the annual probability of invalidity was ·0001, while at age 79 it attained to unity: the intermediate ratios being interpolated by simple geometrical progression between these limits.

ii. Dr. Hülsse next investigated the experience of the Freiberg Miners' Benefit Society, the oldest and most extensive in Saxony. He showed 68 cases of invalidity per 1,000 members: and in

consequence of this result Dr. Heym corrected his table by assuming from Hülsse's figures an average probability of ·002, adopting one-half of this as a purely arbitrary constant (named the "Probability of Invalidity through Accident") and the other half as a variable, reckoning unity at age 79. For the variable part of the probability he assumed that from age 20, for which he placed the value at ·00002, the annual probability increased up to unity at age 79 in a geometrical progression. Dr. Heym's table is of course discarded on account of inadequacy of observations, though its theoretical merits are maintained. Its error (too low a probability value) was to an extent neutralised by the assumption that the mortality of the invalids was similar to that of the general population of Saxony.

iii. A further investigation was undertaken by Dr. Zeuner in his treatise on "Mathematical Statistic", where he endeavoured to form a table of invalidity for miners: but the meagre facts at his command have been proved by subsequent statistics (derived from miners' benefit societies) to show too low a scale of probability of invalidity.

iv. It was not until the year 1868 that improved results were possible, when, under the advice of Dr. Wiegand, director of the life insurance company, the "Iduna", at Halle, the Union of German Railway Administrations completed an extensive enquiry into the rate of invalidity among railway employees: this investigation was extended in 1886 to railway workmen. The investigation was conducted for the first time on scientific principles.

Dr. Behm, of Berlin, after Dr. Wiegand's death, continued the investigation until 1883, when it was undertaken by Dr. Zimmermann, of Berlin.

Subsequent investigations are mentioned relating to coal and other miners by Drs. Küttner, Caron, A. Morgenbesser, and Münscher.

v. In 1884 Dr. Zillmer of Berlin published tables of invalidity for engineers and metal workers in Germany.

vi. An invalidity table in manuscript for workmen of different occupations has also been compiled by Dr. Behm and forwarded to the German Home Secretary.

The following table exhibits for interval ages some of the facts deduced by the preceding statisticians:—

TABLE A.

Age	Invalidity Probability												
	Of the Railway Employees in the Years						Of Miners					Of Engineers and Metal-Workers (according to Zillmer)	Of Workmen in various Trades (according to Behm)
	For the Train Servants		For the Whole Staff		For the Non-Train Staff		In the Prussian Coal Mines (According to Küttner)		In the Prussian Mines		In the Austrian Mines (according to Kaan)		
	1868 to 1873 (according to Behm)	1868 to 1884 (according to Zimmermann)	1868 to 1873 (according to Behm)	1868 to 1884 (according to Zimmermann)	(according to Behm)	1868 to 1884 (according to Zimmermann)	In the Years 1874 to 1878	1869 to 1883	1870 to 1879 (according to Caron)	1868 to 1878 (according to Morgenbesser)			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
20	·00052	·00031	·00022	·00021	·00038	·00021	·00310	·00220	·00295	...	·00037	·00100	·00019
25	·00117	·00118	·00053	·00072	·00044	·00054	·00420	·00320	·00373	·00259	·00092	·00132	·00038
30	·00218	·00281	·00125	·00153	·00084	·00096	·00530	·00600	·00472	·00421	·00215	·00183	·00076
35	·00342	·00147	·00212	·00284	·00172	·00220	·00890	·00900	·00754	·00787	·00362	·00267	·00152
40	·00671	·00740	·00382	·00474	·00326	·00382	·01860	·01650	·01278	·01383	·00652	·00410	·00305
45	·01005	·01129	·00662	·00811	·00662	·00698	·03580	·02460	·02332	·02219	·01130	·00663	·00609
50	·01897	·02159	·01217	·01557	·01272	·01375	·06960	·06650	·04335	·05104	·02076	·01129	·01218
55	·03568	·04163	·02317	·02935	·02558	·02687	·11060	·11200	·08056	·09566	·03953	·02023	·02437
60	·05618	·07623	·03928	·05728	·04651	·05427	·22800	·19710	·10331	·16729	·06709	·03815	·04873
65	·09765	·12207	·06763	·10002	·07426	·09752	·39500	·28660	·13249	·25823	·11564	·07571	·09747
70	·15789	·18165	·10153	·16023	·10633	·15781	·58280	·43230	·25989	·38294	·17330	·15814	·19493
75	·72022	·28962	·13306	·20704	·13782	·20617	1·00000	·58430	·50979	·61885	·22883	·39580	·38986
80	...	...	·40255	·23134	·25822	·23134	...	·91470	1·00000	1·00000	·70291	...	·77972

The facts for railway employees showing generally that the "invalidity" probabilities of Zimmermann are higher than those of Behm are explained by the circumstance that the health of persons of this occupation is said to have deteriorated generally during the last 10 years. The divergence in the results of Küttner's table and those compared with it is explained by the fact that the invalidity of coal miners (as shown by Küttner) has fluctuated. And the higher probabilities in these tables as compared with those of Kaan are, perhaps, explicable on the ground that the former relate only to coal miners, while the latter contain other branches of mining. In other cases of discrepancy no explanation seems feasible.

vii. The purpose of the law requiring a table of invalidity appertaining to various occupations, the authors have selected Behm's Table as the basis of the scheme. (See column 14 of Table A.)

viii. The following table comprises, for interval-ages, the actual cases of invalidity which occurred, compared with the expectation of invalidity according to Behm for the same number of workmen:

TABLE B.

Age	Active Workers One Year under Observation	Number of Invalids expected according to Behm	Actual Number of Invalids
(1)	(2)	(3)	(4)
14 to 20	1,371·8	·229	0
25	2,192·7	·833	3
30	2,287·6	1·74	3
35	2,111·5	3·21	2
40	1,378·8	4·21	4
45	685	4·17	0
50	360	4·38	6
55	199·9	4·87	3
60	121	5·90	5
65	58·2	5·66	4
70	22·5	4·39	3
75	6	2·34	2
80 to 85	3	2·78	1
Sum for all Ages	49,981·1	218·589	206

Thus, of 49,981 workmen, 206 became invalids within the year, while the expected number according to Behm was 218·589, or an excess of 6 per-cent.

By comparison with the statistics of occupation of 1882 (vol. ii of the statistics of the German Empire), it is considered that Behm's Table (although dealing only with railway workmen) may be applied to all the insured. At this occupation census of the Empire—undertaken on the 5th of June 1882—it was attempted to ascertain the number of invalid persons in different occupations where the invalidity was due to illness, accident, and old age—all these causes being combined. Thus the Government statistics exhibit the number of active persons, and those who have become invalids. From these statistics a new invalidity table has been attempted, with which to compare Behm's results. Some revision was adopted with reference to the proportionate numbers at each age: and this has introduced uncertainty. The statistics, too, are admitted to be themselves defective on the question of invalidity: for example, has the enquiry (contained in the schedules issued) —"inability to gain a livelihood"—always been answered in the sense intended by the question? Revision therefore has also been attempted on this point. Again, it would appear probable that in these statistics of the Empire hardly all the cases of invalidity can have been included. If, for example, the miners' benefit societies be adopted as a test of the extent of invalidity, the invalids in the Empire statistics would have to be increased by about 30 per-cent to arrive at the number of invalids really existing in June 1882 according to the experience of these societies. If, then, the occupation statistics, with respect to invalids in the classes of miners and smelters, show so deficient a number it is clear that the deviation of the actual cases of invalidity in other occupations must be much greater still, since in no other calling is there so precise a definition as in the occupations named.

In the statistics of the Empire, then, if employed, the number of invalids which they show must be considerably augmented. To ensure accuracy the increase should be not less than 60 per-cent, which, however, the Imperial Office of Statistics considers to be an excessive adjustment.

In the Empire statistics a total of 11,763,472 working males and invalids over 20 years of age, is shown, among whom the invalids are 554,031. Consequently, the proposed augmentation gives $554{,}031 \times 1{\cdot}6 = 886{,}450$ invalids to be dealt with.

By a further adjustment in respect of survivors (in the mortality table)—the grounds of which are not stated—it is assumed that the proportion of actives and invalids in the survivors of the mortality table for the male population of the

German Empire is about the same as in the ascertained male population: the probability of invalidity on these suppositions is shown in the following table, in which also appear, for comparison, Behm's results:

TABLE C.

Age	Invalidity Probability	
	According to Behm	Calculated according to the Statistics of Occupation
20	·00019	·00024
25	·00038	·00064
30	·00076	·00109
35	·00152	·00145
40	·00305	·00250
45	·00609	·00455
50	·01218	·00860
55	·02437	·01615
60	·04873	·02720
65	·09747	·04575
70	·19493	·07980
75	·38986	·13335
80	·77972	·22540

It is therefore concluded, on the whole, that Behm's table, showing, except at the lower ages, considerably higher values, may be relied upon for the calculations in respect of invalidity and old age. But Behm's table again requires modification in respect of those who become invalids through accident, and who are entitled to annuities by the accident insurance laws: for his results comprise *all* cases of invalidity, without distinction of cause. Thus, Behm's invalidity probabilities must be reduced by the probability of invalidity occurring through accidents. This is effected by multiplying the final result by a coefficient equivalent to unity minus the quotient obtained by dividing the number of annual invalids through accident by the total number of annual invalids. The coefficient deduced from the experience of railway workmen (where, out of 206 invalids 27 cases have occurred from accidents) is $1 - \frac{27}{206} = \cdot 86893$: and multiplying by ·87 the figures of Behm's Table for an "invalidity" annuity, there is obtained the amount to be charged for the insurance of invalidity and old age. This coefficient of ·87 is confirmed by the statistical compilation of the Imperial Insurance Office for 1886.

(II).—*Mortality Tables.*

With regard to the selection of a table of mortality for invalids, similar difficulties and divergence of results exist in the statistics compiled.

The table of Zimmermann shows that out of 1,000 invalids in the district of the German railway administration at age 40, 62·20 persons died in the year, whereas out of the same number of actives of that age 10·46 deaths occurred annually: and in the following table are furnished the results according to various enquirers:

Table D.

Age	Mortality Probability of Invalids			Probability of Death to Actives without a previous Declaration of Invalidity		
	According to Zimmermann (Pensioned Railway Employees)	According to Kaan (Austrian Miners)	According to Morgenbesser (Upper Silesian Miners)	According to Zimmermann (Railway Employees)	According to Kaan (Austrian Miners)	According to Morgenbesser (Upper Silesian Miners)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
25	·08310	·18807	·12655	·00708	·00586	·00679
30	·06560	·10339	·10959	·00690	·00702	·00775
35	·06390	·06559	·09490	·00807	·00844	·00882
40	·06220	·08663	·08328	·01046	·00912	·01206
45	·05300	·06624	·07244	·01204	·01124	·01648
50	·05100	·06115	·06958	·01572	·01341	·01952
55	·04850	·05693	·05911	·02011	·01755	·02058
60	·05120	·07052	·05523	·02642	·01767	·02556
65	·06290	·08340	·07324	·03813	·01871	·03785
70	·07800	·09773	·09637	·05046	·03130	·05874
75	·10680	·14157	·12290	·07897	·04127	·07250
80	·16260	·19328	...	...	...	...

[*Note.*—The decrease shown in the probability of mortality among the invalids up to an advanced age is striking, while among the actives there is throughout an increase from age 30: this points to the presumption that in other occupations the probability of "invalidity" mortality would show similar results. It is clear that, especially in cases of injury, persons only recently becoming invalids are less able to resist noxious influences than others of the same age who have been invalids for a longer period and have recovered: and at the higher ages the reduced mortality of invalids who have recovered over-balances the higher mortality of the newer invalids, while at the younger ages (where the majority of the invalids have only recently become so) the heavy mortality of these recent invalids results in a greater number of

deaths of all invalids at those ages. At the younger ages, too, the invalidity is chiefly due to accidents, and is therefore to be excluded, as before stated, from the calculations, so that on the whole the difference in the mortality of invalids may be disregarded.]

Bearing on the point, the average duration of life is shown in the following table:

TABLE E.

Age	Average Duration in Years — Of the Austrian Miners (according to Kaan): Of Activity	Of Invalidity	Expectation of Life in General	Expectation of Life of Persons already Invalid	Average Duration of Life of the Male German Population, according to the German Mortality Table
20	34·6	6·3	40·9	5·1	38·45
30	26·6	6·6	33·2	12·8	31·41
40	19·2	6·6	25·8	13·4	24·46
50	12·5	6·4	18·9	13·2	17·98
60	7·5	5·2	12·7	10·1	12·11
70	4·1	3·5	7·6	7·1	7·34
80	1·4	2·8	4·2	4·1	4·10

These results show that to obtain the contributions for "invalidity" insurance, the mortality of invalids should be distinguished from that of active persons.

The authors finally selected the "invalid mortality" table of Zimmermann for pensioned railway employees. But as the "invalid mortality" probabilities are throughout lower than those compiled hitherto for other occupations, and from age 69 are less than the values of the general German mortality table, Zimmermann's results from age 69 are abandoned, and the mortality probabilities of the German mortality table* are substituted. The modified results of Zimmermann's "invalidity mortality" are contained in column 3 of Table I in the Supplement.

The authors then seek, as a further basis, a table of mortality probabilities for "actives" who had not, previous to death, declared invalidity, to be employed in conjunction with the table of invalidity. (The "activity period" is the number showing how many of a certain total of actives of the same age remain in the several years of age in a condition of activity.)

It is assumed that on the whole, for old age and invalidity insurance, the same future grouping at each age of actives and

* *Vide* column 5 of Table I in the Supplement.

invalids will be found as in the total German population, and that consequently the mortality table of that population, accurately compiled by the Imperial Statistical Office, may be adopted as expressing the general death-rate of the persons expected to be insured. This table then is selected. Now, if a large number of persons be observed from birth, and no migration occurs, a continual decrease by death takes place till, at a certain age, the last survivor dies. As soon as the survivors enter upon the age in which exposure to the risk of invalidity occurs, they are divided into actives and invalids. At each subsequent year additional invalids increase the number already existing, till, finally, the last of the actives will enter upon invalidity. Actives and invalids combined for each year exhibit the general death period, while the actives taken alone constitute the activity period.

From the general German mortality table can be found, for a certain number of persons living, the most probable number of deaths to be annually anticipated, whether they occur in the period of activity or invalidity. And the number who may be expected annually to become invalids is to be calculated from the invalidity table (page 306), while the number of those invalids likely to die annually can be obtained from the following "invalidity mortality" table:

TABLE F.

Age-Classification	Invalid Workmen One Year under Observation	Expected Deaths	Number of Invalids who Died
(1)	(2)	(3)	(4)
20 to 29	10·08	·777	...
30 „ 39	61·	3·895	6
40 „ 49	83·64	4·547	9
50 „ 59	133·	6·580	9
60 „ 69	207·80	12·763	17
70 „ 79	69·28	7·711	7
80 „ 87	14·52	2·848	2
Totals . .	579·32	39·121	50

Thus there can be ascertained the number of persons living at each period of age, and the number of them who are invalids. If from the survivors at each age be deducted the number of existing invalids, the activity period is determined.*

Against the employment of the German Mortality Table it

* *Vide* Section (I) of the Mathematical Basis and Table I in the Supplement.

may be urged that the mortality probability among workmen is in excess of that of the population of the Empire in general, and that therefore the use of this table exhibits too low a mortality for actives than is contemplated in the Bill. This, it is held, is not the case:* and the following table is adduced in illustration, which furnishes the numbers of cases of invalidity and death among the active workmen in Table B, and those expected according to the newly-calculated activity period:

TABLE G.

Age-Category	Active Workmen One Year under Observation	Expected Cases of Invalidity and Death	Number of Workmen actually Withdrawn through Invalidity and Death
(1)	(2)	(3)	(4)
Under 30	18,416·3	164·99	114
30 to 39	20,563·5	252·28	218
40 „ 49	7,810·6	165·85	133
50 „ 59	2,345·4	112·57	110
60 „ 69	759·3	100·06	88
70 „ 85	86·	34·31	24
Total . .	49,981·1	830·06	687

According to the newly-calculated period, 830·06 actives were expected to be withdrawn in the course of the year—218·589 through invalidity (Table B)—while only 687 actives were withdrawn, among whom 206 cases were due to invalidity. The objection, therefore, against the appropriateness of the German Mortality Table is assumed to be cancelled.†

(III).—*The Age and Number of the Persons to be Insured.*

To ascertain the average contributions to be paid by the insured as a whole, and the settlement of the total financial charge for old age and invalidity, the proportions of age and the number of persons to be insured require to be known. With regard to the former, the "occupation" statistics of the 5th of June 1882 are sufficient: while for the number to be insured, in connexion also with the increase of population, an addition requires to be made to the results of those statistics. The following table contains a comparison of the number and decennial classes of age of the persons subject to compulsory insurance, as ascertained by the Imperial Statistical Office:

* *Confer* columns 5 to 7 of Table D with column 4 of Table I in the Supplement.

† *Vide* Table I in the Supplement.

TABLE H.

Occupation Division	Number of Persons in Employment liable to Insurance at the Age of								
	Under 15	15 to under 20	20 to under 30	30 to under 40	40 to under 50	50 to under 60	60 to under 70	70 and upwards	Total
				(*a*) Males					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
A	117,583	495,919	626,479	460,615	415,641	316,012	200,815	52,822	2,685,916
B	109,995	828,192	1,173,410	746,749	447,652	226,086	97,502	18,235	3,647,821
C	13,399	122,794	205,434	137,728	85,040	41,105	15,868	2,967	624,335
D	1,478	12,349	41,973	58,759	45,560	30,885	18,129	4,613	213,746
E	56	573	3,932	4,077	3,726	2,732	1,974	810	17,880
G	2,436	10,737	17,929	5,682	2,509	1,589	1,106	522	42,510
Total	244,947	1,470,564	2,069,157	1,413,610	1,000,128	618,439	335,394	79,969	7,232,208
				(*b*) Females					
A	55,051	377,179	389,019	157,194	157,193	136,619	83,079	20,403	1,375,737
B	28,634	203,160	189,695	60,090	35,910	19,296	8,479	2,204	547,498
C	2,652	39,088	60,571	19,561	12,077	7,920	4,213	1,205	147,287
D	3,470	18,536	29,293	28,421	36,494	36,539	24,514	6,569	183,836
E	144	2,496	8,584	6,381	4,567	2,839	1,384	360	26,755
G	61,248	475,589	534,679	97,093	50,121	35,857	21,455	6,372	1,282,414
Total	151,199	1,116,048	1,211,841	368,740	296,392	239,070	143,134	37,113	3,563,527

A comprises the occupations of agriculture and forestry, &c.
B ,, ,, ,, mining and building.
C ,, ,, ,, commerce and traffic, with hotel and public-house service.
D ,, ,, ,, domestic service and occupation in no special capacity.
E ,, ,, ,, civil and ecclesiastical service, &c.
G ,, ,, ,, servants living in the houses of their employers.

From these figures the annual age-classes of the insured have been calculated for every division of occupation.*

It is to be remembered, also, that from the total number of those expected to be insured, a deduction must be made for persons serving as soldiers, so that irregularities occur in the results during the years of military service. The ages of these persons, divided into yearly age-classes, were calculated on the basis of the levy of soldiers for the years 1880 to 1882, as follows:

TABLE I.

Age	At the end of 1882 the Number of Levied Soldiers was as follows, from the Years			Total
	1880	1881	1882	
(1)	(2)	(3)	(4)	(5)
20 to 21	...	...	65,153	65,153
21 ,, 22	...	63,023	37,159	100,182
22 ,, 23	64,824	37,603	37,841	140,268
23 ,, 24	37,711	38,749	1,812	78,272
24 ,, 25	36,504	1,613	...	38,117
25 ,, 26	1,502	...	...	1,502
Total	...	...	...	423,494

In the following table are shown at quinquennial stages the age-divisions of the persons existing at the commencement of insurance, which, however, require modification, since all soldiers become liable to insurance only after leaving the service:

* *Vide* Table I in the Supplement.

Table J.

Age	Male	Female
	Persons in Employment on the 5th of June 1882, liable to Insurance	
(1)	(2)	(3)
16	284,625	213,319
20	278,897	236,063
25	212,350	90,508
30	163,935	46,036
35	138,112	34,913
40	117,612	30,970
45	97,909	29,531
50	78,437	27,785
55	59,439	23,561
60	44,659	18,687
65	32,346	13,968
70	19,172	8,310
75	5,286	2,685
80	530	302
84	...	15
Sum for all Ages	6,775,303	3,257,068

Thus, the number liable to insurance on the 5th of June 1882 consisted of

6,775,303 males
3,257,068 females

or 10,032,371 persons.

If it is assumed that the number of persons liable to insurance will increase in the same proportion as the male and female population respectively of the German Empire above the age of 16 (from the end of 1880 to the end of 1885), there would be, in the middle of 1889,

7,092,633 males, and
3,439,100 females

liable to insurance above the limiting age.

To these must be added the persons who, as relations, are employed by self-dependent agriculturists for wages, and there must be deducted the persons who work for wages in no special capacity so far as they are not liable to this law. These elements are problematical. But a mistake will not be material if it be considered that for the calculation of the average contributions the relative age-combinations are only to be regarded, and not

the number of persons to be insured—the number only being necessary to show the amount of the total contributions. Making an adjustment for ascertaining the number with more approximate accuracy, the resulting figures for the middle of the year 1889 would be

	7,322,000	males
	3,696,000	females
or	11,018,000	persons

liable to compulsory insurance. This, then, is the number to be employed for obtaining the total charge, and the amount of the State contribution.

(IV).—*The Division, as to Locality, of the Empire according to the Annual Wages.*

In the calculation of the basis of each wage-category the number of such classes should be as few as possible, and each category should include whole towns, so that average results may be relied upon.

It is important, too, that the age-groupings ascertained for the whole population should be applied to these several categories. And as part of the uniformity of the scheme the wage-categories should be constructed according to local wages.

To effect a limitation of local classes the authors obtain the average amount of the wages paid throughout the Empire. By means of the number of population in the different localities the average customary local wages are ascertained for larger districts; the average wage-rate of each district is then multiplied by the number of persons liable to insurance in that district: the results of the different districts are summed, and the total figure ascertained for the Empire is divided by the number of persons subject to insurance throughout the Empire.

Taking as a basis the preceding figures of 7,322,000 males, with wages of £577,230, and 3,696,000 females, with wages of £189,521, there result the average wages of the Empire of

$$\frac{577,230}{7,322,000} = \text{about } 1s.\ 7d. \quad \text{for males,}$$

$$\frac{189,521}{3,696,000} = \text{about } 1s.\ 0\tfrac{1}{4}d. \quad \text{for females,}$$

or the average wages for females are nearly two-thirds of those for males. Thus, it is only necessary—not to form special classes

of wages for females, but—to assess their annuities at two-thirds of those for males.*

The division into classes of wages should be as limited as possible, so that the number of persons liable to insurance in each class shall be the greatest possible. This is secured without any great increase of the actual average wages of 1*s.* 7*d.* if the following locality-division be selected:

Locality-Class I,	with annual wages of £15,	all the localities with wages up to	1*s.*
" II,	" " £20,	all the localities with wages from upwards of	1*s.* 0*d.* to 1*s.* 4¾*d.*
" III,	" " £25,	" "	1*s.* 4¾*d.* " 1*s.* 9½*d.*
" IV,	" " £30,	" "	1*s.* 9½*d.* " 2*s.* 2½*d.*
" V,	" " £35,	all the localities with wages from upwards of	2*s.* 2½*d.*

The number of males in each locality-category liable to insurance, and their total annual wages, are—

			£
In Class I are to be insured	825,953	persons, with yearly wages of	12,389,295
" II " "	1,880,047	" " "	37,600,940
" III " "	2,928,366	" " "	73,209,650
" IV " "	1,077,884	" " "	32,336,520
" V " "	609,750	" " "	21,341,250
Totals . .	7,322,000	" " "	176,877,655

For this division into five classes the wages throughout the Empire of each male insured amount, on the average, to

$$£\,\frac{176{,}877{,}655}{300 \times 7{,}322{,}000} = 1s.\ 7\tfrac{1}{4}d.$$

This result closely agrees with the real average wages already ascertained for the Empire.

(It will be observed that in the bill only four classes were distinguished.)

For the total financial charge, therefore, it is of no consequence whether the annuities in each of the five classes of locality are based in equal proportion to the rate of wages therein, or whether for all the classes uniform annuities in proportion to the average annual wages of $300 \times 1s.\ 7\frac{1}{4}d. =$ £24. 3*s.* are granted. For calculating this charge it will, however, be necessary, in connexion with the fluctuations of wages of an upward tendency, to start from higher average wages than £24. 3*s.*, and, consequently, the standard amount of £25 is adopted as the average annual wages in ascertaining the complete cost.

* The sterling figures throughout this paper are only fractionally correct, in consequence of the conversion (at 1*s.* per mark) of decimals of marks. They are quite sufficient, however, for all purposes.

(V).—*Calculation of the Contributions.*

The authors recommend that as respects the "period of waiting"—wartezeit—before an invalidity allowance should vest, those who become invalids in the first year of membership should not receive any grant, while those who become chargeable in the second to the fifth year should receive one-half of the lowest amount of "invalidity" annuity. In respect of the calculation of "old age" annuities, no period of waiting has been taken into account.

The "invalidity" annuity for males is to commence with an annual payment of 24 per-cent of the annual normal wages. After the expiration of the "waiting period", the annuity is to increase with every complete calendar year by an additional portion of such wages, *i.e.*, in the next ensuing 15 calendar years, by 4 per 1,000 each: in the subsequent 20 calendar years, by 6 per 1,000 each: and thereafter by 8 per 1,000 each until it attains the maximum of 50 per-cent per annum of the annual wages.

The "old age" annuity for males amounts annually to 24 per-cent of the yearly wages: (this annuity is cancelled when an "invalidity" annuity is paid to the recipient). For females the rates, as already determined, are to be two-thirds of the preceding scales.

The management expenses are to be borne by employers and employed in equal shares. For such expenses the amount of about $8\frac{1}{2}d.$ is charged for each insured during the period of activity.

The contributions are assumed to accumulate at $3\frac{1}{2}$ per-cent compound interest annually.

The contracts can only be fulfilled if contributions from all the insured are paid regularly during the entire period of activity. Any loss of contributions, so far as they are not voluntarily replaced at a later stage, must therefore be compensated in some other mode. Since every contribution represents a part of the subsequent annuity payments, the latter cannot be realised if the former be not received.

The cessation of contributions may occur in consequence of (1) illness, (2) discontinuance of employment, and (3) service in the army or navy. As regards No. (1), the average number of days' illness for each insured at the different years of age, and the duration of the illness, can be ascertained. The most probable

value of the loss of contributions for each person during the period of activity through illness can thus be obtained, and the amount fixed by which the regular contributions of the healthy must be raised in order to meet the liability.

[Thus, to show how the amount of weekly contribution is to be fixed, if it is to compensate the losses due to illness:—a labourer of age 35 is supposed to contribute weekly, so long as he is capable of doing so, $\frac{£\cdot 05}{47}$.

His total contributions,* irrespective of illness, show a capital value of 15*s*. 3*d*.$-6\frac{1}{4}$*d*.$=$14*s*. $8\frac{3}{4}$*d*.

If contributions cease during illness the capital value of the deficiency is about 5*d*. Consequently the actual value of the future contributions at this age, after deduction of the probable losses, is 14*s*. $8\frac{3}{4}$*d*.$-$5*d*.$=$14*s*. $3\frac{3}{4}$*d*.

According to Tables V and VI of the Supplement, the actual value of the reversion to the annuity of £6–£12. 10*s*. of an insured of age 35—*i.e.*, the capital value of his future contributions—is $\cdot 87 \times £14\cdot 941 + £\cdot 40142 = £13\cdot 4$. His weekly contribution, therefore, will, if he pay also for the periods of illness, amount to $\frac{£13\cdot 4}{47 \times 14\cdot 7425} = 4\frac{3}{4}d.$; and, if the contributions during illness cease, to $\frac{£13\cdot 4}{47 \times 14\cdot 3203} =$ about 5*d*. Consequently, to avoid loss to the insurance institute, the regular weekly contribution must be increased from $4\frac{3}{4}$*d*. to 5*d*., or by $\frac{1}{4}$*d*.]

For the appropriate statistics as to illness, reference is made to Dr. Heym's published experience of the Insurance Society "Gegenseitigkeit" (Reciprocity) of Leipzig. (*Vide* Table VII in the Supplement.)

As respects cause No. (2)—want of employment—a different course is required. No adequate bases exist and may never be obtained. But in accordance with the bill, short durations of absence of work will produce practically no effect as regards the contributions, since the yearly contributions, instead of extending over $52\frac{5}{28}$ weeks, are to be discharged within 47 weeks, so that in every year five weeks may be passed without employment, and without entailing any falling-off in contributions. (The conclusion as to five weeks is derived from the experience of illness prevailing in the Benefit Society of German Printers. And, further, the contributions paid for upwards of 47 weeks afford a compensation.)

* See Column 6 of Table VIII in the Supplement.

But absence of employment for longer periods is important apart from strikes, idleness and cognate causes, and are to be traced to commercial crises, whose occurrence and duration cannot be predicted. Again, there is the case of a workman for wages becoming an employer, which is in general an act of free determination, and cannot be statistically determined. For these cases, where the lost contribution is not in some way recouped, the annuity must be diminished. To ascertain the assessment of diminution very different results would follow, if, *e.g.*, age 20 were considered in comparison with age 60. For the reversion of a person aged 20 to an annuity of £·05 from entry upon invalidity, and at the latest after age 70, amounts to 1*s.* $0\frac{1}{4}d.$, while at age 60 the corresponding amount is 1*s.* $0\frac{3}{4}d.$, so that the payment of £·05 at 20 entitles him to an annuity of $\frac{\cdot 0025}{\cdot 051} = 11\frac{3}{4}d.$, while the annuity of 60 is $\frac{\cdot 0025}{\cdot 203} = 3d.$ Hence a neglect to pay contributions entailing the deprivation of annuity produces a very different amount of loss in the two cases. A technical calculation is therefore impossible, and it will be sufficient to accumulate at interest the unpaid contributions up to the date of entering upon the benefit, and then reduce the latter by dividing the accumulated loss by the annuity-value at the age attained, *i.e.*, the capital value of the annuity of 1. The resulting amount will then be deducted from the "old age" and "invalidity" allowance respectively. The sum to be deducted from an annuity is ascertained by multiplying the accumulated loss by a tariff figure in the following table:

TABLE K.

Age	If the Entry on the Annuity begins at the given Age, each £·05 of the omitted Payment of Contributions, with Interest, represents for the Insurance Establishments in respect of the Annuity, the Amount of
(1)	(2)
	d.
20	about $1\frac{1}{2}$
30	„ $1\frac{1}{4}$
40	„ $1\frac{1}{4}$
50	„ $1\frac{1}{4}$
60	„ $1\frac{1}{2}$
70	„ 2
80	„ $3\frac{1}{4}$

As an example: an insured person has, in the course of the year 1890, omitted to pay contributions for 10 weeks of $2\frac{1}{2}d.$ each; and becomes invalided in the course of the year 1920, after 50 complete years. From ordinary compound interest tables can be extracted the loss for 10 weeks for 30 years (1890 to 1920). This will be $5s.\ 7\frac{1}{4}d.$ According to the preceding table, each £·05 of loss, with interest, represents, to a person aged 50, an annuity-value of $1\frac{1}{4}d.$, so that for $5s.\ 7\frac{1}{4}d.$ there is obtained the annuity of $\frac{\cdot 2805 \times \cdot 004845}{\cdot 05}$ = about $6\frac{1}{2}d.$, which could have been granted. Now, during the period of loss of work and, therefore, cessation of contributions, the State contribution will also cease, so that a deduction not of $6\frac{1}{2}d.$ only, but of $1{\cdot}5 \times 6\frac{1}{2}d. = 9\frac{3}{4}d.$, must be made.

With respect to cause No. (3),—military service in time of peace,—the same method may be pursued as that employed for cases of illness, as sufficiently precise materials exist, and the probable loss could be met by increasing the average contributions. For a period of mobilisation and war, another plan must be adopted: and losses must be treated in a way similar to that applied to cases of want of employment.

The case is also fully treated where the institute suffers through loss of interest: and this, generally, is to be provided by an additional tax to the regular contributions. The possible loss of interest, however, is considered to be comparatively insignificant.

(VI).—*The Calculations and their Results.*

In order to calculate the amount of the contributions it is necessary to ascertain the capital value at the beginning of insurance of the expected total contributions of each member on the one side, and the value of the reversion to the annuities on the other.

In column 6 of Table VIII in the Supplement is shown, under B_x, the most probable* capital value of the future weekly contributions at the date of entry upon insurance for each year of life. For example, B_x for age 40 gives a value of $47 \times 12s.\ 8d.$,† showing that it is immaterial (as regards the financial condition of the insurance institute), whether the workman of age 40 at entry pay at once the whole contribution of $47 \times 12s.\ 8d.$, or pay each calendar week a sum of £·05. The annuities of the institute are

* The phrase "most probable", employed here and elsewhere, is that used in the original.

† In the table 12·6576.

calculated separately. In Table V in the Supplement, column 9, ${}^{i}P_x$ furnishes the most probable capital value of the "invalidity" annuities (including the element of invalidity due to accidents) at the date of entry upon insurance. If these values are multiplied by the coefficient ·87, there is obtained the appropriate annuity function to be applied to the calculation in question. If the £16. 11*s*. 9¼*d*. (at age 40) be multiplied by ·87, giving a result of £14. 8*s*. 7½*d*., it is thereby shown that the labourer aged 40 acquires for the single payment of £14. 8*s*. 7½*d*. the right to an annuity of £6 to £12. 10*s*. a year from the commencement of invalidity. Similarly as to the "old age annuity, column 4 of Table VI in the Supplement—under ${}^{a}P_x$—furnishes the most probable capital value for each year of age of the calculated annuity at the date of entering upon insurance when the annuity amounts to £6. If in that case [Table VI, age 40] the figure 10*s*. 2½*d*. applies, it is thereby signified that the workman of that age is entitled, in consequence of a single payment of 10*s*. 2½*d*., to an annuity of £6 for the period of his activity subsequent to age 70.

To ascertain the equal average contribution for all age-categories, the values of B_x, ${}^{i}P_x$, and ${}^{a}P_x$, for all workmen to be insured, must be calculated.* The results appear in the Tables IX and X in the Supplement for males and females, namely:

FOR MALES.

			£
i. Total of all Values of			$B_x = 47 \times 4{,}860{,}242$
ii.	,,	,,	${}^{i}P_x = 92{,}003{,}061$
iii.	,,	,,	${}^{a}P_x = 5{,}067{,}907$

Consequently the present value of the future charge by existing male workmen at the commencement of insurance (when the annuity for invalids is £6 to £12. 10*s*., and for old age £6), including the charge for invalidity due to accidents, is £92,003,061 + 5,067,907 = £97,070,968.

With a weekly contribution of £·05, the present value of all the contributions is 47 × £4,860,242, so that the weekly contribution, b, equal for all age-categories, is obtained from the equation

$$b \, . \, 47 \times £4{,}860{,}242 = £92{,}003{,}561 + £5{,}067{,}907$$

$$\text{with } b = \frac{£92{,}003{,}061 \times \cdot 05}{47 \times 4{,}860{,}242} + \frac{£5{,}067{,}907 \times \cdot 05}{47 \times 4{,}860{,}242}$$

$$= 4\tfrac{3}{4}d. + \tfrac{1}{4}d. \text{ approximately.}$$

* See formula xxiv, page 340.

Now, deducting the portion of the annuity in respect of invalidity through accident, the weekly contribution required for old age and invalidity insurance, the same for all age-categories, is $4\frac{3}{4}d. \times \cdot 87 + \frac{1}{4}d. = 4\frac{1}{2}d.$ nearly: of which $4\frac{1}{4}d.$ appertains to the invalidity annuity, and $\frac{1}{4}d.$ to the annuity for old age. This rate of contribution applies provided the members entering in future are distributed over the different age-categories in the same proportion as at the present state of insurance.

The cases of elderly persons entering insurance (by obtaining employment at a late stage in life) are rare: those who enter will generally do so at the younger ages.

Bearing in mind the preceding facts, the difference in the value of the risk at different ages, the decision that no distinction should be introduced between the contributions of the first members (at the commencement of insurance) and of those entering at a later period, the excess of members from the increase of population, and the cases of discharged soldiers entering employments liable to insurance, it is finally seen that the equal weekly contribution for all males requisite for all time, b_m, is about $3d.$, of which $2\frac{3}{4}d.$ applies to the invalidity annuity, and $\frac{1}{4}d.$ to the annuity for old age. For females the corresponding figures are $1\frac{3}{4}d.$, a trifle under $1\frac{3}{4}d.$, and the difference. Expressed as a percentage of the annual wages the weekly contributions to be paid both by the employer and the employed are ·03355 per-cent for males, and ·01971 per-cent for females.

The preceding contributions do not include the costs of management. The extent to which these will reach is unknown. The experience of Government accident insurance shows a cost for administration in 1886 of about $8d.$ for each insured: and adopting $8\frac{1}{2}d.$ per head annually for this purpose, the weekly contribution with this addition becomes

$2\frac{1}{4}d.$, or ·03653 per-cent for males, and

$1\frac{1}{4}d.$, or ·02269 per-cent for females,

of the annual wages. [Thus: an average cost of $8\frac{1}{2}d.$ in relation to average annual wages of £25 is at the rate of ·140 per-cent; assuming 47 weeks per annum, there results ·00298 per week; and, adding this sum to the preceding amounts respectively, the figures just furnished are obtained.]

The following table expresses the contributions to be levied for each locality-category according to sex:

TABLE L.

Locality Classes	Weekly Contributions, including Management Expenses			
	Males		Females	
	Persons			
	Ascertained according to the Calculations	To be Paid according to the Bill	Ascertained according to the Calculations	To be Paid according to the Bill
(1)	(2)	(3)	(4)	(5)
	About			
	d.	*d.*	*d.*	*d.*
With Annual Wages of £15	$1\frac{1}{4}$	$1\frac{1}{2}$	$\frac{3}{4}$	1
,, 20	$1\frac{3}{4}$	2	1	$1\frac{1}{4}$
,, 25	$2\frac{1}{4}$	$2\frac{1}{2}$	$1\frac{1}{4}$	$1\frac{1}{2}$
,, 30	$2\frac{3}{4}$	3	$1\frac{3}{4}$	$1\frac{3}{4}$
,, 35	3	$3\frac{1}{4}$	2	2

Persons entering the insurance scheme at higher ages obviously obtain a considerable advantage from the plan of equal contribution for all years of life at the expense of the younger entrants: but, on the other hand, the young members are not prejudiced since, for their present contribution, they could not secure the advantages offered by the law from a private insurance company. The member aged 16, which is the most favourable illustration, in order to obtain the given annuity of £6 to £12. 10*s.*, would, in an insurance company, pay a weekly contribution of about $2\frac{1}{2}$*d.*, while he now secures it (through the financial intervention of the State and the employer) for a payment only, including the additional charge, of $1\frac{1}{4}$*d.* per week.

(VII).—*Estimate of the State Contributions.*

The State contribution to the financial objects of the law amounts to a third share of the total sums which, in each year, are actually paid in annuities. Supposing that the basis of the scheme should not be liable to any considerable change, assuming accumulation at $3\frac{1}{2}$ per-cent interest, and omitting the consideration of any possible increase in the insurance amount due to increase of population, it is estimated that if all the persons entering upon insurance are, for the whole period of their activity, in employment subject to insurance, and, assuming further, that the average wages remain constant at £25, the total State

contribution calculated from the commencement of insurance would amount, for the insurance of males, to one-third of £(162,057,399 + 6,842,071) = £56,299,823: and for females to the same proportion of £(96,928,597 + 3,462,702) = £33,463,766: or, together, to £89,763,589.

The average annual charge would therefore be, at the beginning of each year, $£89,763,590 \times \frac{\cdot 035}{1 \cdot 035} = £3,035,484$.

But many persons withdraw from an employment liable to insurance, and in these cases no further State contribution is payable. Adopting an assumption on this point (that future withdrawals occur in the proportion experienced in the past) and omitting for the moment the increase of the possible insurance amount due to augmented population, it has to be admitted, for determining the State contribution, that the insurance amount (with regard to the number and condition as to age of the persons for whom a State contribution is to be paid in the different years) would remain equal to the number of members existing at the beginning of insurance.

The number of insured for the middle of the year 1889 is computed at 7,322,000 males and 3,696,000 females, or 11,018,000 persons. If it be assumed that for each person, on the average, one week's illness occurs, and that the State does not participate in management expenses, the State contribution amounts annually, on the average, to

$$7,322,000\left(\frac{46 \times 3d. \text{ (about)}}{3}\right) + 3,696,000\left(\frac{46 \times 1\tfrac{3}{4}d. \text{ (about)}}{3}\right)$$
$$= £1,412,421 + £418,891 = £1,831,312.$$

[It is pointed out that the total charge devolving upon the State in respect of "old age and invalidity" insurance consists—in addition to the heavier expenditure entailed upon the State Insurance Office and Imperial Post Office, and the State contribution—of amounts to be expended upon (i) annuities for contributions which cease during military service, and (ii) weekly contributions for persons employed in State factories—the State thus being the employer.

An average estimate can be assigned under No. (i), as the number of soldiers liable to insurance by reason of their previous employment may be computed for the middle of the year 1889 at about 295,000. Hence the falling-off in contributions amounts to $46 . \left(2\tfrac{1}{2}d. + \frac{3d. \text{ (about)}}{3}\right) 295,000 = £192,592$ annually. In respect

of the period of military service of the reserves (during training in exercises), the yearly loss of contributions may be assessed at £10,500, so that in respect of military service in time of peace the average annual sum of £203,092 is found, for which compensation must be provided. The additional charge to the State, in consequence of cessation of contributions during military service in mobilisation and in time of war, is incapable of calculation from the absence of data.

In respect of the burden involved in case No. (ii), it is found from the statistical reports of the Imperial Insurance Office, that the number of persons employed in State factories and liable to compulsory insurance amounts (in the several classes of army, navy, post and telegraph services, and State railways) to 42,283. Adding to these the employees in State printing offices, and in other State departments, it is ascertained that the total number employed by the State, for whom the State, as employer, has to defray contributions, is 44,000. The annual charge to the State for the insurance of these persons amounts to $46 \times 1\frac{1}{4}d.$ (about) $\times 44{,}000 =$ £10,120. Combining the several figures and including the State contribution, the annual average State charge is ascertained to be £2,044,524. If certain other sections of persons specified in the law are also included in the scheme of compulsory insurance, the State contribution requires to be met in respect of 8,720,000 males, and 4,640,000 females, or a total of 13,360,000 persons.

The average annual charge entailed on the State for the combined insurance of old age and invalidity—exclusive of the estimated additional expenses of management above stated, and the annuities in respect of loss of contributions during military service in mobilisation and war—amounts to £2,421,200.]

The State has to discharge one-third of the amount actually payable in the form of annuities permitted under the law, and its contribution therefore in the first years of insurance will be small, while with an increasing number of invalids it will augment in such a mode that after (say) 80 years of insurance the amount of the total annual contribution falling upon the State in respect of each person in activity will exhibit thereafter no appreciable change.

To determine the annual amount of annuities, the number of persons becoming invalids in each year through other than accidents in work must be obtained: the number of such invalided persons who annually die, and the number who annually receive annuities. The "period of waiting" must also be admitted into the estimate. If the latter be adopted in the same manner as is

employed in the calculation of the weekly contributions, the State contribution might amount:

						£
i,	in the	1st	year of	insurance,	to about	191,500
ii,	„	2nd	„	„	„	242,500
iii,	„	3rd	„	„	„	338,000
iv,	„	4th	„	„	„	425,500
v,	„	5th	„	„	„	505,500
vi,	„	6th	„	„	„	630,500
vii,	„	7th	„	„	„	794,000
viii,	„	30th	„	„	„	2,637,500
ix,	„	80th	„	„	„	3,961,500

These figures correspond to an annual number of invalids of 115,762 (amongst whom are included 76,929 male invalids), of which, at the end of the year of invalidity, 111,449 (of whom 74,063 are males) are alive: at the end of the subsequent insurance year 102,742 invalids (including 68,277 males): and so on. After the 80th year of insurance the invalids will number 1,251,000 (including 831,510 males and 419,729 females). Accordingly, after the 80th insurance year there will be, out of every 1,000 active persons, 113·5 invalids who have fallen into this condition through other than accidents incurred in work.

(VIII).—*Estimate of the Capital Accumulations.*

The relatively small annuity charges upon the insurance institutes in the early years enable a reserve fund to be accumulated, and the interest yielded by this fund must ultimately effect a balance between the charges and receipts when, at a later stage, the annual annuities payable exceed the yearly contributions. To what extent, then, may this fund be expected to grow? For this object a trustworthy estimate is impossible: for it cannot be predicted how far the regular revisions of contributions as future necessities may require, will introduce a change, and no facts exist again as to possible alterations in the wage-rates of the several localities: nor to what degree those who retire from an employment will avail themselves of the right of retaining the insurance benefit by continuing to pay the full contributions. An approximation alone is feasible. Starting from the same point as that adopted for ascertaining the State contributions, the annuities payable by the institutes may be expected to amount

					£
i,	in the	1st	year of	insurance, to	383,000
ii,	„	2nd	„	„	485,000
iii,	„	3rd	„	„	676,000
iv,	„	4th	„	„	851,000
v,	„	5th	„	„	1,011,000
vi,	„	6th	„	„	1,261,000
vii,	„	7th	„	„	1,588,000
viii,	„	80th	„	„	7,923,000

The annual income from contributions—management expenses being deducted—from 11,018,000 persons may be estimated to amount to

£3,117,342 from males,
£893,508 „ females,

or a total sum of £4,010,850.

Hence, the capital, accumulated at 3½ per-cent compound interest, will probably amount

		£
i,	at the end of the 1st year of insurance, to about	3,691,500
ii,	„ „ 2nd „ „ „	7,408,000
iii,	„ „ 3rd „ „ „	11,060,500
iv,	„ „ 4th „ „ „	14,663,000
v,	„ „ 5th „ „ „	18,228,500
vi,	„ „ 6th „ „ „	21,664,500
vii,	„ „ 7th „ „ „	24,888,000
viii,	„ „ 80th „ „ „	115,688,000

And, possibly, at about the 17th year of insurance, the capital may attain to £50,000,000.

V.—Mathematical Basis of the Scheme.*

It seemed to me, also, that it would prove of interest to actuaries if I compiled from the official document which accompanied the introduction of the bill some of the mathematical methods adopted in the various elements forming the basis, and in the manipulation of the statistics of which I have furnished an abstract. These methods, probably, will not attract much attention here by reason of any novelty of analysis or expression, or on the ground of striking ingenuity, but I think their exposition will be useful on account of their constituting the basis of an attempt to found this imposing legislative scheme upon scientific elements and processes, more especially as an explanation of them has not previously been published in England.

This portion of the elaborate preparation for the law will show the ingenious and laborious mode in which, with meagre materials and the incorporation of wide hypotheses, the scientific advisers of the Government have proceeded, though I fear it will exhibit a very *doctrinaire* character.

I have introduced many details which have no very essential relation to the main scope of the law; but I have been guided in my decision by the consideration I have already expressed, that for the effective criticism of some future enquirer I may provide an arranged collection of materials which will spare him the toil and time of minute and difficult research.

* I have not, unfortunately, found it feasible to convert the system of notation into that adopted by the Institute of Actuaries.

(I).—*To ascertain the Period of Activity or Health.*

Let A_x=the actives of age x in the course of the $(x+1)$th year of life;

J_x=those who have become invalids;

S_x=those who die without a previous declaration of invalidity.

Then the probability of remaining active at age $(x+1)$, or a_x

$$=\frac{A_x-J_x-S_x}{A_x}=1-\frac{J_x}{A_x}-\frac{S_x}{A_x} \quad . \quad . \quad . \quad . \quad \text{(i)}$$

and $a_x.A_x=A_{x+1}$=the actives of age $(x+1)$,

$a_{x+1}.A_{x+1}=A_{x+2}$=the actives of age $(x+2)$; and so on.

As the statistics from which the values of these functions would be derived do not throw any light upon the number of deaths, the following plan is pursued in order to obtain the period of activity:

Let x_0=the age at which invalidity begins;

A_{x_0}=the number of actives at that age;

i_{x_0}=the probability of becoming invalid in the course of the (x_0+1)th year of life;

${}^is_{x_0}$=the probability that an invalid will die in the course of the (x_0+1)th year of life.

Then, in the course of the (x_0+1)th year of life, $A_{x_0}.i_{x_0}$ persons will become invalids.

And of these $A_{x_0}.i_{x_0}\left(1-\frac{{}^is_{x_0}}{2}\right)$ persons will attain the age of (x_0+1).

If P_{x_0+1} be the number of these surviving invalids, and A_{x_0+1} the number of actives living at that age, the probability of dying in the course of the (x_0+1)th year of life, or

$$s_{x_0}=\frac{A_{x_0}-(A_{x_0+1}+P_{x_0+1})}{A_{x_0}}$$

consequently $1-s_{x_0}=\dfrac{A_{x_0+1}+P_{x_0+1}}{A_{x_0}}$

$$\therefore \quad A_{x_0+1}+P_{x_0+1}=A_{x_0}(1-s_{x_0}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

$A_{x_0+2}+P_{x_0+2}=(A_{x_0+1}+P_{x_0+1})(1-s_{x_0+1})$, and so on:

and, consequently,

$A_{x_0+1}=A_{x_0}(1-s_{x_0})-P_{x_0+1}=$ the actives of age (x_0+1),

$A_{x_0+2}=(A_{x_0+1}+P_{x_0+1})(1-s_{x_0+1})-P_{x_0+2}=$ the actives of age (x_0+2), and so on.

For the values s_{x_0}, s_{x_0+1}, &c., the facts of the German Mortality Table, published by the Imperial Statistical Office, are employed. The survivors in this table for the years of life x_0, x_0+1, &c., are L_{x_0}, L_{x_0+1}, &c., and, if $A_{x_0}=L_{x_0}$ be inserted, that function becomes $A_{x_0}(1-s_{x_0})=L_{x_0+1}$, &c.

For the calculation of the period of activity, there are $A_{x_0}=L_{x_0}$, $A_{x_0+1}=L_{x_0+1}-P_{x_0+1}$, &c., by which the values P_{x_0+1}, P_{x_0+2}, &c., can be obtained from the equations:

$$P_{x_0+1}=A_{x_0}\,.\,i_{x_0}\left(1-\frac{{}^is_{x_0}}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(iii)}$$

$$P_{x_0+2}=P_{x_0+1}(1-{}^is_{x_0+1})+A_{x_0+1}\,.\,i_{x_0+1}\left(1-\frac{{}^is_{x_0+1}}{2}\right); \text{ and so on.}$$

[From this formula it follows generally that

$$i_x=\frac{P_{x+1}-P_x(1-{}^is_x)}{A_x\left(1-\frac{{}^is_x}{2}\right)}\qquad]$$

The unknown quantities ${}^is_x$ and i_x are extracted from the "Invalid-Mortality" Table of Zimmermann, and those for workmen in different occupations from the Invalidity Table of Behm.

[As the latter table has not been published, but simply exists in manuscript, the following statements are furnished as an indication of the statistical material which forms its basis. For the average amount of "workmen" invalidity, Behm employed the observations of the Invalid Pension Societies. The figures referred to (taking only intervals of 10) are $A_{20}=452$: $A_{30}=2{,}511$: $A_{40}=3{,}181$: $A_{50}=2{,}293$: $A_{60}=1{,}062$: $A_{70}=208$: while $J_{20}=0$: $J_{30}=0$: $J_{40}=13$: $J_{50}=22$: $J_{60}=50$: and $J_{70}=48$. With regard to the course of the invalidity-probabilities, Behm assumes that the invalidity among workmen will not in general prove different from that of railway employees who are not engaged in the actual train service. This assumption has been confirmed by the investigations of Zimmermann. The indication—that for such employees the invalidity-probability between the years 25–60 is doubled every five years—was, after examination, applied by Behm

to labourers in general. The general expression for the invalidity-probability at age x is thus taken by Behm:

$$i_x = i_{20} \,.\, 2^{\frac{x-20}{5}} \,.$$

If for age x the number of observed invalidity cases is J_x, and if A_x represent the number of actives, the calculated number of invalidity cases is equal to

$$A_x \,.\, i_{20} \,.\, 2^{\frac{x-20}{5}} \,;$$

consequently the divergence between calculated results and observed facts is

$$A_x \,.\, i_{20} \,.\, 2^{\frac{x-20}{5}} - J_x \,.$$

The expression $-\Sigma(A_x \,.\, i_{20} \,.\, 2^{\frac{x-20}{5}} - J_x)^2$—must therefore be rendered a minimum: or, in the present case, by the differential calculus,

$$i_{20} = \frac{A_{20} \,.\, J_{20} + 2A_{25} \,.\, J_{25} + \ldots\ 2^{12} \,.\, A_{80} \,.\, J_{80}}{A^2_{20} + 2^2 A^2_{25} + \ldots\ 2^{24} A^2_{80}}$$

must be employed.

Inserting the numerical values there results

$$i_{20} = \cdot 00019036 \text{: and}$$

$$\therefore\ i_x = \cdot 00019036 \,.\, 2^{\frac{x-20}{5}}\ ].$$

In Table I, in the Supplement, the given figures, as well as those which have been newly deduced as a basis for the calculations, are summarily furnished.

The difference which probably exists between the mortality probabilities of invalids immediately after becoming invalid and those appertaining to their later years of life, when the surviving invalids have become again restored, cannot be determined as the requisite statistical observations do not exist.

(II).—*The Calculation of the Value of the Reversion to Annuities.*

By the expression "Reversion to invalidity-annuity" is intended the most probable value of the whole of the annuities which the insurance institutes will be liable to pay to actives from the date when they become invalid, and during the remaining duration of their lives.

To estimate its value it is first necessary to ascertain the capital value of the annuities for invalids.

Let R_x = the value of the expected annuity of amount 1 for an invalid aged x,

and q = the factor of discount.

Then if $A'_x = \frac{A_x}{q^x}$ represents the discounted number of the living from age x of the death-period for invalids, and $x+y$ the highest age of that period,

$$^iR_x = \frac{\sum_{h=0}^{h=y} A'_{x+h}}{A'_x} = 1 + \frac{\sum_{h=0}^{h=y} A'_{x+1+h}}{A'_x},$$

whence it follows that

$$^iR_x = 1 + {}^iR_{x+1} \cdot \frac{1 - {}^is_x}{q} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(iv)}$$

This formula renders it possible to calculate the invalidity-annuities without discounted values by commencing at the highest age of the death-category, for which, as

$1 - {}^is_{x+y} = 0$, there results

$^iR_{x+y} = 1$.

The annuity, payable monthly in advance of annual amount 1, if indicated by $^iR_x^{\frac{1}{12}}$, becomes, with the element of interest,

$$^iR_x^{\frac{1}{12}} = (a + q\beta) \cdot {}^iR_x - q\beta$$

in which

$$a = \frac{1}{144}\left(12 + \frac{11}{q^{\frac{1}{12}}} + \frac{10}{q^{\frac{2}{12}}} + \ldots \frac{1}{q^{\frac{11}{12}}}\right)$$

$$\beta = \frac{1}{144}\left(\frac{1}{q^{\frac{1}{12}}} + \frac{2}{q^{\frac{2}{12}}} + \ldots \frac{11}{q^{\frac{11}{12}}}\right).$$

For $q = 1{\cdot}035$ ($3\frac{1}{2}$ per-cent interest), there may be approximately employed as the value

$$^iR^{\frac{1}{12}} = R_x - {\cdot}4630 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(v)}$$

The numerical calculations of the values ^iR_x, $^iR_x^{\frac{1}{12}}$ are contained in Table II in the Supplement.

It is necessary still, for determining the value of the reversion, to ascertain the proportion of a given number of actives of an equal age who gradually become invalids, and the amount of annuities entailed by the invalids during each year.

Out of A_x actives there become invalids in the course of the $(x+1)$th year of life—first year of insurance—$A_x \cdot i_x = J_x$ persons, of whom $J_x\left(1 - \frac{^is_x}{2}\right)$ are surviving at the end of the year.

If the annual invalidity-annuity be provisionally taken at unity, the present value (at the commencement of insurance) of the annuities to be paid to invalids of the 1st year of insurance, at the end of the year, amounts to

$$J_x\left(1-\frac{{}^{i}s_x}{2}\right)\cdot\frac{{}^{i}R_{x+1}^{(12)}}{q} \quad . \; . \; . \; . \; . \; . \; . \quad \text{(vi)}$$

To this must be added the annuities which have already fallen due in the course of the year. Approximate accuracy will be attained if it be supposed that in the middle of each year $\frac{J_x}{12}$ persons become invalids, of whom $\frac{J_x}{12}\left(1-\frac{1}{24}\cdot{}^{i}s_x\right)$ are still surviving at the end of the month. Accordingly, there are payable in each month $\frac{J_x}{12}$ half-monthly annuities: and, further, at the beginning of the second month, $\frac{J_x}{12}\left(1-\frac{1}{24}\cdot{}^{i}s_x\right)$ full monthly annuities: at the commencement of the third month

$$\frac{J_x}{12}\left(1-\frac{3}{24}\cdot{}^{i}s_x\right)+\frac{J_x}{12}\left(1-\frac{1}{24}\cdot{}^{i}s_x\right)$$
$$=\frac{J_x}{12}\left(2-\frac{4}{24}\cdot{}^{i}s_x\right)$$

complete months' annuities: and so on.

The pension-rates already due in the year of invalidity accordingly amount, at the beginning of the year, to

$$\frac{1}{24}\cdot\frac{J_x}{12}\left(\frac{1}{q^{\frac{1}{24}}}+\frac{1}{q^{\frac{3}{24}}}+\frac{1}{q^{\frac{5}{24}}}+\ldots+\frac{1}{q^{\frac{23}{24}}}\right)$$
$$+\frac{1}{12}\cdot\frac{J_x}{12}\left(\frac{1-\frac{{}^{i}s_x}{24}}{q^{\frac{1}{12}}}+\frac{2-\frac{4}{24}\cdot{}^{i}s_x}{q^{\frac{2}{12}}}+\ldots+\frac{11-\frac{12}{24}\cdot{}^{i}s_x}{q^{\frac{11}{12}}}\right)$$
$$=J_x(\cdot4893-{}^{i}s_x.\cdot1428) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(vii)}$$

So that the present value of all annuities to invalids amounts, in the first year, to

$$J_x\left(\cdot4893-{}^{i}s_x.\cdot1428+\left(1-\frac{{}^{i}s_x}{2}\right)\cdot\frac{{}^{i}R_{x+1}^{(12)}}{q}\right. \quad . \; . \quad \text{(viii)}$$

If for abbreviation the expression within brackets be designated by C_x, the present value of the charge entailed for the payment of annuities to invalids in the second year amounts to $\frac{J_{x+1}.C_{x+1}}{q}$: and so on.

The values summed to the highest age, $x+z$, show the present value of the payments to all invalids arising out of A_x actives in the several years of insurance.

The reversion to the annuity for each active, aged x, accordingly possesses the value of

$$\frac{1}{A_x}\left(J_x . C_x + \frac{J_{x+1} . C_{x+1}}{q} + \ldots + \frac{J_{x+z} . C_{x+z}}{q^z}\right)$$

$$= \frac{\sum_{h=0}^{h=z} \frac{J_{x+h} . C_{x+h}}{q^{x+h}}}{\frac{A_x}{q^x}} \quad : \text{ for which may be put } \frac{Z'_x}{A'_x} \quad . \quad . \quad \text{(ix)}$$

The numerical application of the formulas viii and ix is shown in Tables III and IV in the Supplement. And the values of C_x are contained in column 8 of Table III, while the values of $\sum \frac{J_{x+h} . C_{x+h}}{q^{x+h}} = Z'_x$ are inserted in column 3 of Table IV.

For the calculations in question, the probability of invalidity during the period of work must be omitted, since invalidity through accident occurring in employment has been otherwise provided for. If this probability be indicated by u_x, $i_x - u_x$ must be substituted in the calculations for i_x.

The quantity u_x is difficult to determine: some consider that u_x is independent of age: recent enquiries have shown, however, that it increases with the age, but the statistical material available is insufficient to yield its value with exactitude. It is probably close enough to truth to assume that u_x increases in the same proportion as i_x, namely, $u_x = k . i_x$. Then for formula ix the expression is obtained

$$\frac{\sum_{h=0}^{h=z} A'_{x+h} . i_{x+h} . (1-k) C_{x+h}}{A'_x} = (1-k) \cdot \frac{Z'_x}{A'_x} \quad . \quad . \quad \text{(x)}$$

The "accident-invalidity" probability can so be dealt with that the numerical calculations are conducted with the value i_x, and the final result be multiplied by the coefficient $c = 1 - k$. Formula x applies where the right to an annuity in invalidity is not dependent upon a "period of waiting."

According to the law the full invalidity-annuity is only payable where the contributions have been received during at least five years. If invalidity occur prior to this period, only one-half of the annuity is claimable, and on condition, further, that

contributions have been paid for one year at least. No annuity, therefore, is payable where invalidity occurs during the first year of membership: and half the annuity applies, as stated, in respect of invalids of the second to the fifth year of membership.

It follows from formulas ix and x that the present value of the reversion to an invalidity-annuity to an active aged x is

$$\frac{c}{A'_x}\left(\frac{Z'_{x+1}-Z'_{x+5}}{2}+Z'_{x+5}\right)=\frac{c}{A'_x}\left(\frac{Z'_{x+1}+Z'_{x+5}}{2}\right) \quad . \quad . \quad \text{(xi)}$$

The annual invalidity annuity to males is to be equivalent to $\frac{24}{100}$ths of the yearly wages, and after the termination of the "period of waiting" an annual increment is to count for the next 15 calendar years of $\frac{4}{1000}$ths; $\frac{6}{1000}$ths in the ensuing 20 calendar years: and, subsequently, $\frac{8}{1000}$ths annually until the maximum is attained of $\frac{50}{100}$ths of the yearly wages.

If, in place of these fractional amounts, the absolute values of the annuities for the average yearly wages throughout the Empire of £25 be taken into account, so that the annual invalidity-annuity amounts to £6 to £12. 10*s*., and the yearly increments to 2*s*., 3*s*., and 4*s*. respectively, the effect of the increments upon the value of the reversion may be thus defined: by formula xi the reversion to an invalidity-annuity of 1 is $\frac{c}{A'_x}\left(\frac{Z'_{x+1}+Z'_{x+5}}{2}\right)$: if the invalidity-annuity be £6 the value of the insurance obviously becomes

$$6\,.\,\frac{c}{A'_x}\left(\frac{Z'_{x+1}+Z'_{x+5}}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(xii)}$$

After the expiration of six years, *i.e.*, on completion of age $x+6$, the increase in the value of the annuity commences in the mode specified, so that at age $x+50$ the maximum annuity (£12. 10*s*.) is reached. The value of formula xii must therefore be increased by

$$\left\{\begin{matrix} \frac{\cdot 1c}{A'_x}\cdot Z'_{x+6}, & \frac{\cdot 1c}{A'_x}\cdot Z'_{x+7}, & \ldots & \frac{\cdot 1c}{A'_x}\cdot Z'_{x+20} \\ \frac{\cdot 15c}{A'_x}\cdot Z'_{x+21}, & \frac{\cdot 15c}{A'_x}\cdot Z'_{x+22}, & \ldots & \frac{\cdot 15c}{A'_x}\cdot Z'_{x+40} \\ \frac{\cdot 2c}{A'_x}\cdot Z'_{x+41}, & \frac{\cdot 2c}{A'_x}\cdot Z'_{x+42}, & \ldots & \frac{\cdot 2c}{A'_x}\cdot Z'_{x+50} \end{matrix}\right\} \quad . \quad . \quad \text{(xiii)}$$

It appears, therefore, that in respect of the actives aged x the present value of the reversion to an invalidity-annuity of £6 to £12. 10*s*. annually—symbolized by $c\,.\,{}^{i}P_x$—becomes

$$c.{}^{i}\mathrm{P}_x = \frac{c}{\mathrm{A}'_x}\Big[6.\frac{\mathrm{Z}'_{x+1}+\mathrm{Z}'_{x+5}}{2} + \cdot 1(\mathrm{Z}'_{x+6}+\mathrm{Z}'_{x+7}+\ldots+\mathrm{Z}'_{x+20})$$

$$+\cdot 15(\mathrm{Z}'_{x+21}+\ldots+\mathrm{Z}'_{x+40})+\cdot 2(\mathrm{Z}'_{x+41}+\ldots+\mathrm{Z}'_{x+50})\Big]$$

$$= \frac{c}{\mathrm{A}'_x}\Big[3(\mathrm{Z}'_{x+1}+\mathrm{Z}'_{x+5})+\cdot 1\sum_{h=6}^{h=z}\mathrm{Z}'_{x+h}+\cdot 05\sum_{h=21}^{h=z}\mathrm{Z}'_{x+h}$$

$$+\cdot 05\sum_{h=41}^{h=z}\mathrm{Z}'_{x+h}-\cdot 2\sum_{h=51}^{h=z}\mathrm{Z}'_{x+h}\Big] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(xiv)}$$

The numerical application of this formula is shown in Table V in the Supplement. And for females, $c.{}^{i}\mathrm{P}_x$ is to be multiplied by $\frac{2}{3}$.

(III).—*Reversion to the Old Age Annuity.*

The annual annuity is based upon $\frac{24}{100}$ths of the yearly wages on the average throughout the Empire, and consequently to the extent of £6 to males and $\frac{2}{3}$rds thereof to females who complete the 70th year and remain active, and to continue during the remainder of their lives. The old age annuity is consequently a deferred activity-annuity. Placing the annual amount of the annuity at unity, A_x actives entail an annuity charge of A_{70} at age 70, A_{71} at age 71, and A_{x+z} at age $x+z$. The present value of this charge amounts to

$$£\left(\frac{\mathrm{A}_{70}}{q^{70-x}}+\frac{\mathrm{A}_{71}}{q^{71-x}}+\ldots+\frac{\mathrm{A}_{x+z}}{q^{z}}\right)$$

and the value for each insured becomes

$$\frac{1}{\mathrm{A}_x}\left(\frac{\mathrm{A}_{70}}{q^{70-x}}+\frac{\mathrm{A}_{71}}{q^{71-x}}+\ldots+\frac{\mathrm{A}_{x+z}}{q^{z}}\right)={}^{a}\mathrm{R}_{70}\cdot\frac{\mathrm{A}_{70}}{\mathrm{A}_x\cdot q^{70-z}}$$

$$={}^{a}\mathrm{R}_{70}\cdot\frac{\mathrm{A}'_{70}}{\mathrm{A}'_x}\ \text{(multiplied into the annuity)},$$

in which ${}^{a}\mathrm{R}_{70}$ represents the activity-annuity in advance of 1 annually at age 70. In respect of monthly instalments in advance, for ${}^{a}\mathrm{R}_{70}$, the annuity-value is to be substituted of ${}^{a}\mathrm{R}_{70}{}^{\frac{1}{12}}={}^{a}\mathrm{R}_{70}-\cdot 463$. (Vide formula v.)

The annuity being £6, the value of the reversion to the old age allowance for the years of age x to 69 in respect of males is

$$£6.{}^{a}\mathrm{R}_{70}{}^{\frac{1}{12}}\cdot\frac{\mathrm{A}'_{70}}{\mathrm{A}'_x}:\ \text{for higher ages, } £6.{}^{a}\mathrm{R}_x{}^{\frac{1}{12}} \quad . \quad . \quad \text{(xv)}$$

For females the proportion is two-thirds. The numerical results of these formulas are furnished in Table VI under the symbol ${}^{a}\mathrm{P}_x$.

The "period of waiting" does not affect the calculations, since (section x, page 300) the insured who, at the date of the law had completed their 40th year, are at once entitled to old age annuities if they have been continuously in one employment during the preceding three calendar years for at least 47 weeks per annum, or have been barred from work for the specified number of weeks through illness. The extent, however, to which the non-fulfilment of this condition will prevail cannot yet be ascertained. For the sake of safety, let it be assumed that *all* the insured who, after the adoption of the law, have completed their 70th year, will fulfil it and remain active, and receive an annuity for old age. The addition, then, of the expressions xiv and xv will indicate the present value of the reversion of actives aged x to invalidity and old age annuities. Representing this by Ξ there is obtained

$$\Xi_x = c \,.\, {}^i\mathrm{P}_x + {}^a\mathrm{P}_x \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(xvi)}$$

(IV).—*Calculation of the Contributions.*

Equal contributions are to be levied from employers and employed for all age-categories for each calendar week, until invalidity occurs, during which the insured has been engaged in an employment subject to compulsion to the provisions of the law. Designating the annuity by 1, and A'_x as the discounted number of actives of age x and $x+z$ (the highest age of the activity-category), the most probable capital value of the annuity for actives aged x is

$$\frac{\sum\limits_{h=0}^{h=z} \mathrm{A}'_{x+h}}{\mathrm{A}'_x} \cdot (a + q \,.\, \beta) - \left(q \,.\, \beta + \frac{1}{52}\right) \quad . \quad . \quad . \qquad \text{(xvii)}$$

in which

$$a = \frac{1}{52^2}\left(52 + \frac{51}{q^{\frac{1}{52}}} + \frac{50}{q^{\frac{2}{52}}} + \ldots \frac{1}{q^{\frac{1}{52}}}\right)$$

and

$$\beta = \frac{1}{52^2}\left(\frac{1}{q^{\frac{1}{52}}} + \frac{2}{q^{\frac{2}{52}}} + \ldots \frac{51}{q^{\frac{1}{52}}}\right)$$

For $q = 1{\cdot}035$, the approximate value for the expression is obtained of

$${}^a\mathrm{R}_x = {\cdot}5154 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(xviii)}$$

Now the insured is entitled to the full annuity on completion of contributions for 47 weeks per calendar year, so that the annual contribution is to be collected for 47 terms. If the contribution

for each term be 1, the annual contribution $= 47$, so that from expression xviii the value of the total contributions of those aged x is

$$47\,({}^{a}\mathrm{R}_x - \cdot 5154) \quad . \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{xix})$$

And as the contributions are not to be required during the period of certified illness, the value of formula xix is to be modified in respect of these probable cessations.

This modification can be deduced from the statistics published by Dr. E. Heym, of Leipzig, in his table of the "Number and Duration of Illnesses among a Mixed Population", issued in 1884. Some results, derived from these materials, are contained in Table VII in the Supplement, showing the average number of days' illness to be expected for each of the insured in the several years of age. From these there can be calculated for each insured the most probable capital value lost in consequence of illness throughout the entire insurance.

If an insured at age x is unfit for work for K_x days, there will be lost, in respect of a weekly contribution of 1, an amount of contributions, in the different years of age, of

$$\frac{1}{7}\cdot \mathrm{K}_x, \quad \frac{1}{7}\cdot \mathrm{K}_{x+1}, \; . \; . \; . \; \frac{1}{7}\,\mathrm{K}_{x+z}.$$

Assuming that the annual losses through illness are equally distributed over the whole year, the insurance institutes will suffer in contributions by each person aged x

$$\frac{1}{7\,.\,q^{\frac{1}{2}}\,.\,\mathrm{A}_x}\left(\mathrm{A}_x\,.\,\mathrm{K}_x + \frac{\mathrm{A}_{x+1}\,.\,\mathrm{K}_{x+1}}{q} + \; . \; . \; . \; \frac{\mathrm{A}_{x+z}\,.\,\mathrm{K}_{x+z}}{q^z}\right)$$

$$= \frac{1}{7\,.\,q^{\frac{1}{2}}}\,.\,\frac{\sum\limits_{h=0}^{h=z}\mathrm{A}'_{x+h}\,.\,\mathrm{K}_{x+h}}{\mathrm{A}'_x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{xx})$$

If the expression xix be reduced by this value, there is obtained the annuity-value necessary for the calculation of the weekly contributions. Designating it by B_x, there results

$$\mathrm{B}_x = 47\,({}^{a}\mathrm{R}_x - \cdot 5154) - \frac{1}{7\,.\,q^{\frac{1}{2}}}\,.\,\frac{\sum\limits_{h=0}^{h=z}\mathrm{A}'_{x+h}\,.\,\mathrm{K}_{x+h}}{\mathrm{A}'_x} \quad . \quad . \quad . \quad (\mathrm{xxi})$$

The numerical calculations by this formula are contained in Table VIII in the Supplement, and the values of B_x are furnished in column 6.

Formula xxi also expresses the capital value of contributions if, under these suppositions, each insured pay 1 weekly.

If b_m represent the weekly contribution equal for all age-categories required for the provision of old age and invalidity insurance, the present value of all the contributions from actives aged x is, according to formula xxi, $b_m . B_x$. If N be the total number of persons to be insured, of which n_x belong to age x, so that $N = n_x + n_{x+1} + \ldots n_{x+z}$, then the present value of all the contributions which the institutes are expected to receive amounts to

$$b_m(n_x . B_x + n_{x+1} . B_{x+1} + \ldots + n_{x+z} . B_{x+z})$$

$$= b_m . \sum_{h=0}^{h=z} n_{x+h} . B_{x+h} \quad \ldots \ldots \ldots \quad \text{(xxii)}$$

whereas, according to formula xvi, the value of all future payments which the institutes have to provide for N persons amounts to

$$n_x . \mathbb{K}_x + n_{x+1} . \mathbb{K}_{x+1} + \ldots n_{x+z} . \mathbb{K}_{x+z}$$

$$= \sum_{h=0}^{h=z} n_{x+h} . \mathbb{K}_{x+h} \quad \ldots \ldots \ldots \quad \text{(xxiii)}$$

Consequently the weekly contribution, b_m, equal for all age-categories, for the present insurance amount, is obtained from the equation

$$b_m . \sum_{h=0}^{h=z} n_{x+h} . B_{x+h} = \sum_{h=0}^{h=z} n_{x+h} . \mathbb{K}_{x+h},$$

namely,

$$b_m = \frac{\sum_{h=0}^{h=z} n_{x+h} . \mathbb{K}_{x+h}}{\sum_{h=0}^{h=z} n_{x+h} . B_{x+h}} \quad \ldots \ldots \quad \text{(xxiv)}$$

The numerical values of formulas xxii and xxiii are given in Table IX in the Supplement, and for females in Table X.

The average contribution, b_m, applies on the supposition that the workmen entering upon insurance in future will be distributed over the several years of age in the same proportion as the present insurance amount. It cannot be assumed, as regards obligatory invalidity and old age insurance, that future entrants, as a rule, will belong to the higher ages. On the contrary, insurance will almost exclusively take place at the younger ages. Hence, for the later years, a considerably smaller contribution than that calculated in b_m will be sufficient to meet the charges for annuities.

This fact must be taken into account in fixing the equal payments, since no distinction can be made between the *first* entrants (at the commencement of insurance) and later members in respect of contributions.

Out of the existing workmen, too, at the commencement of insurance, a great number of persons are in military service: many of whom become liable to the provisions of the law on obtaining their discharge and resuming employment. The increase on this account must be allowed for. With reference to the first class of increase of members there cannot exist any objection against the assumption, if the continual increase of the population be considered, that in each subsequent insurance year the numbers of the insured at the ages 16, 17, and 18 will not change. There will be admitted each year into the scheme the new members n_{16} at the age of 16 complete years, $n_{17}-n_{16}.a_{16}$ at the age of 17, and $n_{18}-n_{17}.a_{17}$ at age 18. Hence the present value of the receipts is augmented by the sum of

$$b_m\{n_{16}.B_{16}+(n_{17}-n_{16}.a_{16})B_{17}+(n_{18}-n_{17}.a_{17})B_{18}\}\frac{\sqrt{q}}{q-1} \quad . \quad \text{(xxv)}$$

and the value of the payments by the institutes raised by the sum of

$$\{n_{16}.K_{16}+(n_{17}-n_{16}.a_{16})K_{17}+(n_{18}-n_{17}.a_{17})K_{18}\}\frac{\sqrt{q}}{q-1} \quad . \quad \text{(xxvi)}$$

Now, with regard to the increase of the insured derived from the military service, it may be assumed that at the end of the first three years of insurance the following numbers are discharged from the army, and return to work entailing upon them compulsory insurance, namely, m_{23} at age 23, m_{24} at 24, m_{25} at 25, m_{26} at age 26.

The augmentation of the present value of the receipts produced thereby amounts to

$$b_m(m_{23}.B_{23}+m_{24}.B_{24}+m_{25}.B_{25}+m_{26}.B_{26})\left(\frac{1}{q}+\frac{1}{q^2}+\frac{1}{q^3}\right) \quad . \quad \text{(xxvii)}$$

and the increase of the payments to

$$(m_{23}.K_{23}+m_{24}.K_{24}+m_{25}.K_{25}+m_{26}.K_{26})\left(\frac{1}{q}+\frac{1}{q^2}+\frac{1}{q^3}\right) \quad . \quad \text{(xxviii)}$$

By combining the values xxii, xxiii, and xxv to xxviii, there is obtained the equal average weekly contribution permanently necessary for all age-categories to meet the invalidity and old age insurance,

$$b_m=\frac{\sum\limits_{h=0}^{h=z}.n_{x+h}.K_{x+h}+(n_{16}.K_{16}+\ldots)\frac{\sqrt{q}}{q-1}+(m_{23}.K_{23}+\ldots)\left(\frac{1}{q}+\frac{1}{q^2}+\frac{1}{q^3}\right)}{\sum\limits_{h=0}^{h=z}.n_{x+h}.B_{x+h}+(n_{16}.B_{16}+\ldots)\frac{\sqrt{q}}{q-1}+(m_{23}.B_{23}+\ldots)\left(\frac{1}{q}+\frac{1}{q^2}+\frac{1}{q^3}\right)}$$

. . . . (xxix)

(V).—Tariff for the Reduction of Annuities, in case of the occurrence of Arrears of Contributions.

If in any calendar year contributions are paid for less than 47 weeks, a deduction in the annuity is to be made of $1\frac{1}{2}$-fold the value of the yearly annuity, as explained on page 321.

The most probable value (formula viii) of the charge for invalids which is created in respect of J_x in the course of the $(x+1)$th year of age (the annual annuity of 1 being adjusted for monthly instalments in advance) is

$$J_x\left[\cdot 4893 - {}^i s_x \,.\, \cdot 1428 + \left(1 - \frac{{}^i s_x}{2}\right) \cdot \frac{{}^i R^{\frac{1}{12}}_{x+1}}{q}\right]$$
$$= J_x \,.\, C_x \,.$$

The most probable value of the charge in connexion with each invalid aged x is, therefore, C_x: *i.e.*, at the date of entering upon receipt of the annuity, the life-annuity to each invalid aged x (taking the annuity at £1 and adjusting for its actual mode of payment) represents a capital entailed upon the institutes of £C_x.

For the capital amount of £1 the institutes could, therefore, pay an annual annuity of £$\frac{1}{C_x}$.

For every unit of the omitted payments of contributions, with interest thereon, the institutes would require to make a reduction in the annuity of £$\frac{1}{C_x}$, so that the total annuity deduction for each unit of the omitted contributions, with interest, amounts annually to $1\cdot 5\,.$ £$\frac{1}{C_x}$; and the values of $\frac{1}{C_x}$ are tabulated in Table XI in the Supplement.

It will be observed that some of the recommendations submitted in the memorandum were modified during the progress of the Bill.

I will not dwell on the difficulty I have experienced—often, I admit, with imperfect success—in expressing the German in clear and intelligible English.

VI.—Observations upon the Statistical and Mathematical Basis.

It is at present difficult adequately to criticise the preceding methods, and I might almost fairly leave the exposition itself to form its own comment. I venture, however, to add a few observations. . . . .

i. The data, it will be noticed, are essentially imperfect, frequently irrelevant, and generally unsafe as the foundation of so elaborate and ambitious a superstructure, especially when its necessarily permanent obligations are considered. And I submit this remark after full admission of the fact that all large social innovations must greatly assume the character of pure experiments, obscurely and deficiently guided by the experience of the past. But I have insisted on the paramount necessity in such reforms of attending not so much to the actual experience itself as to its significance and interpretative value for the future in connexion with the enquiry whether that experience forms an organic stage or otherwise in the development of the nation. And here, after a careful study of the subject, I diffidently though clearly venture to point out that this significance, apparent on close analysis, is not a genuinely natural one in respect of the evolution of the people, and therefore fails to justify this radical innovation. On the contrary, the significance appears to be indicative, on the one hand, of the growing demand of the people, fostered by previous legislation, for the substitution of State aid—*i.e.*, the compulsory assistance of other sections of the community—in place of the ennobling habit of fruitful individual self-dependence and development, and indicative, on the other hand, of official recognition, grounded on simple expediency, and erroneously directed, of the urgent need of averting social disruption, springing with distracting possibilities from the conjoint oppressive burden of the military and protectionist incubus which now enthrals the nation.

While the Bill was under discussion in the Reichstag, many members objected to the inadequacy of the data, and on this ground the National Liberal Party desired the postponement of the Act until more relevant and substantial statistics had been collected. The Government, however, contended that a measure of so vast a character could not be perfected at once, and that the most efficient mode of testing the adequacy of its working was at once to set it into execution. But the amplitude of the Bill and its incalculable and practically irreversible influence upon national life and character would rather appear to form the strongest arguments for a sagacious and wise delay.

(I have ascertained that the only bases for the statistics of sickness and survivance collected consisted of tables prepared by benefit clubs and other associations limited to districts or vocations which could only furnish returns of a restricted character. The

scheme therefore may almost be figured as a pyramid built up from the apex!)

ii. I need not further point out the entirely arbitrary assumptions: the doctrinaire hypotheses: and the tentative methods (based again upon suppositions) that stud the official memorandum in every part. We can, unfortunately, merely regard the teachings of this document in general as gained and promulgated in statistical and economic obscurity.

iii. A reference may be made to the difficulties involved in so vast a range with respect to "malingering." Our persistent trouble in the investigation of our friendly societies turns upon this central pivot: if the society is sufficiently limited to enable an effective supervision to be exercised on this vital question, the numbers tend to become inadequate to display average results, while if the latter condition be averted the extent of the society prevents an efficient check upon feigned illness, and renders our calculations to an extent precarious. This point is not distinctly dealt with in the memorandum and the law: but the complicated and expensive machinery devised involves it. I need not refer in detail to the serious financial cost which such supervision and proof entail in so wide a province, nor the consequent effect upon the estimates, and the minute and intrusive enquiry into private and social life involved, fraught with danger of general discontent. We find, therefore, implicit in the law, the necessity of a universal official espionage, involving, undoubtedly, social disturbance to probably a serious extent. In fact, an original proposition of the Government in relation to the receipt cards encountered so decided an opposition on the part of the people, as introducing offensive personal inspection, that a modification in the subject had to be adopted during the passage of the Bill.

iv. The essential difficulty connected with future wars was avoided as impossible of estimate, with its resulting and vital dislocation of the calculations. And the military tension on the Continent liable at any moment, especially in practical autocracies, to disruption, if only to obtain relief from the intolerable social pressure on the nation, is a significant comment upon this portion of the memorandum. Even apart from the occurrence of widespread wars, the military and naval expenditure continuously augmenting by habit, and from observation of neighbouring nations, until the tension expands into rupture, has not been considered with sufficient seriousness in connexion with increased taxation and the contributions lost during military service.

v. The element of emigration, again, does not appear to have received due recognition. I find, as some indication on this point, that during the 69 years—1820 to 1889—the total emigration to the United States alone (which absorb the superior classes of emigrants) amounted to 3½ millions, while, during the concluding 12 years, this national loss exceeded three quarters of a million; and it is officially calculated that each person represented an average monetary value of £10, so that the financial loss attained the sum of £35,000,000. It is needless to add that the financial burden entailed by this law will be principally avoided by the vigorous and enterprising, who will seek an arena for their industrial and unfettered capacities under ampler and more propitious conditions for their exercise in other lands. This omission—which seems to me to be a fatal defect—was, no doubt, influenced by Bismarck's favourite theory, that increasing emigration is one of the evidences of the prosperity of a country.

vi. It will be admitted, I think, looking to the permanent duration of the scheme and the range of securities appropriate for investments under the law, that the rate of interest of 3½ per-cent is excessive.

I have ascertained that the average rate of interest realised on German Government stocks has been 3½ per-cent to a fraction less. For example, the Imperial German 4 per-cent stock varies in price from 107·20 to 108: and the 3½ per-cent stock from 99·50 to 100·50.

The Government stocks of the German States yield about 3½ per-cent.

Loans issued by towns and provinces or counties in Germany (not guaranteed by the State) yield about 3⅝ per-cent.

On first mortgage of real estate the rate obtained is about 3⅞ per-cent (upon freehold buildings in the best positions in Berlin): this, however, may be regarded as exceptional, and 4 per-cent may be considered the current rate on such securities in Berlin, and 4½ per-cent in all the other principal towns.

For loans on agricultural properties the rate varies from 4 to 5 per-cent, according to situation.

There is the further important point to be considered in connexion with the rate assumed, that as the capital accumulated under the law increases and additional purchases of stock therefore require to be made, the price, of course, will successively rise, and the rate of interest obtained will consequently diminish.

VII.—General Observations upon the Scheme as a Whole.

In endeavouring to test the scheme by an application of the true principles of Government (so far as these have been approximately established in their fundamental features), it is to be observed that to some extent our deductions will vary, according to the nature of the form of Government concerned,—that form being to a large extent a natural function of the stage of social development to which the nation has attained; whether it be an autocracy or a constitutional system, and whether, in either case, it be approximately complete or imperfect. A scheme, to adopt an extreme example, adapted and adjusted to a simple patriarchal era, is obviously unfitted for an advanced and complicated condition of civilisation. Hence, our conclusions must be somewhat qualified according to the particular case: still, general primal considerations apply to all periods of national progress, and my remarks will, I trust, be so far limited as to involve only the essential elements of rule.

Without entering into the theories and minutiæ of the *quæstio vexata* of the functions of Government, it may be admitted that at any stage the primary elements of Government control—being the restrictions imposed by the corporate body for the general benefit of its constituents—relate to protection of the community, subject to its control, or rather to its defensive function: protection from external force, so that the progress of civilisation or true national expansion may securely proceed: protection from internal disorders and factions, so that the same end may be promoted, and the members held in co-operative and interdependent unity without disintegration into opposing sections which would otherwise ensue. Involved in the latter condition, the enforcement of contracts between individuals and associated bodies by an expeditious and economical process, within the ready reach of all, and emanating from a recognised body, to the end that sections may not contend confusedly and irregularly for maintenance of their common rights, with consequent disruption of the society. The object of such protection is, as I have stated, the provision that progress in civilisation—the development in all qualities and relations which must, in an increasing ascent, tend to produce the "bright consummate flower" of perfected individual and social character—may be achieved; the care, therefore, that each member shall possess the amplest right to his special and original development, consistently with the co-existence of

an equivalent right to all; and that all those high qualities which constitute completed humanity shall harmoniously and continuously be secured. In short, as government in any form is a sign and index of individual and social imperfection, and not an evidence of social vigour and spontaneous completeness, the ideal and, therefore, the practical aim of national rule is the dispensing with external government altogether; the substitution of spontaneous laws and feelings *internal* within the individual, for rules and processes imperfectly established *externally*. Fuller and freer forms of government; the successive removal of restriction after restriction from the governed; the gradual and continuous allowance, therefore, of ampler opportunity for unfettered individual freedom, are the just historical eras in the course of social and national expansion. It will, I think, be conceded that the formation of an elevated self-reliance, associated with social justice and regard, constituting in the totality a righteous national character, is the appropriate aim of all institutions which, themselves transient, should universally include in ever brighter outline this perpetually immanent issue.

And it follows, as a corollary, that all attempts by governments at administrative and executive power which are inconsistent with this fundamental principle, are to be sedulously and scrupulously criticised before the individual surrenders any additional portion of his freedom. It is not contended that all government intervention, beyond its narrower confines, is dangerous or useless: many expressions of subordinate control are needed in the adjustment of the conditions of different periods of civic and national progress, with a view to ensuring the primal end of equal freedom. Government, in its true sense, is simply the body corporate, protecting its constituent members, and so maintaining and expanding the unity they compose: but all excursions beyond the provision and enlargement of equal rights, all processes, therefore, which tend to affect detrimentally the individual, and consequently the national, character, are to be judged by a severe and far-reaching standard. Though authority, I think, is needless, I may be pardoned a quotation from the *Journal Intime* of Amiel, that "the test of every religious, political, and educational system is the men whom it forms." Does the system tend to develop generally the individual, so that he possesses the power of orginal growth, which at the same time essentially implicates the practical recognition of vital and just relationships to his fellows? And the philosophical historian, Mr. Lecky, has often pointed out that

the national type of excellence is, for the most part, the expression or net moral result of the national institutions and circumstances.

The progress of civilisation and the expansion of its gifts essentially lie in personal and social ethics, so that the rightful march of social effort commences in a finer cultivation of the moral sense of the community as the herald and guide of any political acts which that feeling may demand as its general expression.

Viewed in this light, the scheme I have described fails to sustain the test; for, far beyond all its predecessors, it forms a more coercive and a deeper interference with individual liberty, and a more serious restraint upon a free and original expansion of national character and life. Some of its detailed evils I may briefly enumerate.

(1) It restrains the development of the individual, and consequently of the national, character. Men only advance in civilisation by toil and patience.

The social and industrial character and aptitudes are created and nourished by self-dependence and self-action in such spheres as those embraced by this law: by the consequent teaching of successful and painful experience: by the discovery of more effective modes of combined action and enterprise which defective experiments have taught through the discipline of failure, and by that spontaneous adjustment of relations between themselves, which is the distinguishing evidence of a genuine and organic national life. Hence, I doubt not that the advancement of social and national character, the effective educational training of the community by experience of their own labours and sufferings, are seriously imperilled by the wide-reaching nature and minute intervention of this scheme: and the natural evolution of the people to a higher stage of progress crippled and embarrassed. For history is not a mere chronological sequence: it is a chain of causes linked indissolubly with effects, and of effects in turn assuming a causal power.

At the risk of being tedious, I would repeat the contention that the fact of any tyranny and minuteness of Government control dwarfing the individual character and the freer education of the people by the only sure method of their own toilsome and perplexed experience, marks the fitness or unfitness of a scheme for general acceptance.

(2) A government, by such needlessly detailed and elaborate supervision, holding itself implicitly as possessing the sole remedy

for all social ills, still more radically tends to demoralise and impede national life by confirming and widening the social habit of appealing to the State at every crisis, real or imagined. External aid is substituted for internal and inspiring effort. To speak in the language of natural history, the homogeneous and gradual evolution of the social organism into higher stages of being, its more intimate adjustment of existence to environment won through painful toil, dwindles down into the degradation of parasitism, with atrophy of the activities which triumphantly imply progress, and a concurrent degeneration of the entire national system. For, as in the universal organic kingdom, the decrepitude and paralysis of any section of the corporate body involve the progressive decay of the complete fabric of which it constitutes a vital part, especially in respect of the nobler aptitudes, or the parasitic degeneration of the part impoverishes and destroys the vitality of the vigorous sections, until the whole organism wastes away and relinquishes its high functions and destinies in the order of civilisation. And where is the supreme standard that shall assign the limits to such widening scope? If the appeal to Government be not responded to, and an ever-increasing burden of oppression entailed upon the nation, what other end is possible in the ramifying evil but national disintegration or the supreme revulsion which may, like storms and convulsions in nature, either purify and enlarge through bitter and long-enduring eclipse and conflict, or simply lapse into decadence and decay?

In the discussion in the Reichstag it was forcibly pointed out by many members that the intervention of the State subsidy was liable to attach a pauperising tendency to the Bill, and that the additional indirect taxation requisite would be principally incident upon the poorer classes, so that the subsidy, designed as a blessing, might ultimately be regarded by the people as a curse.

(3) If we image to the mental eye the vast and intricate system which the scheme necessarily involves: the disciplined and coherent army of officials in opposition frequently to popular views and feelings: the attraction of the capable and intelligent, the gifted and sagacious, from the general community to this privileged class: the consequent impoverishment in intellect and the higher qualities of service of the constantly denuded populace: the severe taxation, with its absolute restraint upon popular liberty, popular comfort, popular education in life, popular development in general, popular taste, culture, and civilisation,

produced by limitation of the requisite means and of capacity to procure them: the practically illimitable cost of administration: the additional burden of official salaries and pensions to those in place and power: we perceive a *tout ensemble* which surrounds the picture with a gloomy setting.

(4) I need not dwell, too, on the notorious fact that Government supervision in any extended form is always attended by defective work, maladjustment of simple and effective means to ends, with further resulting loss to the community in character, hope, and enlarged scope of action, crippled again as these are, through the modes I have briefly sketched, by the absence of the opportunities and ability for unfolding the nature and qualities which constitute effective manhood.

(5) The payment of a portion of the contributions by the employers will, I fully apprehend from the teachings of economic history, involve a reduction in the nominal amount of wages, and consequently a further restriction of personal and social life. The employers will obviously seek to transfer a portion of their burden of enforced charges to the labourers' gains: hence reduced wages with higher prices: all acting in combined power against the workman's impoverished condition. In our old poor law history the taxes in aid of wages invariably meant a diminution of wages and a degradation of the position of the labourer. And political economy is the teaching of the repetition of effects, when similar causes and conditions exist.

Hence, with augmented taxation and a reduction of the means of life, we shall, I fear, find the prices of the labourers' essentials of life—to omit regard altogether of possible comforts—gradually increase: so that we obtain the relation of advancing cost and diminishing means of purchase, involving, by restriction of the necessaries of livelihood, defective and inefficient work, and the disastrous enfeeblement of individual enterprise. It is sufficient in intensification of this picture to advert to the serious expenditure which the execution of the scheme, especially in its official aspect, will entail.

I need only again refer to the probability of extended emigration of the more vigorous and intelligent classes, induced by this financial and social pressure.

(6) The application of a State subsidy has been hailed with delight, though some sagacious members of the Reichstag, as I have mentioned, discerned the true features of this aid and prophesied a revulsion of popular feeling respecting its permanent

national value. It is a truism to characterise the genuine nature and meaning of a government subsidy; but history has always shown that admitted facts lose their real significance from the very circumstance of the depth and reality of their truth. In our own country we perceive, unfortunately, a similar fiction and delusion surrounding our Consolidated Fund. The people, and, too frequently, their teachers also, appear on this question to regard the State as a distinct entity, possessed of extraneous and intrinsic powers of financial supply. State funds are viewed as external reservoirs of opulence; the financial rains, whose descent is implored, are not observed to be simply the accumulation, as in nature, of financial currents ascending from the earth, though, unlike the natural shower, leaving in their ascent the earth permanently impoverished, without the re-endowment of recovery and fruitful gain.

(7) I venture also to express the view that, in every social and political project, the more especially as its scope is the more enlarged, a sagacious reformer should take into approximate account and incorporate the consideration into the frame of his scheme—however difficult and precarious the process may be—the altered relations and social modifications which the reform will certainly and eminently produce. This element is important, in order to cause the minimum amount of friction and ensure the gradual readjustment of the measure with existing conditions at the outset, until it is firmly established without abrupt and hasty dislocation of the relations that prevail. A familiar example, I think, may be adduced from the consideration of the element of Secessions in our investigations into friendly societies. If that element be taken fully into account in our calculations, may it not tend, for obvious reasons, to *reduce* the future rate of secession, and thus partially defeat our object and the basis of our valuations? I venture to think that a consideration of this character has not adequately been included in the conception of this law; and, to repeat a special concrete illustration, I do not perceive that the probable stimulus which the increasing burdens of the scheme will afford to emigration has been seriously observed. The abstraction of the energetic men of disciplined enterprise and commercial initiative from the country must impair the financial resources required for the complete execution of the law, and must tend to leave a larger and larger number of dependent and inferior labourers appealing to its provisions and to its reducing sources of supply. Will not the very scheme

itself, again, help to develop the extent of invalidity by producing habits which lend themselves to that condition, and hence augment the range to which the funds must apply? And will not these grants in aid, as shown so constantly in economic history, tend to promote marriage among the lower classes, and thus further intensify the oppression through the larger grants to children, the aided growth of a comparatively pauper population, and (as Mr. Francis Galton has elsewhere pointed out as a general consideration upon another subject) a deterioration of the national character by a wider leavening from inferior races?

(8) If I apply the test of simplicity in administration and execution—which a practised economist and statesman, Professor Fawcett, deemed to be a valid measure of the utility and adaptation of social arrangements—I should be simply repeating the analysis I have furnished already as evidence of the failure of the law to fulfil this criterion. I have indicated the ingenuity of the elaborate machinery which has been devised: the balanced counterpoises: the minute and laboured adjustments and relations between body and body and the public: admirable devices and correlations for inorganic machinery governed by external nature, but adapted from their very multiplicity and intricacy to defeat the facile working of a machine where the operators and materials are human beings endowed with passions and hopes. Unfortunately, in human arrangements ingenuity is far from being everything: skilful schemes of equilibrium do not realise the whole requirements: and friction and irregularity are involved essentially in the vast mechanical array. When one surveys the wide and complicated official organisations: the hierarchy of official bodies: the limitless scope for official memoranda, reports, delays, interrogatories, references, decisions, and appeals, the legal powers conferred upon 12,000,000 individuals with characters showing every conceivable gradation of self-assertion and litigiousness (without the safeguard of any *direct* financial deterrent upon their acts), the paradise thus revealed to the official mind and habit is blotted out by the possible spectacle of financial oppression, social disenchantment, and wide-spread delusion to popular hopes.

In the glow and preliminary invigoration of a mighty scheme, the intrusive present fills the reformer's entire horizon and hopefully excludes the dim regions beyond, but it is true that no more fruitful principle was ever announced in social and political

spheres than that of the economist Bastiat, that, in the consideration of all social and government proposals, the "things that are not seen" at once as natural results are even more impressive, and more to be scrupulously scanned and weighed, than the "things that are seen" and are obvious to all.

(9) I may, perhaps not unfittingly, digress for a moment to point out that the scheme appears to me to form an endeavour to attack a *symptom* merely of national disquietude, instead of a wise and cautious investigation of the nature of the ailment and an attempted removal of the *cause*. History everywhere, and at every stage, has shown that this proceeding is a universal mischief, especially where the malady is severe, and the reformer is filled with the righteous pity which too often suppresses the calm serenity of judgment. The arm of Briareus is there: but the blows become too often random and ineffective, for the watchful eyes of Argus are partly veiled. It is but common prudence—a truism—that the future should be as much regarded in all reforms as the clamorous and necessitous present, so that righteous effort be not defrauded of her legitimate reward through the sleep of vigilant foresight. If the system of industrial protection which so deeply prevails: if the searching results of a vast military system were exhaustively explored, a gradual cure would be discovered there, and a free national life be rendered possible, which the additional official and executive machinery will now only more effectually thwart.

The scheme is clearly a valuable adjunct to the resources of autocracy, however beneficently disposed, for besides a *military* power searching into every grade of life, a *social* and *civil* power is now evoked into being, entering into every remaining channel and intensifying and extending the grip that already exists. It is comment enough to add that government exists solely for the governed, and to the extent of the benefits and services to them in their just development which otherwise could not be attained.

(10) A vital consideration in the examination of this subject is the fact that a widely organised scheme like the present is practically final as regards any essential modification, except under the danger of radical social disturbance, and the serious discontent involved in any alleged breach of national contract, with its possibilities of disruption approaching to the dimensions of a revolution and a shaking of the bases of a country's life. It forms a burden, a tax, on posterity for which the Present should rather

assume the functions of trustee, which cannot in any appreciable measure be removed: and the probable effects which I have indicated—increased and increasing taxation with consequent restriction of the means of social comforts, of individual culture and self-reliance, the nourishing of popular clamour for augmented State assistance, and the resulting impoverishment and decay of combined personal efforts—all seem likely to operate in gradually enfeebling the national habits and consequent character, and the progress of the people in the scale of highest development.

It is needless, I hope, to indicate that these criticisms are limited to the contemplation of the scheme in its economic and political aspects, and are accompanied by a clear recognition of the belief that the amelioration of the race is dependent upon personal sympathy and direct individual and social aid, where faculties of service and social aptitudes are conceived as a personal trust for enlargement of the life and hopes of the less gifted and less happy.

In taking a final retrospect over my labours I seem to myself —and with genuine regret—to have sustained the reversed *rôle* of the Prophet Balaam. I appear to have started with a deep impression of the benevolent motives and probably beneficent effects of the measure: but as my enquiry extended I seem to have merged the prophetic benediction into a regretful prediction of social disunion, national disillusionment, and financial loss. I retain my admiration of the value of the object, and of the careful and ingenious though doctrinaire methods by which the achievement of that end has been sought. I can perceive, however, though at the moment dimly, that a symptom of national disease has been temporarily prescribed for in place of a direct attempt at gradual extirpation of the cause: and though I have ventured to this tentative conclusion, I await the advent of larger experience and clearer light to enable me to pronounce a more securely-based decision upon the efficiency of this legislative remedy.

I now leave the subject to a future student, for whom I have toilfully amassed these details, to survey with completer knowledge and wider grasp the probable result of a measure which, from the various points of view I have described at the outset, demands authentically the deepest and most patient attention and thought.

SUPPLEMENT.—TABLES.

I.—Where interest is involved, the results are expressed in marks, and the sterling equivalent is obtained by multiplication by ·05.

II.—Results for interval-ages alone are furnished, but the complete Tables have been placed in the Library of the Institute of Actuaries.

III.—I regret that I have not found it possible to substitute the system of notation adopted by the Institute of Actuaries.

Table I.

Age x	Invalidity Probability (according to Behm) (i_x)	Mortality Probability for Invalids (according to Zimmermann) $({}^is_x)$	Mortality Probability for Actives, without previous Declaration of Invalidity (newly calculated) $({}^as_x)$	Mortality Probability for the Male German Population (s_x)	Activity Probability (newly calculated) $\left(a_x=\frac{\mathrm{A}_{x+1}}{\mathrm{A}_x}\right)$	Out of 60,657 Males (aged 16) entering Insurance, there remain at Age x: Existing in all (L_x)	As Actives (A_x)	As Invalids (P_x)	Number of Invalids occurring Annually $(\mathrm{J}_x=\mathrm{A}_x \,.\, i_x)$	Number of the J_x Invalids surviving at Age $x+1$ $\left[\mathrm{J}_x\left(1-\frac{{}^is_x}{2}\right)\right]$	Number of the P_x Invalids surviving at Age $x+1$ $[\mathrm{P}_x(1-{}^is_x)]$	Discounted Number of Actives $\left(\lambda\frac{\mathrm{A}_x}{q^x}=\lambda\mathrm{A}'_x\right)$ $(q=1·035)$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
16	·00011	·1182	·00451	·00451	·99538	60,657	60,657	...	6·67	6·28	...	4·54384
20	·00019	·1020	·00744	·00750	·99237	59,287	59,260	27	11·26	10·69	23·86	4·47396
25	·00038	·0831	·00835	·00848	·99127	56,892	56,815	77	21·59	20·69	70·55	4·38096
30	·00076	·0656	·00908	·00928	·99016	54,454	54,281	173	41·25	39·90	161·72	4·28644
35	·00152	·0639	·01061	·01101	·98787	51,815	51,456	359	78·21	75·71	336·41	4·18853
40	·00305	·0622	·01282	·01363	·98413	48,775	48,073	702	146·6	142·1	658·1	4·08429
45	·00609	·0530	·01554	·01680	·97837	45,272	43,918	1,354	267·5	260·4	1,282·7	3·97033
50	·01218	·0510	·01921	·02145	·96861	41,228	38,674	2,554	471	459	2,423·6	3·84040
55	·02437	·0485	·02434	·02790	·95129	36,544	31,946	4,598	778·5	759·6	4,374·8	3·68270
60	·04873	·0512	·03270	·03820	·91857	31,124	23,432	7,692	1,142	1,113	7,298	3·47339
65	·09747	·0629	·04584	·05520	·85669	24,802	13,600	11,202	1,326	1,284	10,498	3·16242
70	·19493	·08108	·07322	·08108	·73185	17,750	4,807	12,943	937	899	11,893	2·63605
75	·38986	·12004	·09561	·12004	·51453	10,743	568	10,175	221·4	208·1	8,954	1·63383
80	·77972	·17410	·10400	·17448	·11628	5,035	3·3	5,032	2·57	2·35	4,153·3	9·32329

TABLE II.

Annuity Calculation for Invalids.

$$\left({}^{i}R_x = 1 + {}^{i}R_{x+1} \cdot \frac{1 - {}^{i}s_x}{q}\right)$$

$$(q = 1{\cdot}035).$$

Age x	Probability of Survivance for Invalids $(1 - {}^{i}s_x)$	${}^{i}R_{x+1} \cdot \frac{1-{}^{i}s_x}{q}$	${}^{i}R_{x+1} \cdot \frac{1-{}^{i}s_x}{q} + 1 = {}^{i}R_x$	${}^{i}R_x - {\cdot}463 = {}^{i}R_x^{\frac{12}{2}}$
(1)	(2)	(3)	(4)	(5)
100	·48070	·4644	1·4644	·0014
95	·59783	1·2287	2·2287	1·7657
90	·68098	1·7242	2·7242	2·2612
85	·75637	2·3377	3·3377	2·8747
80	·82552	3·1869	4·1869	3·7239
75	·87996	4·2989	5·2989	4·8359
70	·91892	5·6485	6·6485	6·1855
65	·9371	7·0418	8·0418	7·5788
60	·9488	8·3131	9·3131	8·8501
55	·9515	9·3536	10·3536	9·8906
50	·9490	10·0006	11·0006	10·5376
45	·9470	10·3193	11·3193	10·8563
40	·9378	10·2241	11·2241	10·7611
35	·9361	9·9288	10·9288	10·4658
30	·9344	9·7255	10·7255	10·2625
25	·9169	9·0981	10·0981	9·6351
20	·8980	8·0466	9·0466	8·5836
16	·8818	7·1085	8·1085	7·6455

Table III.

Calculation of the Value $C_x = \cdot 4893 - \cdot 1428 \,.\, {}^{i}s_x + \left(1 - \frac{{}^{i}s_x}{2}\right)\frac{{}^{i}R^{1\frac{1}{2}}_{x+1}}{q}$

$(q = 1\cdot 035)$.

Age x	${}^{i}s_x$	$1 - \frac{{}^{i}s_x}{2}$	${}^{i}R^{1\frac{1}{2}}_{x+1}$	$\left(1 - \frac{{}^{i}s_x}{2}\right)\frac{{}^{i}R^{1\frac{1}{2}}_{x+1}}{q}$	$\cdot 1428 \,.\, {}^{i}s_x$	$\cdot 4893 - \cdot 1428 \,.\, {}^{i}s_x$	$\cdot 4893 - \cdot 1428 \,.\, {}^{i}s_x + \left(1 - \frac{{}^{i}s_x}{2}\right)\frac{{}^{i}R^{1\frac{1}{2}}_{x+1}}{q} = C_x$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
16	·1182	·94090	7·8805	7·1640	·0169	·4724	7·6364
20	·1020	·94900	8·8122	8·0790	·0146	·4747	8·5537
25	·0831	·95845	9·8070	9·0818	·0119	·4774	9·5592
30	·0656	·96720	10·3096	9·6343	·0094	·4799	10·1142
35	·0639	·96805	10·5148	9·8346	·0091	·4802	10·3148
40	·0622	·96890	10·8208	10·1297	·0089	·4804	10·6101
45	·0530	·97350	10·8152	10·1726	·0076	·4817	10·6543
50	·0510	·97450	10·4139	9·8334	·0073	·4820	10·3154
55	·0485	·97575	9·7063	9·1507	·0069	·4824	9·6331
60	·0512	·97440	8·6054	8·1016	·0073	·4820	8·5836
65	·0629	·96855	7·3145	6·8449	·0090	·4803	7·3252
70	·08108	·95946	5·8990	5·4685	·0116	·4777	5·9462
75	·12004	·93998	4·5933	4·1716	·0171	·4722	4·6438
80	·17448	·91276	3·5326	3·1151	·0249	·4644	3·5798

Table IV.

Calculation of the Reversion to an Invalidity Annuity of the Amount of 120–250 Marks annually.

Age x	$A'_x \,.\, i_x \,.\, C_x$	$\Sigma A'_x \,.\, i_x \,.\, C_x = Z'_x$	$\Sigma Z'_x$
(1)	(2)	(3)	(4)
16	30·07	32,005	1,260,512
20	48·40	31,863	1,132,683
25	87·33	31,550	973,921
30	148·66	30,999	817,156
35	242·01	30,086	663,806
40	392·92	28,601	516,062
45	606	26,233	377,378
50	870·04	22,691	252,776
55	1,130·63	17,792	148,579
60	1,244·11	11,816	71,289
65	1,037·79	5,858·5	24,461
70	501·38	1,657·27	1,614·9
75	77·91	151·46	271·5
80	·5876	·6626	·7376

TABLE V.

Calculation of the Reversion to an Invalidity Annuity of the Amount of 120–250 Marks annually.

Age x	$60(Z'_{x+1} + Z'_{x+5})$	$2\Sigma Z'_{x+6}$	$\Sigma Z'_{x+21}$	$\Sigma Z'_{x+41}$	$60(Z'_{x+1} + Z'_{x+5}) + 2\Sigma Z'_{x+6} + \Sigma Z'_{x+21} + \Sigma Z'_{x+41}$	$4\Sigma Z'_{x+51}$	Col. 6–Col. 7 $=A'_x \cdot {}^iP_x$	λ^iP_x
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
16	3,827,340	2,138,012	603,877	114,125	6,683,354	55,128	6,628,226	2·27756
20	3,801,840	1,884,742	487,461	59,473	6,233,516	11,831	6,221,685	2·31995
25	3,747,720	1,572,314	351,145	18,603	5,689,782	480	5,689,302	2·37410
30	3,656,220	1,267,410	230,085	2,958	5,156,703	...	5,156,703	2·42593
35	3,506,640	974,922	130,787	120	4,612,469	...	4,612,469	2·47541
40	3,266,460	702,290	59,473	...	4,028,223	...	4,028,223	2·52082
45	2,899,080	460,170	18,603	...	3,377,853	...	3,377,853	2·55831
50	2,376,780	261,574	2,958	...	2,641,312	...	2,641,312	2·58142
55	1,708,680	118,946	120	...	1,827,746	...	1,827,746	2·57922
60	985,830	37,206	...	...	1,023,036	...	1,023,036	2·53650
65	388,678	5,915	...	...	394,593	...	394,593	2·43373
70	78,441	240	...	...	78,681	...	78,681	2·25982
75	4,453	...	...	...	4,453	...	4,453	2·01483
80	4·5024	...	...	...	4·5024	...	4·5024	1·33015

TABLE VI.

Calculation of the Reversion to an Old Age Annuity of the Amount of 120 Marks annually.

Age x	${}^aR$ (Table 8)	${}^aR^{1\frac{2}{3}}$	$\lambda \frac{120A'_{70}}{A'_x} \cdot {}^aR^{1\frac{2}{3}} = \lambda^aP_x$
(1)	(2)	(3)	(4)
16	2·8505	2·3875	·54934
20	...	..	·61922
25	...	...	·71222
30	...	...	·80674
35	...	...	·90465
40	...	...	1·00889
45	...	...	1·12285
50	...	...	1·25277
55	...	...	1·41018
60	...	...	1·61979
65	...	...	1·93076
70	...	...	2·45712
75	...	...	2·21224
80	...	...	1·89489

TABLE VII.

Calculation of the Capital Value of the Losses of Contributions in consequence of Illness when the weekly Contribution amounts to $\frac{1}{47}$.

$(q=1{\cdot}035)$.

Age x	Number of days of Illness per Annum (K_x)	$A'_x . K_x$	$\Sigma(A'_x . K_x)$	k_x
(1)	(2)	(3)	(4)	(5)
16	5·681	198,729	4,766,223	·4071
20	5·875	174,970	4,007,887	·4021
25	5·167	124,221	3,237,613	·4023
30	5·335	103,175	2,676,064	·4134
35	5·944	91,750	2,179,771	·4222
40	7·174	87,107	1,738,053	·4277
45	7·991	74,633	1,317,308	·4214
50	8·885	61,526	973,256	·4199
55	12·273	59,109	677,638	·4204
60	17·114	50,903	398,570	·4004
65	22·821	33,171	173,329	·3563
70	33·204	14,363	47,177	·3258
75	49·455	2,128	4,078	·2831
80	68·764	14·48	16·281	·2311

Note: $\lambda \frac{\Sigma(A'_x . K_x)}{A'_x} + \lambda \frac{1}{7 . 47 . q^{\frac{1}{2}}} = \lambda k_x .$

TABLE VIII.

Calculation of the Capital Value of Contributions of $\frac{1}{47}$ weekly, with reference to the Losses in consequence of Illness

$(q=1\cdot035)$.

Age x	A'_x	$\Sigma A'_x$	${}^aR_x$	k_x	${}^aR_x - \cdot5154 - k_x = \frac{B_x}{47}$
(1)	(2)	(3)	(4)	(5)	(6)
16	34,981	703,568	20·1127	·4071	19·1902
20	29,782	571,575	19·1919	·4021	18·2744
25	24,041	434,564	18·0758	·4023	17·1581
30	19,339	324,128	16·7601	·4134	15·8313
35	15,436	235,516	15·2579	·4222	14·3203
40	12,142	165,139	13·6007	·4277	12·6576
45	9,340	110,211	11·8004	·4214	10·8636
50	6,925	68,478	9·8889	·4199	8·9536
55	4,816	38,181	7·9277	·4204	6·9919
60	2,974·3	17,889·9	6·0147	·4004	5·0989
65	1,453·5	6,220·2	4·2794	·3563	3·4077
70	432·57	1,233·04	2·8505	·3258	2·0093
75	43·035	78·39	1·8215	·2831	1·0230
80	·2105	·2352	1·1172	·2311	·3707

TABLE IX.

Age x	Males to be Insured according to the Age n_x	$n_x \cdot \frac{B_x}{47}$	$n_x \cdot {}^iP_x$	$n_x \cdot {}^aP_x$
(1)	(2)	(3)	(4)	(5)
16	284,625	5,462,010	53,930,500	1,008,360
20	278,897	5,096,670	58,263,500	1,160,560
25	212,350	3,643,520	50,252,300	1,094,650
30	163,935	2,595,300	43,712,400	1,050,540
35	138,112	1,977,810	41,270,500	1,108,880
40	117,612	1,488,690	39,019,000	1,200,440
45	97,909	1,063,640	35,410,700	1,299,190
50	78,437	702,290	29,918,300	1,403,770
55	59,439	415,590	22,557,300	1,529,500
60	44,659	227,710	15,360,600	1,860,780
65	32,346	110,230	8,781,130	2,757,900
70	19,172	38,522	3,487,260	5,492,780
75	5.286	5,408	546,960	861,720
80	530	196	11,340	41,610
Sum of all Ages	...	97,204,836	1,840,061,210	101,358,140

TABLE X.

Age x	Females to be Insured according to the Age n'_x	$n'_x \cdot \frac{B_x}{47}$	$n'_x \cdot {}^iP_x$	$n'_x \cdot {}^aP_x$
(1)	(2)	(3)	(4)	(5)
16	213,319	4,093,630	40,419,500	755,740
20	236,063	4,313,910	49,315,200	982,310
25	90,508	1,552,950	21,418,600	466,560
30	46,036	728,810	12,275,300	295,010
35	34.913	499,960	10,432,700	280,310
40	30,970	392,010	10,274,600	316,100
45	29,531	320,810	10,680,500	391,860
50	27,785	248,780	10,598,100	497,260
55	23,561	164,740	8,941,480	606,280
60	18,687	95,283	6,427,450	778,620
65	13,968	47,599	3,791,960	1,190,950
70	8,310	16,697	1,511,530	2,380,810
75	2,685	2,747	277,820	437,710
80	302	112	6,460	23,710
Sum of all Ages	...	50,396,747	812,912,310	42,705,190

TABLE XI.

Tariff for the calculation of the Annuity Deduction for cessation of Contributions.

Age x	If the Entry upon receipt of the Annuity begins at the Age indicated, each Mark of the omitted Contributions, with Interest, represents for the Insurance Institutes, in respect of the Annuity, the Amount of $\frac{1}{C_x} =$
(1)	(2)
16	·1310
20	·1169
30	·0989
40	·0943
50	·0969
60	·1165
70	·1682
80	·2793

ADDENDUM.

Since writing this paper, I have seen an article from the *Norddeutsche Zeitung* (the official organ, I believe, of the German Government) of the 12th of March 1890, explaining to workmen the necessary procedure for securing the benefit of those special provisions of the Law which apply to the years immediately following its enactment.

It will be remembered, in the exposition of the Law, that pensions in invalidity and old age are only claimable after the lapse, from the date of the Law, of 5 and 30 years respectively. "Transitory" provisions, however,* dealt with cases of invalidity occurring within the first 5 years, and of the attainment of age 70 within the first 30 years, by reduction of the "wartezeit", or probationary period: and the *Zeitung* points out to labourers how the advantages of these provisions are to be secured.

The process in respect of both invalidity and old age consists in obtaining certificates—for which the authorities supply the appropriate forms—relating to the employment, rate of wages, illnesses, and military or naval service. The legal formalities in authentication of the certificates are then explained in detail.

The *Gazette* illustrates the procedure by the following useful examples:

i. A workman—at the date when the Law comes into operation, about, say, January 1891—is engaged in employment entailing compulsory insurance; remains in it for at least 47 consecutive weeks; discharges the required contributions; and then becomes seriously ill, about the 52nd week, and incapable of earning a living. He would then, according to the Law, possess no right to an allowance for invalidity, as his contributions would not have extended over the

* *Vide* section ii under B, page 285; and section x on page 300.

entire probationary period of 5 contributory years of 47 weeks each, or 235 weeks in all. Notwithstanding this defect, however, an allowance will still be granted if he can prove that, before the Law came into force, and within the last 5 years prior to incapacity to work—which in this case would be the interval between the beginning of 1887 and the end of 1890—he was in an employment specified by the Law, or in a condition corresponding thereto (namely, illness, military service, &c.), during a period of so many weeks as are wanting to complete the necessary number of contributory weeks (*i.e.*, 235) of the probationary period—amounting, in this example, to 234−47=188 weeks in all (since during the 47 weeks he had paid the contributions). If a certificate be presented to this effect, he would receive, according as contributions were paid for him—after the Law came into force—from the 1st, 2nd, 3rd, or 4th wage-class, an annual allowance during invalidity of £5. 10*s*. 11½*d*., £5. 12*s*. 10¼*d*., £5. 14*s*. 3*d*., or £5. 16*s*. 1¼*d*., respectively, although he had actually only contributed the respective sums of

$$47 \times \tfrac{14}{2} \text{ pfennigs} = 3s.\ 3\tfrac{1}{2}d.$$
$$47 \times \tfrac{20}{2} \quad ,, \quad = 4s.\ 9d.$$
$$47 \times \tfrac{24}{2} \quad ,, \quad = 5s.\ 7\tfrac{1}{2}d.$$
$$47 \times \tfrac{30}{2} \quad ,, \quad = 7s.\ 0\tfrac{1}{2}d.$$

This considerable advantage would thus be lost if, through carelessness, the proper certificate should not be procured and preserved.

ii. If a workman, who is over 40 years of age at the same date (say, January 1891) attains the age of 71—when he is entitled to an allowance for old age—about January 1894, after he has been in employment (defined in the Law) for a period of about 100 weeks subsequent to the Law coming into operation, and has paid the required contributions, he will then have failed to complete the probationary term of 30×47=1,410 contributory weeks, and would therefore possess no right to an allowance. But the allowance will nevertheless be accorded if he can show that during the 3 calendar years immediately prior to the enforcement of the Law he was engaged—for at least 141 weeks altogether—in one of the prescribed employments, or was in a condition corresponding thereto (illness, &c.). If the workman is unable to prove the average amount of his actual wages during these 141 weeks, the lowest wage-class will be adopted as the basis of calculation with respect to the period before the Law came into effect. The annual grant for old age—if after the Law came into force contributions have been paid from the 2nd wage-class—would amount to only 50 marks+(100×6) pfennigs +[(1410−100)×4] pfennigs=£5. 8*s*. 5*d*. But if the insured can

prove that his average annual wages during the 141 weeks in question came within the 2nd wage-class and not within the lowest, the amount of old-age allowance, to which he would then be entitled, would be 50 marks + (100 × 6) pfennigs + [(1,410 − 100) × 6] pfennigs = £6. 14*s*. 7*d*. Thus, if he omit to obtain and preserve the requisite certificate as to the duration of his employment, he damages himself to the extent of an annual allowance of 108 marks + 40 pfennigs = £5. 8*s*. 5*d*.; and if he forget to procure a certificate attesting the average annual wages he received, he forfeits 134 marks + 60 pfennigs − 108 marks − 40 pfennigs = 26 marks + 20 pfennigs = £1. 6*s*. 2½*d*.

T. E. YOUNG.

DISCUSSION.

The PRESIDENT (Mr. B. Newbatt) having asked the two referees, Mr. W. Sutton and Mr. R. P. Hardy, to open the discussion,

Mr. W. SUTTON said that the paper dealt not only with the subject with which he was officially connected, but also with social topics which were not improbably about to become prominent political topics in this country and elsewhere. That being so, in his capacity as a servant of the State, it followed that, with regard to what might be called the essence of Mr. Young's contribution, they must find him practically dumb. He must express the very high value he attached to Mr. Young's paper. It was a curious coincidence that 100 years ago the topic of national insurance in some form or other was very common. He possessed a short summary of one of the schemes which were then put forward. It was called "A plan for ameliorating the condition of man by creating in "every nation a national fund to pay to every person when arrived "at the age of 21 years the sum of £15 sterling, to enable him "(or her) to begin the world, and also £10 sterling per annum "during life to every person now living of the age of 50 years, and "to all others when they arrive at that age, to enable them to live in "old age without wretchedness, and go decently out of the world." The fund was to be created by taking, on the death of every individual, 10 per-cent of his property as "*due to society*", and from 5 to 12 per-cent more if there were no near relations, in proportion as the next-of-kin was nearer or more remote. That proposal, as given to him, bore the name of Thomas Paine, and the date of 1795. He thought he should be correct in saying that during the last 70 or 80 years there had been a vast improvement in the social condition of the working classes of this country, and those classes had perhaps never been in so prosperous a state as at the present time. It was, therefore, remarkable to find the Actuarial Society of Edinburgh, a short time ago, discussing Mr. Robert Stirling's paper on this subject, and now the Institute of Actuaries was engaged in discussing a scheme of national compulsory insurance for old age and invalidity, just adopted in Germany, and which, so far as he could learn, was being followed to

some extent in nearly all the countries on the Continent. He thought that much of what was contemplated to be done in Germany by means of the State had been going on in this country in an unostentatious manner for many years. They already had superannuation funds connected with railways and other large industries: many had been recently established, and many more were about to be established. They had well-conducted miners' accident funds throughout the country, and, in some of the manufacturing districts, amalgamated accident funds had been at work for many years more or less effectually. It would greatly improve the paper for readers who were not conversant with the German language if it could have attached to it some definite account of the actual financial constitution of the friendly societies' system in Germany. He had not the remotest idea whether they went on the principle of accumulating funds to meet future claims, or whether they were of the nature of what were called slate clubs, or dividing societies. Then there was the question of expenses. He did not see any mention by the author of what, in his official experience, he found to be an extremely important matter, namely, the cost of medical aid. He did not know whether that item was included in the estimate of expenses of management, and the author hardly made it clear. Coming to the general question of deferred annuities, it was perfectly true that at the present time the average working man of this country did not take kindly to the idea of beginning to pay at age 20 for what he could not possibly receive any benefit from for the next 45 or 50 years. One striking case came before him officially in connection with what was the largest, and perhaps the richest, of the affiliated orders. They had between 600,000 and 700,000 members in their various lodges, and some 10 or 12 years ago they made a most praiseworthy attempt to establish a deferred annuity fund. He believed that in the course of that 12 years some 12 to 15 members joined, and the fund now contained two members only. Actuaries would, he was sure, be pleased if all friendly societies could be induced to give up their sick pay benefit at the age of 60 or 65, and then give a permanent pay of some kind or other. This would assist them in their work, and, in addition, actuaries would be able to speak with much more authority as to the stability of a society. He hoped to see the day when the large affiliated orders and some of the larger independent societies would co-operate, and undertake that in future members should not receive sick pay after a certain age—60 or 65, with the deferred annuity clause. If the friendly society system of this country was ever to receive State aid, the form he should like to see it take would be that when members of friendly societies (financially and otherwise well conducted) were out of employment for weeks, or perhaps months, and could not find the money to keep up their contributions, State aid might be introduced. Some years ago, when the Mansion House Relief Fund was inaugurated, he was hon. secretary to a local branch of the Charity Organisation Society. The money was left to his administration, and when the trouble had been tided over he had a very considerable surplus. He obtained the consent of his committee to appropriate that surplus in paying the arrears of friendly society contributions of all the applicants for the Mansion House Relief Fund in the district. In carrying out that work he

learnt that many of the societies were apparently only too glad to get rid of their members. He also learnt that many of the members who applied to the Relief Fund were very heavily insured, a number of them being insured in two or three societies. One was insured in no less than five; in fact, he would be much better off financially when he was ill than when he was well. He could not find an exact definition of invalidity given anywhere in the paper, and that was a very important matter, with regard to the German Government, for their own practical financial purposes. His experience was that there was very little difference in practice in many cases between the phrase "got no work to do" and "want no work to do."

Mr. Ralph P. Hardy said that while they all must regret, they must at the same time fully appreciate, the reasons which compelled Mr. Sutton to forbear from giving his views upon a subject which was peculiarly his own. This threw upon him (Mr. Hardy) a larger responsibility than he had contemplated, for he relied upon the matter being fully discussed by Mr. Sutton. After carefully studying the paper as a referee, he had come to the conclusion that it was one eminently worthy of being submitted to the Institute, and that their thanks were due to Mr. Young for putting together in a convenient and acceptable form all the main facts that bore on this important social question. They were to be congratulated that the paper indicated to some extent a reversion to an old practice of the Institute. Its founders never considered that they were to meet for the purpose of discussing such small problems as could be best solved in their studies, but they believed that the training and study of the actuary generally made him peculiarly well fitted to approach, from the larger point of view, some of those legal and social questions which must from time to time arise. Mr. Young had succeeded in giving, in not too strong terms, a most instructive object-lesson. He had pointed out what must inevitably take place in the endeavour to re-arrange society by drastic methods of legislation, instead of allowing it to develop itself by the ordinary process of evolution. He did not sketch too strongly the evils of that vast incubus of officialism which would both oppress and depress society, nor the eternally irritating regulations to which they would all be subject, such as not being able to change their address without giving some sort of notice to the police, and as they knew the working classes change their address on an average once a year, they might see what that would involve. Again, on every occasion when a claim was made, let them look at the trouble and expense of identifying the claimant, and the delay that must necessarily take place between the delivery of the claim and its satisfaction. What the unfortunate invalid on a sick bed was to do in the meantime he did not know. The scheme would leave untouched a vast area of distress, which was to a large extent untouchable, except by private charity. In considering the principles upon which these schemes rested, they must enquire by what they were promoted. Were they founded upon any religious impression? Did they represent the tardy recognition of the principle of the universal brotherhood of mankind, or did they accept the doctrine that their private means and wealth were held upon stewardship? If these were the views that actuated the founders, he would say that these schemes should and

would command their universal commendation. So far as the German Empire was concerned, they knew that no such high and humane views had ever entered into the mind of Bismarck. Neither did they form any expression of the practical code of ethics that governed the conduct of the men of physical force who now held in their iron grip the destinies of that highly-educated German people. They knew that it was terror and political terror alone that had induced the Government to put forward this sop, for it was that and nothing more. As the author had properly pointed out, the people had been demoralised by an outrageous conscription. Endeavours had been made to conciliate small interests either by bounties on exports or protective tariffs, but at last they were faced with the red spectre, and this was one effort, and it would be an ineffectual effort, to put that down. In this country there were not wanting gentlemen who were anxious, and had made great efforts, to bring the question practically before the public, and he desired, in any observations he made, in no way to reflect upon Canon Blackley, who they all knew was actuated, not only by the purest motives, but with a desire to exercise the duties pertaining to his sacred office. It was with deep regret that he, after full consideration, had to differ from Canon Blackley as to the means he proposed to adopt, but he was more than with the Canon in his desire to alleviate any distress to which their fellow human creatures were subject. Unfortunately, in this country the question became tacked on to the political wire-pullers, who were on the look-out for a cry to go to the electors. He should not recommend any Government to put forward any proposition muzzling the people of this country with the principle of State aid without having very seriously counted the inevitable cost. Supposing they allowed their friend Canon Blackley to have his own way, for he only wanted a modification of what was now at work in Germany, what would be the result? Were they going to make men better citizens, for that was one of the great objects of all legislation? Personal character was the result of an evolution which had been carried forward for generations through personal effort, and nothing else would alter the character of a man, or teach him truly to believe that, as a reasonable being, he must make provision for his inevitable need; and any plan that made a jump to avoid this necessary evolution was destined to failure. Before beginning to talk of compulsion, he should like to ask any gentleman present where it was that private enterprise had failed. Private enterprise had produced innumerable investment societies suitable for the collection and investment of small savings; it had studded the country with building societies—they could scarcely go into a small town where there were not several at work. It had created the most unique system of friendly societies that the civilised world had ever seen, and those societies were the legitimate historical successors of the ancient guilds, which could at least be traced to Saxon times, and he believed very much earlier. Had it not also called into being those large associations which were called "orders"? Had it not created some of those societies which Mr. Sutton had referred to, and which might be described as models of organisation and administration? He had often been present at meetings of the governing body of one of those institutions, and the

order, the regularity, the common sense and wisdom of those debates were not exceeded in Parliament itself. Had this been effected by State aid? Had any Prime Minister or any Order in Council called the Hearts of Oak Society into being, and provided the people with a cut-and-dried institution? No, it grew; it had its roots in the common sense of the people. It would expand, as it was expanding. He might refer to an observation made by Mr. Sutton, that he would like to see some Poor-Law aid towards maintaining the contributions of members while sick. Several would be pleased to learn that, left to themselves, the members of that body had, as an *imperium in imperio*, initiated such an arrangement; that was to say, the moment the want became recognised, the unstimulated action of the people supplied it, and that was how it would always be. Each further development of society, as soon as it was recognised thoroughly, would be satisfied, and that as the result of private enterprise, and not from any of those odious State-aid schemes. He did not propose to discuss at any length the details of the scheme. He was aware that the regulations must be very minute, and he had no quarrel with them, but he would say a word as to the scientific basis upon which they professed to rest. That basis was purely imaginative. The tables that were put forward were evolved from the German inner consciousness. They must do justice to their actuarial friends in Germany: those actuaries were extremely thorough men. They had done their best with the scanty material at their command, but the scheme had been ordered by the German Parliament, and it had been their duty to find some means of starting it. He did not quarrel with them, but let no gentleman imagine that the tables in question were any contribution to knowledge—in fact, they darkened knowledge. The scheme could not completely reduce distress. It did not apply to all classes. There were a large number of persons who drifted down who did not commence life as artizans. Then it did not deal at all with the great mass that General Booth called the residuum, and they were the dangerous classes of society. Those were the persons who must be dealt with if they wished to keep the body politic in good humour. The allowances were utterly insufficient for the real needs of the people, and he doubted if they would do much more than pay rent; they certainly would not present a man with any moderate living of a substantial character. He would, therefore, to sum up, say, that while they were all pleased to have the opportunity of considering this scheme, no State meddling with the private arrangements of the people was required, so far as this country was concerned. Private enterprise had followed the course of civilisation, which had brought the country to its present unique position. Private enterprise, if let alone, would carry the people still further, until they arrived at that ideal day when they would live in the perfect liberty which was the object of all their aspirations.

Sir JULIUS VOGEL said that he did not think there was any general knowledge in England at all to approach that which would be the result of the exhaustive and analytical investigation which had been placed before them. He agreed very much with the author that the measure, as they might fully understand it, now that its complex details were explained, was not suitable at any rate

to this country. The friction of the many wheels within wheels which had been revealed to them must raise a very strong conviction that even were the measure desirable it would not work satisfactorily, but he did not think it was desirable, and for this reason:—His sympathies were largely with the labouring classes, and he believed that the measure was framed more in the interests of the employers than of the employed. Indeed, it seemed to him that this could be proved in very few words in an undeniable manner. If the Government had a system of an enormous kind dependent upon the continuity of work of the individual labourers, did it not follow that they were founding an excuse for the Government to insist upon labourers working, whether they desired to do so or not; in other words, to prevent strikes, not by such legislation as they might more or less desire—the legislation which would bring the employers and employed into conciliatory relations—but in the Government saying: "Your "ceasing to work injures a large Government system and a large "Government, therefore you must be compelled to work." That brought them back to the early part of the century, when it was illegal for labour to combine to secure larger wages. He was certain that there were very few persons in this century who would broadly advocate a power being given to the Government to legislate in this direction, and yet they must see that if the State was interested, as it would be if this fund were accumulated, in preventing the labourer from discontinuing industrial employment, then they gave the State a handle: they would drive in the thin end of the wedge by which the State would assert its right to compel labourers to work and to stop strikes. He feared to encroach upon their patience to explain, even in a rudimentary manner, his opinions upon this subject, but he would say that in his opinion the labouring classes had made enormous advances during the last few years: they had essentially done this with the assistance of the unionist system, and that system was yet in its infancy. The labouring classes were beginning to realise that labour was dormant capital, as readily converted into capital as heat into motion. When the unions reared up—as they would—managers, organisers, financiers, it would be found that the labouring classes would have the charge of a great many of the large industrial institutions of the country. Capitalists, as capitalists, would supply capital, receiving a minimum rate of interest and a contingent share in the profits. For the personal assistance they rendered they would receive a separate remuneration. That, he believed, was the goal which they were approaching. With the power which these unions possessed, was it likely that they would give an excuse to the Government to legislate in the direction he had already mentioned. He believed, even now, that were the unions fortunate enough to have amongst their managers competent members of the Institute of Actuaries, who would show them what enormous powers of capital they possessed, their progress would be greater than it was. He thought they were approaching to a system in which the labouring classes and the employers would alike see that it was desirable to repress those disastrous movements of which they had lately heard so much—movements of strikes. He was not one of those who believed that the progress of the unions meant in any sense anarchy

or revolution: he believed their conservative instincts were inseparably connected with the rise over power and influence and wealth, and those who might look upon the matter from the individual point of view of particular businesses, might rest quite content with what would be worked out in a more or less speedy and natural manner. He would conclude by saying that the researches made by the author were equally interesting, whether one approved of the measure or not.

Canon BLACKLEY said that they were there to discuss the German system. He did not believe that the German system was actuarially sound, although of that, not being an actuary, he was not a judge, nor did he think it wise from a political point of view. He had opposed Prince Bismarck's proposal for sick pay, which also implied a Government subsidy, in an article in the *Contemporary Review*. He held that the subsidy proposed would have the result of making every member of that insurance scheme a compulsory pauper. He made a strong objection to the clamour for State aid. There had been some reference to a proposal of the same sort in England; but he never had asked for State aid as far as money went, but the State might, he believed, without a single farthing of expense, give them unalienable security for their money. A State subsidy was absolutely distinct from State establishment and guarantee, and they might have the enormous boon that the investment of the savings of the poor might be secured without any cost to the State, and in a way absolutely inconceivable at the present time. Mr. Sutton was chiefly concerned in supervising the affairs of their friendly societies, but the people who were concerned with one particular branch of social questions of this sort were very apt to forget that there were more branches in existence than one. Supposing, for argument sake, that every lodge of Oddfellows and Foresters was perfectly solvent at the present time—and he believed it was common knowledge that they were not—those societies, together with some others, embraced only about two millions out of a population of forty millions. In spite of the great work that had been done by their friendly societies and by industrial insurance societies, it must be remembered that in this great prosperous nation they had to deplore the fact that 45 per-cent of all the population that reached 60 years of age died in a condition of pauperism. Therefore, if they could devise, as they had attempted to devise in Germany, a system which should prevent this miserable want of nearly one half of the population, and certainly one half of their working population, they ought to be thoroughly earnest in doing so. Mr. Hardy had told them something about the work done by societies, which he said grew by man's own work. He (Canon Blackley) had no objection to them. He honoured highly this work of independent men, but there were a number of members of the great friendly societies who, in spite of themselves, became paupers. Mr. Hardy said that the German scheme did not apply to all classes, and that it left out the residuum: but a scheme providing that everybody should have a pension in old age would provide for the residuum, and it was for them that he pleaded. He (Canon Blackley) did not think they should give everybody a pension, they

should make a person provide it for himself. He would prophesy that before a great many years were over there would be a scheme established in this country. He looked forward to a time, it might be a generation or two beyond them, when independent-minded working men in England might feel during their working lives that they would not become paupers in the end, that no change in their own mind, no change of membership, no breach of contract, should deprive them of a little rest from hard work in their old age.

Mr. M. N. Adler said that the English knew the value of self-help, and our institutions were allowed gradual growth. The German, on the other hand, was accustomed to rely on State initiative and upon official control. Hence it followed that in England compulsory insurance schemes administered by the State were not likely to flourish. In drawing general conclusions, he thought they should not view the subject of the old age pension scheme by itself, but should take, in connection with it, the other two schemes, the sickness insurance and the accident insurance. From what he could learn as to the sentiment with which these three institutions were regarded in Germany, he found that the sickness insurance, including, as it did, medical attendance, medicine, and medical appliances, was looked upon by the general population with a good deal of favour. He might say, parenthetically, that when the sickness insurance scheme was introduced in 1883, it took over the sickness benefit societies which were then existing. As regards miners, more especially, there were organised friendly societies existing in Germany for many years. All those were made part of this grand sickness benefit scheme. Similarly the old age and invalidity scheme commenced this year made necessary the transfer of pension and friendly societies which were already in existence. With regard to the accident insurance, he did not think that it found quite so much favour, although the employers of labour had to pay the piper. The administration, though costly, was not effective. It appeared as if the working people, knowing that they received a fair indemnity in the event of accident, were somewhat less careful in avoiding accident than they were in former days, before the scheme existed. With regard to the scheme that had been so carefully analysed by Mr. Young that evening, it was too early to speak of the manner in which it was appreciated by the German nation at large; but there were two or three features in it which had not been specially adverted to by Mr. Young, but which were important defects in the well-working of the scheme. To begin with, all women that came under that large category of working in a dependent position for regular hire must join the scheme, but as soon as they married, and most likely the great majority of those women would marry, they would lose the benefit of the scheme, only about one half of their contributions being returned. Those women had to save from their wages as much as 6 per-cent, which was about the average deduction from the weekly wages for joining the three schemes. Then, again, the apprentices and agricultural labourers looked forward to becoming eventually masters or small landowners. It was a pull upon their resources to have to pay the 6 per-cent per annum with the view of making provision under the three several branches, and as soon as they became masters or earned £100 they lost the benefit, and only

a trifling amount was returned to them. Again, the provision for widows and children of a capital sum at the death of the bread-winner, had not been dealt with by the German Government. He believed the introduction of a death-pay scheme was very remote at present, but in England that was considered one of the most important things to provide for. Mr. Hardy and Canon Blackley had wished to know what were the motives which inspired the German Government in bringing this Act forward. He might, in reply, refer them to *The Times* of 19 June 1889, which dealt very fully with this question. The article was headed "State Socialism in Germany." To sum up, old age pensions and provision for disablement cannot be enforced in England, the land of the *laissez faire*, and Germany adopted this scheme, originated, as it was, by Bismarck, whose maxim was *laissez moi faire*.

Dr. Hunter, M.P., said that Mr. Young had done good service to all who were desirous of knowing the character of the German scheme. He felt that especially, as one who had struggled through such papers as the Government had provided to enable them to understand that scheme, but it seemed to him that neither Mr. Young nor those who followed him had kept clearly in view the different ways in which the State might assist or interfere with reference to this question of insurance. In the first place, the State might act as an annuity insurance office, to receive money and to pay annuities. The English Government did that already. It was quite true that their annuities had not been generally taken advantage of by the working classes or by any other classes; it nevertheless remained the fact that the function of providing annuity insurance was one which, with the assent of all parties, had long been part of the functions of the State according to English ideas. There was another function of the State, which had been recently recommended by Mr. Chamberlain, who proposed that the Government should give a subsidy, which should take the form of 5 per-cent interest on such sums as working men chose to invest towards their pension, instead of the $2\frac{1}{2}$ per-cent which was the present value of money. The third form of State interference was compulsion, in which the State not only subsidised pensions, but made it compulsory on a certain class of persons in Germany—a class embracing twelve to thirteen millions—to become insured under their scheme. It was obvious that the arguments which might be fairly directed against compulsion would not necessarily be directed against such a scheme as Mr. Chamberlain had shadowed forth, and that, of course, those arguments had no application to the very limited sphere which the Government at present occupied in the field of insurance. Years ago, in addressing his constituents, he spoke upon the desirability of a better system of pensions for old age, by way of insurance, than at present existed, and throughout his remarks, having a holy horror of the word "compulsion", he spoke of something which must be purely voluntary. What was his surprise to find that the very first objection the working men took to that arrangement was that it was not compulsory. Their view was that no scheme of pension, whether arranged by voluntary assistance or by the State, would really meet the necessities of the case, having regard to the circumstances of

working men, unless it was a compulsory scheme. He was not at all competent to express an opinion upon the questions which had been so ably discussed by Mr. Sutton, Mr. Young, and Mr. Hardy. No doubt they were perfectly correct in their views. He meant that, from what he knew of German and of English scientific men, he had such a preference for the latter that he should be disposed to say *a priori* that they were right, and that their German brethren were wrong. But then, of course, the whole question was, how far wrong they might be: although they were not absolutely right, they nevertheless might not have committed so great an error as to involve any serious consequences; but he agreed with what had been said, that, as to that, the future alone could determine. He was glad to hear the remarks made by Mr. Adler, pointing, as they did, to considerations affecting the value of the German scheme in its present form, and pointing out undoubted defects. His impression was that something very different from what they had at present would be necessary, and that this would be in the direction of further interference on the part of the State. His reason for saying so was this: he did not believe, when they came to deal with the question of pensions, that they could have any private society sufficiently strong in its finances to justify a working man in setting aside his money, at the age of 20, with the expectation that 40 or 50 years hence that association would be solvent and able to pay what it promised. Then there was another immense difficulty with regard to private societies, namely, cost of collection; and one of the happy ideas which the Germans had adopted, and which was part of their scheme, was that of the collection of money by means of postage stamps from the employers, which rendered that part of the scheme extremely cheap. They knew by experience that the cost of collection in some societies in this country was a very large part of the income.

Mr. R. P. Hardy—Pension societies?

Dr. Hunter—Friendly societies and insurance societies.

Mr. R. P. Hardy—The cost of collection in friendly societies is not large.

Dr. Hunter said he was speaking of those associations which collected insurance money. Mr. Hardy had pointed out very truly that one serious defect of the German scheme was the inadequacy of the pensions. When they went to any actuary to ask what would be the cost of an adequate pension, they found that the price was so high as to be practically beyond the means of the great bulk of the working classes. That was really the problem which, in some form or another, the State would ultimately have to solve. For his own part, he was at present in a state of being educated: he had no final scheme, but he hoped the Institute of Actuaries would throw light upon the great problem, whether it was possible to obtain from such sources as the State might draw upon, sums to assist the working man in accomplishing the desired result, because by their own unaided efforts it was impossible that they could do so. One of the objections which Mr. Young properly brought against the German system was, that the money contributed by the State was, to a large extent, taken out of the very same pockets that provided the premiums. He

should say it would be a *sine quâ non* of State aid, whatever it be, that the contribution to be given to the working man should be a contribution deliberately made from property, and not from labour; otherwise, the obvious result was, they were taking money from labour, first, directly in the shape of premium, and, secondly, indirectly by the expensive system of indirect taxation. He was glad to observe that there existed on the part of employers in this country a most hearty disposition to contribute. For example, nearly every one of their leading railway companies had obtained powers from Parliament to establish pension schemes, excellent in regard to the sums which they offered to the employés and the smallness of the contributions exacted, and in every case the railway company subscribed one half of the premium. There was a Bill now pending in the House of Commons—he was sorry to say that it would not be read a second time this year—which was brought in by the mine owners of Scotland to create a pension fund for the miners of Scotland. This was a Bill coming solely from capitalists, and, if he recollected aright, they proposed that the premium of insurance for pensions in old age should come, as to one-third from the workman, one-third from the colliery proprietor, and one-third from the gentleman who received the royalties. When the employers of labour and the owners of property came to consider the enormous social value of relieving workmen from the great terror of their lives, the uncertainty of their future, and when they came to ascertain at what cost that could be accomplished, he did not at all despair that the owners of property in this country would voluntarily come forward on a most liberal scale. The problem was essentially one for actuaries in its earlier stages, and not for property owners.

Mr. A. H. Bailey said there were one or two remarks made by Dr. Hunter which required some notice. Having had some experience in pension funds, he might mention something of the results that he had gathered. There seemed to be a perfect craze in this country for forming superannuation schemes, and the difficulty in working them arose from the necessity of obtaining contributions sufficient to provide the pensions; the trouble being that twice two would make four, while they were constantly asked to make twice two five. The question, of course, resolved itself into this, were the contributions to be derived exclusively from the members of the societies, or were they to be subsidised by the State? He gathered from Canon Blackley that he believed that a sufficient deduction could be made from the wages of working men to provide for themselves reasonable provision in old age. He believed this to be quite impracticable. It was quite true, as Dr. Hunter had remarked, that the railway companies had established schemes of superannuation. Mr. Hardy and himself had had experience of some of these which had become into a state of considerable complication. The usual practice was for $2\frac{1}{2}$ per-cent to be deducted from the salaries, the companies contributing an additional $2\frac{1}{2}$ per-cent. The problem presented to actuaries was, what superannuation allowance would that provide to commence between the ages of 60 and 65? Speaking in the roughest possible manner, they had found out that those contributions would yield

about one-third of a man's salary at the time of retirement, and the experience of most of them was that it was very difficult to make one pound do the work of three. Consequently the men did not take the superannuation allowance, but clung on as long as they possibly could to their work, so that the chief object of the fund was not attained, and the result was anything but satisfactory. That was not all. He would not be far wrong in saying, that of 100 men of the age of 20, about 40 died before reaching the age of 60. In these cases there was an outcry made about the deductions from the salary, and it was said that of those 40 per-cent who had died, some provision must be made out of their contributions for their widows and children. That was not all: The men who between 60 and 65 had paid for 30 years or more claimed their pensions. If death occurred in two or three years afterwards, it was urged that, having paid so much more than they had received, some provision must be made for their widows and children out of the surplus, and the rules were continually being altered in those directions. The effect was, that the contributions resolved themselves very much into savings' bank deposits—that was to say, every man expected to have what he had contributed, with compound interest, which upset actuarial estimates and did not bring about the results that were expected, which seemed to be, that every man's contribution would be sufficient to provide, not only for himself in old age, but also for his family at his death. Most of those present had found that deferred annuities formed a very small proportion of insurance business, and very properly so. The man who had a wife and children dependent upon him had many things to do before providing for himself in old age, and it was much better to educate his children and endeavour to put them forward in the world, than to make such a provision for himself. It was not too much, as a matter of Christianity, to say that, if fathers and mothers had provided for their children and put them forward in life, in their own old age they might expect to be provided for by their children. There was an interesting article in the current number of the *Contemporary Review* advocating an eleemosynary provision by the State for old age, which is a different view. He should rather like to know what authority there was for Canon Blackley's statement that 45 per-cent of the inhabitants of the country were, after the age of 60, claimants upon the poor rate. If it were so, he was sorry to hear it, but the contention of the article in the *Contemporary* was that, instead of being provided for from the poor rate, they should be provided for from the public exchequer. In either case the provision was to be made by others, not by the parties themselves.

Mr. George King wished to say one word as to the position which such a body as the Institute of Actuaries should, in his opinion, take with reference to these questions. He did not think they should form any foregone conclusion, but should keep their minds open to be ready for eventualities. They must all agree with what Mr. Hardy said as to the great benefit that the country had derived from allowing evolution to have its full effect, and no one would for one moment call in question the immense benefit the nation had derived from the gradual development of friendly societies and other institutions fostering thrift. But it was one thing for an iron chancellor with an

army at his back to impose a scheme upon a nation, and another thing for a democratic people to evolve these methods of thrift, and in the fulness of time to pass on to the Government the necessary arrangements for carrying out what the nation required. It therefore seemed to him quite possible that, without in any way disturbing the evolution of which they were all so proud, and of which Mr. Hardy had spoken so eloquently, a democratic government might see its way to take up these questions and work them out to greater perfection and with greater usefulness towards the community than mere individual societies could do. In former times education was left entirely to private enterprise, and the nation became great, powerful, and renowned. Some years ago, by universal consent, elementary education passed into the hands of the government, and was made compulsory, and now they had seen a Conservative Government and a Conservative Chancellor of the Exchequer making it also free. At one time that would have been thought socialistic to the last degree, but they did not seem to think so now. He therefore could not help imagining that some day it might come about that the most convenient and the best way for the populace to have pensions and provisions for old age would be to let the Government work the scheme, not forcing it on the people, but working it for the people and by the people. It seemed to him, therefore, that they should not come to any foregone conclusion whilst criticising particular schemes, and that even if they said that the German scheme was not good, they should keep their minds open and allow their unbiassed judgment to be passed on any scheme that came before them, and not simply say that everything of the kind, carried out by means of Government machinery, was of itself an evil.

Mr. John Lloyd, L.C.C., said he wished to call attention to a scheme proposed in 1772. No doubt members of the Institute of Actuaries remembered the names of Baron Massiéres, and Dr. Pryce, the founder of the Equitable Society, and Mr. Burke. Those men were the means of bringing into Parliament a Bill in that year, 1772, which provided for giving pensions to all old people after their frugal and industrious lives. That scheme passed the House of Commons, and when it went to the House of Lords from some cause or other it failed. The fact that in 1772 legislation was proposed was a fact which should be well known, and studied as a precedent which they might follow in some shape or another to-day. Two millions of money would now be placed in the pockets of the working classes by the remission of school fees. They might very well call upon them to set a considerable portion of that towards providing pensions for themselves in their old age. With regard to statistics of pauperism, he could take them to workhouses in England and show them within those workhouses, as well as outside, in the receipt of relief, 245,647 paupers over 65 years of age, and it was that great army of paupers that they wanted to attack. It was stated that the country was studded with building societies and so on. Yes, but this country alone in the world was studded with workhouses. They wanted to give the people a home and independence, free from the curse of pauperism. To that end he hoped the Institute of Actuaries, instead of throwing cold water on

schemes of that sort, would endeavour to help any such scheme forward.

Mr. W. J. H. Whittall said the discussion seemed to have proceeded on such very broad lines that they had almost lost sight of the actuarial side of the paper. He wished to ask one or two questions. The author had said that the State subsidy was always a fixed sum of £2. 10*s*., added to the allowance; but in another part of the paper it was represented as being one-third of the contribution. Then, again, in deciding how the scale of contributions was to be settled for the first contributory period of 10 years, he could not make out in the description of the various causes for which the contribution was required, any provision for a reserve fund for claims which were to come on to the fund after the expiration of the first contributory period. It seemed, in reading the paper, that the scale of contributions was much lower at the outset than it would ultimately be, and as those who were paying the low scale would be the first to come on the fund, it appeared—somewhat paradoxically—to be the case that, in many instances, those who paid the smallest contribution would get the greatest benefit. With regard to the amount of old age allowance, they were told that it was to be according to an ascending scale, which was clearly explained, but later on in the paper the initial old age allowance was referred to as being 24 per-cent of the wages. Perhaps that might be capable of further explanation. Then, again, Mr. Young had shown how very enormously the Government subsidy would increase as years went on. Could he state whether any arrangement was made by the State for starting a sinking fund towards meeting this very large charge in years to come, or whether they were going to leave it as a charge upon posterity? He wished to ask, having regard to the interesting nature of the subject from the actuarial point of view, whether further particulars could not be given of the tables of mortality and invalidity on which the calculations were based. It might be worth while to print the tables *in extenso*, so that at least they might see one set of the data on which the calculations were based.

The President, in asking the Institute to pass a cordial vote of thanks to Mr. Young, said that he was personally specially indebted to the author for having, at his instigation, undertaken this work. In asking Mr. Young to write the paper he was desirous of realising, as far as might be, one of the ideas to which he gave expression in his presidential address, and which had been referred to by Mr. Hardy, namely, that by services of this kind the Institute would successfully assert its claim to a larger public recognition. The good service which Mr. Young had rendered was sufficiently obvious. The mere collection of the facts in the authentic and extended form in which they had been presented, now for the first time taking an English garb, and so being ready to the hands of the political and economic student, was in itself a great service. Their orderly arrangement in logical sequence was a still greater service. These facts would enable them to understand, and, it might be, to master, the many problems needing solution before practical Englishmen could be asked to embark on an enterprise similar to that in which the Germans, with, apparently, so light a heart, had embarked. The paper demonstrated with tolerable

certainty the impossibility of this particular scheme, at all events, ever being adopted in the free air of England. They had done much of late, and it was quite possible that under democratic domination they might do still more hereafter to restrict individual freedom, but not till Englishmen ceased to be Englishmen would they submit, under compulsion, to a system which brought in its train such irritating supervision and interference as might be possible to a people steeped to the eyes in militarism, and used from their birth to a system of State control so persistent and universal that a youth might scarcely grow into manhood without bureaucratic permission. He had arrived at that conclusion with great reluctance. Though he never sought to act the part of Balaam, he should have been glad, like the author, if he had felt constrained to bless this particular scheme, for he had to the abstract idea of compulsion no necessary repugnance. But the importance of compulsion consisted in its application, and he agreed with the author that compulsion after this fashion was for Englishmen an impossibility. If they were to have compulsion it must apply only to a limited period, to the tractable stages of life, and he was not without hope that the future might be more pregnant of a practical scheme than the past had yet proved to be. He agreed with Mr. Hardy to the full in reference to the natural doctrine of evolution. No one recognised more fully than himself the way in which the manliness of Englishmen had provided in a large measure for their own needs, but he could not also forget that, much as had been done, there was a great problem still unsolved He did not care to quarrel with Mr. Bailey in criticising Canon Blackley in regard to the proportion of persons who died paupers, but it was within the knowledge of them all that a very large proportion of the population most indubitably did die paupers. Was that a state of things which was never to have a remedy? Coming back for a moment to the paper, it was pretty clear that the Germans looked at the position from a point of view totally different from that of Englishmen. Prince Bismarck had apparently two purposes. He desired, first of all, to fight socialism. If Englishmen's ideas had any solid foundation, they rested, he believed, on the recognition of Christian socialism, if he might use the expression, which had had its outcome in the clearer recognition in the present day of a common humanity, and a truer view of the relation of man to man. Again, Prince Bismarck, he believed, desired to destroy the capitalist. It had been said that the scheme was in the interests of the employer. That might possibly be in one sense, but on the whole Mr. Young accurately represented the position when he said "Bismarck has also "frequently expressed the hope that the State should exclusively "possess a monopoly of the insurance system in its entirety, so that, "as he phrased it, capitalists might not, in opposition to reason, "conduct insurance companies for their own benefit at the expense of "the community." We in this country were more tolerant, and probably wiser. By way of contrast with that feeling on the part of Prince Bismarck, let them look at one great industrial organisation that flourished near that hall, an undertaking—a sort of *imperium in imperio*—so vast that it would have provoked State

jealousy in Germany, and probably have led Prince Bismarck to destroy it. Though lying ready to the hand of any ambitious or unscrupulous statesman who desired to create a State department, he did not think that in England that institution need have to tremble on that account for its own safety. Mr. Bailey had referred to a paper in the *Contemporary Review*, written by Mr. Ede. If he might venture to say so without disrespect, and if he was not wrong in his criticism of some of its figures, it was a very fallacious paper in some respects. It stated, perfectly truly, that the cost of an immediate annuity, to be granted by the Post Office, of 5*s*. a week or £13 a year, would be at age 65 about £125 or £126. It then gave a table of the payments at various ages, beginning at 15 and going up to 50, for a deferred annuity to commence at the age of 65. Unless he (the President) quite misunderstood that table, the whole of the figures given were wrong; but, without further discussing the accuracy of the table, he might point out what was the fact, that at age 25 the Post Office would require, for a deferred annuity to commence at 65, £20. 11*s*. 8*d*., instead of £125 or £126, which was the cost of an immediate annuity at 65. He referred to that for the purpose of making a criticism on another project, propounded by one who certainly was not defective in clearness of vision, and that was the right hon. gentleman the member for one of the divisions of Birmingham, in which he put forward as a panacea that the State should not ostensibly, by the payment down of so much money, aid those who would be thrifty, but should aid them by giving them a more liberal allowance of interest, 5 per-cent being the rate he named. If they looked to see what would be required from a man of 25, to provide £125, the cost of an immediate annuity of 5*s*. a week, commencing at 65—*i.e.*, after an interval of 40 years—it needed only about £8 if interest were reckoned at 5 per-cent; so that in reality the contribution of the State, under the form of interest, would be the difference between £20. 11*s*. 8*d*., which was the rate the Post Office actually charged, calculated at 2½ per-cent, and the £8 which alone would be needed if interest were reckoned at 5 per-cent. He was not quarrelling altogether with the suggestion that that rate of interest should be allowed, but he should like to point out that it would create an increasing burden on the State of a very considerable amount. For example, if they invested £1 at interest for 40 years at 2½ per-cent, it would amount to about £2. 13*s*., if at 5 per-cent to £7, and the difference, being about £4. 7*s*., represented the amount contributed by the State; but that was not quite to the point, because much, of course, of that amount would be the natural product of its own accumulations. What he wanted particularly to point out was, that as each year the amount to be invested became larger, in the later years of the series, instead of the simple extra 2½ per-cent on the original deposit, which in the case of £1 deposit would be 6*d*., it would grow till, in the final year, it would be 3*s*. or 3*s*. 6*d*.—*i.e.*, 6 or even 7 times the original amount. That was a burden of a very serious character, and none the less serious because it was so insidious. It was quite possible that that difficulty might be avoided by payment by the State of a fixed annuity over the period of equal amount to

the sum represented by the suggested allowance of £5 per-cent interest, but all the schemes required to be very closely sifted. He was not without hope that the skill of that Institute, and the public spirit of the people, might some day, though it might not be in the immediate present, produce a scheme to which no serious objection could be made. Clearly the basis of it must be self-reliance and self-aid in the largest sense. The problem of how to deal with the undoubted residuum of pauperism was the outcome of many generations: they could not hope to solve it in a day. It was a function of evolution—it was an equal function of that process which sought to work back to an ancient type—that the work must be done slowly, through long ages. They were seeking to work back to an ancient idea, he would not call it an ancient type. It was an ancient idea of the grandest kind, though it could never be realised, of having all things in common. But there was a point at which they might aim, and which they might hope to reach, and the point was, that they should be able, by giving State aid at the beginning instead of the end of life, to put the people, after having lived an honest and laborious life, in the way of retaining what should be a common heritage of self-respect and self-dependence till the end of their days.

Mr. Young said at that late hour he would not do more than thank the members very cordially for the extremely kind manner in which they had received his paper.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

Some Notes on Laws affecting Policies of Life Assurance. By WILLIAM KENT LEMON, F.I.A.

[Read before the Institute, 23 February 1891.]

IN response to the suggestion of our President, in his inaugural address for the present session, I commenced to put together some notes which seemed suitable for a paper to be read before one of the provincial institutes. In this I proposed to deal with some practical legal questions which are encountered by officials engaged in various positions outside the head offices of life insurance companies.

Mentioning my object to one of the leading and much-consulted members of the Council, for the purpose of obtaining certain information, I was met with the advice to offer the result of my work, in the first instance, to this Institute. I was further influenced in this direction by the recollection of the emphatic desire reiterated by our President, in the words "we want more popular papers." Amongst the semi-professional subjects which he suggested might form the bases of papers of interest and value, I found mentioned "the numerous miscellaneous questions of law * * * * which so nearly touch the interests of assurance companies." With some of these questions I now propose to deal.

I recognize and experience very fully the feelings of diffidence that have been expressed more or less audibly by authors and speakers on the different occasions on which legal subjects have been brought under the consideration of the meetings of this Institute. Law seems by its nature a subject little fitted for discussion by minds not trained in its subtleties, or before a body constituted as we are. I remember, however, when joining the Institute, to have asked one of its members "What are the qualities required for an actuary"? The reply, from an eminently practical man, was: "The perfect practical and "theoretical actuary would be a combination of the Lord "Chancellor, the Senior Wrangler, and the President of the "College of Physicians." Such, perhaps, is the ideal—lofty and unattainable—but there can be no room to doubt that some knowledge of law is an absolute necessity for all, even purely theoretical, actuaries. My own short experience has impressed upon me the great importance of the effect upon the actuary's work of certain of the laws of this country, and the friction to which, without skilful handling, such laws give rise between the officials of insurance companies and the public. On these grounds, I am emboldened to ask your indulgence whilst touching upon some of these matters from the point of view rather of the actuary or official of an insurance company than of its legal adviser.

My object will be, not so much to give my opinion as to what the law is, or should be, but rather to collate the opinions of legal advisers on some points of difficulty which have come under my personal notice, and some of which are not fully treated, or even referred to, in the text books on the laws of life assurance.

This paper will take more the form of excerpts from a common-place book than a systematic discussion of the law of life assurance, and I do not anticipate that I shall be able to place before you anything strictly new or original. Indeed, it may be admitted that pure originality in law is undesirable, if not dangerous. If I shall be found to have accurately put together, in a succinct form, ideas or decisions upon actuarial or business questions, their expression without technical legal phraseology may, I hope, be useful as tending to provoke discussion. On some of the subjects a somewhat full discussion in the first instance, followed by united action in practice, would seem highly desirable.

Take first the question of INSURABLE INTEREST. This presents some anomalies. The Act of 14 Geo. III, chap. 48, known as the Gambling Act, forbids any insurance to be made on the life of any person or on any other event wherein the person for whose use and benefit such policy shall be made shall have no interest, and every such insurance, if made, shall be void, and it further provides that if such policy is made no greater sum shall be recovered than the value of the interest of the assured in such life. The Act requires that the name of the person interested in the proceeds of the policy shall be inserted therein, and it applies to Scotland, and to all policies issued in Ireland after 1 November 1866. It has been decided that the insurable interest here referred to is pecuniary.

The question of what constitutes an insurable interest would seem to be determined most readily by regarding all such policies at the date of issue as contracts of indemnity. It must always be borne in mind that though for many years it was held that, like every other insurance valid in law, life insurance was essentially a contract of indemnity, it was in 1854 unanimously decided by six judges in the Exchequer Chamber that a life policy was not to be considered a contract of indemnity merely. (Dalby *v.* India and London Life Assurance Company, 15 C.B., 365). The Master of the Rolls (Sir G. Jessel) gave the following definition of life assurance in his judgment in Fryer *v.* Morland (3 Ch. D., page 685). "It is a purchase of a reversionary sum "in consideration of a present payment of money, or, as is "generally the case, on the payment of an annuity during the "life of the person insuring."

If a digression may be allowed here it may be pointed out that Sir G. Jessel used the phrase "person insuring" to denote the person whose life is assured; not, as the words would appear to imply, the person granting the insurance. It seems strange, that though the difficulty of denoting the various parties to an insurance has so long been felt and so frequently referred to, we are no nearer a settlement. The Institute has settled a notation known and accepted amongst its members, and it might, one would think, also settle this point. Mr. Curtis (*J.I.A.*, xix, 434) suggested "assurer" for the company granting the assurance or insurance, "assuree" for the person in whose favour it was granted, and "assured" for the person whose life was the subject of the contract.

Further—*In*surance may be taken as applicable to a contract

of indemnity where the sum insured is not certainly payable, as in the case of fire insurance, marine insurance, or insurance against issue or death of A before B, and similar cases. *As*surance would then refer to contracts where the sum assured is certainly payable when the contract matures, thus we have a whole-life assurance or an endowment assurance.

An exception to the general rule of insurable interest is the case of a wife assuring her husband's life. The power to do this appears to have existed prior to the Married Women's Property Acts, the point having been raised and decided in the case of Cook *v*. Royal Exchange (Peake's *Additional Cases,* 1870), and seems also to have been recognized in the Income Tax Acts, 16 and 17 Vict., c. 34, s. 54.

The Married Women's Property Acts not only give a statutory power enabling a married woman to effect a policy on her husband's life, but go further by permitting her to assure her own life for the benefit of her husband. Difference of opinion has arisen as to the construction of this enabling clause. It has been contended that it practically enables a husband to insure the life of his wife, inasmuch as it gives the wife power to insure her life for the benefit of her husband without making it obligatory that before doing so she should have a separate estate, out of which to pay the premiums, thus leaving it open for the husband to supply the necessary funds.

In the discussion on Mr. Sprague's paper and Mr. Bunyon's note (*J.I.A.*, xxiv, 77, 82) on the grant of settlement policies under the Married Women's Property Acts, 1870 and 1882, reference was made to this subject, and Mr. Bunyon seemed to be of opinion that if the wife had no separate income such an assurance as that of a wife on her own life for the benefit of her husband was not valid. Although Mr. Bunyon expressed a hope that the question might be decided by a court of law, there has not, so far as I am aware, been any decision thereon.

One other point of interest in this portion of the subject is the right of a debtor to assure the life of his creditor, and the amount for which such insurance can legally be effected.

Where, as in a recent case that came under my notice, A lent B a sum of money to enable B to start in business, and it was agreed that A would not ask for repayment of his capital for a period of 10 years, but that at A's death the loan was repayable; it appeared that a simple promise not to ask for repayment was insufficient to give B an insurable interest in the life of A (*vide*

Hebdon *v.* West, 32 L.J., Q.B., 85), but that a consideration for such promise must be clearly shown and the promise made binding on A.

The general statement of the rule, however, seems to govern and include, as a valid policy, the case of an assurance entered into for the protection of an intending lender who had contracted to lend a sum of money, even though the money had not been actually lent, owing to the death of the proposed borrower before completion of the loan. A creditor may assure the life of the debtor to the extent of the indebtedness, but not of indebtedness and premiums for insurance.

In practice it has, I believe, been usual to accept, as binding on all parties, the mere statement of the person desiring to effect a policy on the life of another, as to the existence of an insurable interest. To this there seems no objection, since the onus of proof of interest, at the date the policy was effected, rests upon the claimant and not the office. A statutory declaration setting out the nature of the pecuniary interest has, however, on occasions, been required by an office granting the assurance. With such a precaution as that just referred to, there would seem to be no objection to mark the policy "Interest admitted", or to insert a clause in the policy stating that the proof of interest has been accepted by the company as satisfactory. This custom prevails in marine insurance in the case of honor policies, and would have the effect of removing one ground of dispute when the payment of the sum assured became due. We must recognize as a fact, that any action against a company on a policy, however necessary it may be to defend it, and whatever its result, is damaging to business.

Turning to the question of ASSIGNMENTS, it is remarkable how soon after its issue a policy becomes the subject of assignment, or some other dealing, either by way of settlement on marriage, by mortgage, or other proceeding. Doubtless the experience of different offices varies largely as to this, and such variation is probably due to the class of business transacted by the various offices.

The transfer of a policy from one person to another takes many forms, from the simple but absolute assignment in the form given in the schedule to the Policies of Assurance Act, 1867, to the settlement on trust effected by some impressive looking deed, a monument of the conveyancer's skill, and containing clauses to meet a great variety of contingencies, and winding up

with exonerating the trustees from keeping the policy in force, with or without a covenant binding the assured to do so.

A policy of assurance is assignable, and such assignment, if carried out by anyone conversant with even the elementary law on the subject, is probably as simple, plain, and straightforward a piece of legal work as can be found. Experience, however, shows that in so simple a matter blunders often arise. To prevent the repetition and recurrence of troubles from this source—so well known to the managers of life offices—the public must learn, though it is slow to learn, that the cheapest method of procedure is, to adopt the form given in the Schedule to the Policies of Assurance Act (30 & 31 Vict., c. 144), where such form is clearly applicable, but in all other cases to consult a lawyer. Quite recently, a City business man brought me a policy for a considerable amount, on which he proposed to place an endorsement assigning it to a friend as security for a temporary advance. He had found on his policy of fire insurance an endorsement:

> "It is hereby declared that the interest of the within-named A. B. in this policy is transferred to C. D.
>
> "Registered in the books of the company, &c."

This endorsement it was proposed to copy, and the only question that arose in the minds of the parties interested was as to registration in the company's books. It never occurred to them that such a document was not in any sense a transfer or assignment, but a declaration of something done outside itself, and that it had no operative power, not being signed by either party. Cases of equally informal documents are by no means rare.

A covenant to effect a policy by way of security will not of itself operate so as effectually to vest the policy in the person entitled to claim the benefit of the covenant, and it would seem not even to create an equitable assignment of the policy, or give such person a lien thereon.

From the decisions of the Courts, conflicting as they no doubt are, and failing to disclose even to a careful student any fixed principle, we may yet assume we are not far wrong in coming to the conclusion that almost any written memorandum, signed by the person entitled to the benefit of the policy, and showing on its face some valuable consideration, would operate so as to convey the policy and the money payable thereunder.

Now, since the passing of "The Customs and Inland Revenue Act, 1888", the stamp on such a writing has become a necessary

element to make the assignment valid. This will be dealt with later on.

Such an endorsement as that copied from the fire policy above referred to might, if stamped and accompanied by letters or other documents showing the intention of the parties, be held to be good as an equitable assignment or mortgage, according to the nature of the evidence showing the intention of the parties.

Coming now to VERBAL ASSIGNMENTS, we find principally two classes of such assignments which call for remark.

An assignment made in anticipation of death is technically called *donatio mortis causâ*. This may or may not be subsequently carried out by deed. If a memorandum or deed of assignment be executed no special difficulty arises with the office, but in the majority of cases there is no subsequent writing—this leaves the question of proof of the donation to be determined. It may be here noticed that although an executor or administrator cannot recover the policy given away by the holder thereof during his lifetime without any written instrument, such holder cannot compel the company assuring to pay him the amount of the policy. As such donations are most frequently made to near relatives, and generally in the presence of members of the family, the difficulties of proof which arise are not formidable, but it is important for the company to insist on evidence, so as to prevent payment to a person not entitled.

To render a *donatio mortis causâ* valid, the following conditions must exist—(1) the gift must be made in view of the donor's death; (2) it must be conditional upon the donor's death from the illness from which he is then suffering; and (3) the subject of the donation must be delivered over.

Mr. Bunyon, in his *Law of Life Assurance*, lays it down that "such a gift differs little from a legacy, except in not requiring "the assent of the executor; it is not complete until the death of "the donor, and is of no avail against creditors in case of a "deficiency of assets, and, by 36 Geo. III, c. 52, s. 7, is subject "to legacy duty." Now by the joint operation of 44 Vict., c. 12, s. 38, and 52 Vict., c. 7, s. 11, such donation, if made less than 12 months prior to the death of the donor, is liable to (account) duty as a legacy.

I observe that neither the liability to duty under the Act of 36 Geo. III, nor the failure of the gift on deficiency of assets, is mentioned in Porter or Crawley on life assurance law, and as to the efficacy of the gift in the event of insufficiency of assets,

I have seen a legal opinion in which the life insurance company was advised that if the declaration setting out the facts of the gift could be made to include a statement that the donor's assets exceeded his debts, or that his debts had been paid, the case would be strengthened, but that under any circumstances the gift was good as against creditors. Whether the gift was subject to legacy duty or not under the Act 36 Geo. III, c. 52, s. 7, there is no doubt that it is now liable to duty. Fortunately for insurance companies, in the case of legacy or account duty no liability to see to its payment rests on the companies.

We proceed to consider the effect of the DEPOSIT of a policy under ordinary conditions, but without any letter or memorandum in writing to show the nature of the deposit or its objects. Such a deposit, even where notice has been given to the insurance company, does not give the person with whom it is deposited power to require payment from the insurance company in the case of the death of the life assured, or to deal in any way with the policy-moneys. It would appear to have the effect only of preventing the depositor, his executors, administrators, or assigns, from so doing.

If a debtor deposit a policy with his creditor by way of security, and at the same time request him in writing to instruct his solicitor to prepare an assignment, this will not amount to an equitable assignment within the provisions of the Policies of Assurance Act, and in such a case written notice to the insurance company would not enable the depositee to sue for or to recover payment from the company and give a receipt for the policy-moneys.

A mere deposit made without notice to the insurance company would not be of any effect as regards the company, and would not enable the depositee upon payment to give a good discharge. Persons giving an indemnity against all claims on an insurance company, in the case of a payment by the company on a policy which has been lost, run little risk, because, if the policy has been deposited and not lost, the claim of the person with whom the deposit was made would be against the person who made the deposit.

It does not appear that in England the actual delivery of a policy to the assignee is necessary to constitute an assignment, though it has been so held in America.

Turning from the case of informal assignment of a policy, a question arises how far it is necessary to make, in a legal form,

the RE-ASSIGNMENT of a policy so informally assigned, and whether any and what informal method will suffice.

The creation and legal operation of a re-assignment do not materially differ from those of assignments, but informal re-assignments are, perhaps, of more frequent occurrence than informal assignments. This may arise because the mortgage or other assignment is more commonly made to a capitalist or to one acquainted with the rules of law and anxious to fully protect himself, while the person who has discharged the obligation to secure which the policy was assigned or deposited is satisfied with getting back his policy.

Nor can we be surprised at this when it is a well-known fact that mortgagors of freeholds and leaseholds have considered that they have done all that is necessary to get back their property, held by a mortgagee under a legal mortgage, if they pay the money due and get a receipt endorsed and receive back the deeds. An idea, possibly traceable to the statutory operation of a receipt by trustees of a building society, which re-vests the property in the persons entitled to the equity of redemption without the execution of any conveyance.

The deposit of a policy, with notice of such deposit to the insurance company, would, in England, amount to an equitable mortgage, and on the depositee claiming the policy-money which had become due, the consent of the depositor or his personal representative or trustee in bankruptcy, as the case may be, would be necessary.

Take one instance of an informal re-assignment—In the case of the deposit of a policy, with notice to the company, a letter signed by the depositee, merely stating that the objects of the deposit had been satisfied and that it was at an end, would be a re-assignment in equity. If such notice were duly given to the company, this would, so far as the deposit alone is concerned, appear to rehabilitate the title of the depositor, but whether or not the position of the company is affected depends in some measure on the effect of the now constantly-recurring question of the payment of stamp duty on assignments, to be referred to subsequently.

Let us now consider some of the EFFECTS OF ASSIGNMENTS AND RE-ASSIGNMENTS, so far as they relate to the companies and their obligations to discharge the liabilities created by policies. The officials of a company have not usually to concern themselves with the drawing-up of assignments, and even in complicated

cases they have to deal only with the payment of the money, not with its ultimate disposal, provided the power of the trustees or other person to whom the payment is made to give a good and valid discharge be clear, ample, and sufficient.

The form of assignment given in the Schedule to the Policies of Assurance Act, is that of an absolute assignment. Occasionally, however, this form has been used for a mortgage, by stating the consideration as a loan. This would probably be considered a mortgage, as having its inception in a loan and being governed by the old rule of law "once a mortgage always a mortgage." If this be so, the statutory power of sale, &c., conferred by "The Conveyancing and Law of Property Act, 1881" (s. 19), would be imported into such an assignment. This Act gives to mortgagees a power of sale, but an important question affecting the business of life offices has often been asked and differently answered, namely, does a POWER OF SALE include a power of surrender?

Crawley on Life Assurance, page 81, has the following:

> "In view of the decision in Dyson *v.* Morris (1 Ha., 413), an express power of sale, extending to a sale by surrender to the office, should be inserted in a mortgage of a policy, or the power of sale given by the Act should be expressly incorporated in the mortgage."

From this the author would appear to be of opinion that the power of sale given by the Act does include a power of surrender.

One difficulty arises from the fact that, although a surrender of a policy is, in fact, a sale to the office, it is also something more, as such surrender puts an end to the existence of the policy. If the policy were sold to a third party, the possibility of repurchase would remain so long as the policy was in the hands of such third party. It has probably on that ground been considered by some that power of sale does not include power of surrender.

Let me call your attention to an illustration taken from another subject, which may serve to throw light on this question.

An occupier being an absolute owner when selling a house by auction usually inserts a condition that the "tenant's" fixtures shall be taken at a valuation; but a mortgagee, when selling thus, should not make this stipulation. The objection is to a sale of any part of the property by valuation, and the surrender of a life policy is clearly a sale by valuation, and that a valuation made by an official of the purchasers.

Instances have come under my notice of an absolute sale by a mortgagee, whose powers were thought not to extend to a surrender, such sale being made to third parties in order that a surrender to the office granting the assurance might be effected by the purchasers as absolute owners after sale. Indeed, some companies purchase their policies and have them conveyed to the company by deed, as in the case of sale to third parties, the company then, as absolute owners, cancelling the policy.

A mortgagee selling under a power of sale can give a complete title, and such title will then include the power of surrender, which the vendor himself may not have been entitled to exercise. No doubt the nature of the powers conferred on the purchaser has an effect on the price given.

It seems, however, to me, that the difference between sale and surrender is more imaginary than real, and partakes somewhat of the nature of a legal subtlety, and that the mere receipt for the surrender-value or sale-moneys is in either case sufficient to vest the policy in the company, and enable them, as true owners, to cancel it. I am not, however, aware of any legal decision on this question, but as the failure to pay the premium, which is the natural consequence of the surrender, will cause the policy to lapse, the danger of any claim is narrowed in point of time to that within which the policy may, by its conditions, be revived or kept in force.

In the case of a mortgage or such a mortgage without special powers to give a receipt for moneys payable under the policy, "The Conveyancing and Law of Property Act, 1881", empowers a mortgagee not only to sell the security, but also to give a good discharge for any money coming due under it. In the case of the policies becoming claims, it is therefore sufficient for the company to take the discharge of the mortgagee without enquiring as to the amount of the mortgage debt, and the company is not concerned to enquire whether or not any money remains due under the mortgage, or to see to the application of the money.

Having regard to these powers mortgagees occasionally claim to be empowered to surrender reversionary bonuses for cash. Until, by the death of the assured, the reversionary bonus becomes payable they cannot, however, give a good discharge for it, and in the event of a surrender of such bonus for cash, the consent of the mortgagor is necessary.

As another instance of informal re-assignments, it is not uncommon to find in the mortgage of a policy a clause to the

effect that a receipt for the debt shall release the policy from the charge. This would appear to make payment off of the mortgage debt operate as re-assignment, and such a release or re-assignment has in practice often been acted upon.

Here again, however, "The Customs and Inland Revenue Act, 1888", which provides for the due stamping of all assignments, appears to raise another difficulty. The receipt, as such, for the debt does not require a stamp of higher value than a penny, but does the payment off do more than vest in the person paying off the debt the beneficial interest in the policy? If, as would seem to be the case, the estate-at-law in the policy remains vested in the mortgagee, the re-assignment is incomplete. If the estate-at-law pass to the person paying the debt, then, unless stamped as an assignment, the memorandum acknowledging payment is invalid, and the company cannot properly pay under it.

It is the custom for insurance offices, on the re-payment of a loan advanced on their own policies, to give a receipt, stamped as such, but it is open to grave doubt whether this receipt, intended to operate as a re-assignment, and being a link in the chain of title, should not be duly stamped as a re-assignment. An assignment stamp would appear to be specially desirable where the policy has, during the currency of the mortgage, been sold or has passed absolutely into other hands.

Before leaving the question of assignments, I desire to draw attention to a valuable paper recently read before the Actuarial Society of Edinburgh, by Mr. G. C. Stenhouse, "What constitutes a valid Conveyance of a Policy, and how should such Conveyance be discharged", which will, I hope, be printed. The author has been good enough to allow me to peruse the MS. of his paper, and, having regard to the complete way in which he has dealt with this subject, I have felt it right to condense very considerably my remarks on this subject.

The Stamp Acts have always been a source of anxiety, and although the companies, or some of them, gave considerable offence in the past by insisting on the proper stamping of deeds, the present position of the companies, though much more satisfactory, is equally, if not more, unpleasant.

"The Customs and Inland Revenue Act, 1888", has imposed on the companies not only a duty, which many discharged before to protect the revenue, but a paltry penalty for neglecting so to do. We are confronted with the exceptional difficulty that the deeds

must be properly stamped, whether executed before the passing of the Act of 1888 or not, in order to give a complete title.

The words of sections 19 and 20 of the Act are as follows:

"**19.** (1) No assignment of a policy of life assurance shall confer on the assignee therein named, his executors, administrators, or assigns, any right to sue for the money assured or secured thereby, or to give a valid discharge for the same or any part thereof unless such assignment is duly stamped, and no payment shall be made to any person claiming under any such assignment unless the same is duly stamped.

"(2) If any payment shall be made in contravention of this Section, the stamp duty not paid upon the assignment, together with the penalty payable on stamping the same, shall be a debt due to Her Majesty from the company or person by whom such payment is made, and shall be recoverable as such accordingly.

"**20.** Every condition of sale framed with the view of precluding objection or requisition upon the ground of absence or insufficiency of stamp upon any instrument executed after the passing of this Act, and every contract, arrangement, or undertaking for assuming the liability on account of absence or insufficiency of a stamp upon any such instrument, or indemnifying against such liability, absence, or insufficiency, shall be void."

An important variation from former enactments here appears in the inability of a person holding under a deed not duly stamped to give a valid discharge for the moneys payable thereunder. Hitherto the omission to stamp affected only the right to use the deed or document as evidence, and did not affect its validity.

A discharge for money paid was good though the deed or document was unstamped, and the only disability imposed on the office acting upon an unstamped document arose when it had to be produced before a court, judge, or arbitrator: in which case, if attention were called to the absence of a stamp, the document would have to be stamped and the penalty paid by the party producing it.

The second alteration is, that a penalty is now placed on the insurance company for paying under an unstamped assignment.

Shortly after the passing of this Act, it was argued that if the last deed of assignment was duly stamped (that is, the deed conveying the property to the person claiming to receive the money from the insurance company), the prior deeds, if executed before the passing of the Act, need not be stamped, *i.e.*, that the payment was made only under the last assignment. In reply to this argument, it may be said that the Act takes from the holder

of a policy under an assignment not duly stamped the rights of ownership except the actual power to hold or to stand possessed of the policy and the power to transfer it to another.

This reading of the Act was, I understand, acted upon; but, on closer examination, it will be found that no assignment not duly stamped conveys power to give a good discharge to an assignee, or his executors, administrators, or assigns: consequently, it is clear that all the deeds through which the title of a claimant descends must be duly stamped.

On this point, the opinion of counsel recently obtained by the Life Offices Association contains the following:

> "Every assignment which forms a link in the chain of title of the claimant must, in our opinion, be duly stamped."

Counsel were not, however, asked as to the special case of the following facts, on which the opinion might have been different:

> A assigns his policy to B by way of mortgage, and the deed is unstamped or insufficiently stamped. On repayment, B re-assigns by a duly-stamped deed. Subsequently A asks the office to pay him the surrender-value.

The assignment from A to B, and re-assignment, may be regarded as links in the chain of title, or the second as entirely cancelling the first, so that the two form a loop, the weakness of which has no effect whatever on the strength of the chain.

In this case there are, however, still two points to consider:

1. If the re-assignment, B to A, is made for the purpose of defeating the operation of the 19th section of the Act, it would, in the case of a deed executed after the passing of the Act, be void.
2. The re-assignment being complete in form, should the original and imperfect assignment be perfected, the re-assignment becomes immediately operative, cancelling it. So that power to give a good discharge, if not perfect, is of such a nature that it cannot be successfully impugned. If, then, the original assignment was executed prior to the passing of the Act, an indemnity being good perfectly protects the company making payment.

From the counsel's opinion already referred to, it appeared that an unstamped deed of assignment would confer actual ownership

but no powers; and that if the deed is in order in all respects except stamp, no other considerations or subsequent events will affect the title.

In the case of a deed executed after the passing of the Act and lost, counsel considered that the authorities would not inflict the penalties if the office acted *boná fide;* but it appears that they have the power to do so. The declaration which would, it is assumed, be required as to the loss of the deed should show that it was duly stamped.

Counsel considered that the execution of a new and similar deed duly stamped would not remove the difficulty of the unstamped condition of an original deed. There seems to be, in short, no method by which the defect of an unstamped deed can be remedied except that of stamping.

Under section 20 of the Act it is clear that no undertaking to indemnify on account of absence or insufficiency of stamp on a deed executed after 16 May 1888 is valid, such indemnity in connection with deeds executed before that date is, however, good.

It seems somewhat strange that when placing a penalty on the insurance offices no attempt was made to put one on both the parties to the original deed. Some banks were great offenders in former days. It was a common practice for them to accept assignments and deposits of deeds by unstamped instruments, and to take an undertaking from the depositor to pay the stamp duty and penalty if called upon. The legality—to say nothing of the commercial morality—of such a proceeding, which approaches a conspiracy to defraud the Revenue, is open to grave comment.

Whilst discussing questions of stamp duties, there are one or two points unconnected with assignments that seem to require attention.

In the case of a settlement of a participating policy, where the settlor contracts to keep the policy on foot, the stamp duty payable is *ad valorem* on the amount of the policy and vested bonus additions, but subsequent bonus additions do not render the stamp insufficient. I have seen many cases of hardship from want of attention to this rule at the time of making the settlement.

It has been admitted by the Inland Revenue authorities, in May 1877, that:

"Receipts endorsed on policies, either on their becoming claims or previously by way of surrender, do not require a penny Inland Revenue stamp if the policy itself be duly stamped." (*Vide* Act 33 and 34 Vict., cap. 97—Schedule, Title Receipt Exemption 11).

Nothing was, however, said as to the stamp on similar receipts not endorsed on the policy itself, as the question was not raised in the letter to which the reply was given.

As a simple receipt for money it would appear that, in the case of payment of a claim or surrender, a receipt not endorsed on the policy would require only a penny stamp. The wording of the Customs and Inland Revenue Act would not appear to touch the question of the stamp on a receipt for surrender of a policy, except so far as this, that if the receipt is not duly stamped the company is liable to a penalty.

Some time since a trustee, appointed by the Court of Chancery to receive the proceeds of a lost policy, was called upon by the office paying the claim to pay stamp duty of 10*s.* on the receipt, and I know that in the offices of some companies this is held to be the law.

I have not been able to trace the authority for this interpretation of the Stamp Act, but probably it arises from treating the receipt, though in common form, as a declaration of trust. It may be that in the archives of some office there exists a letter from the Inland Revenue authorities to this effect.

A receipt (which bore a penny stamp) for the sum assured under a lost policy was recently submitted to the authorities at Somerset House for adjudication of the stamp duty. The receipt appeared to be in common form, and contained the following words, which are, I believe, by no means uncommon:—"Of "which sum, and of all claims and demands under and by virtue "of the said policy, the said company is hereby and for ever "discharged, and the said policy is now cancelled." It was considered that this receipt amounted to an agreement, and the stamp duty payable was accordingly adjudicated at sixpence. This view is supported by a dictum of the late Baron Huddleston, that a receipt "in settlement of all claims" was liable to duty as an agreement.

"The Policies of Assurance Act, 1867", s. 3, provides that no assignment made after the passing of this Act shall give the assignee the right to sue for policy-moneys until a *written notice* of the date and purport of such assignment shall have been given to the assurance company, at their principal place or places of business, and that the date on which such notice shall be received shall regulate the priority of all claims under any assignment, and that a payment *bonâ fide* made in respect of any policy by an assurance company before the date on which such notice

shall have been received shall be as valid against the assignee giving notice as if this Act had not been passed.

The Act goes on to provide for a payment of a fee not exceeding 5*s*. for each such notice. There is a difference of practice as to the fees, some offices not charging any, some a fee for each notice of a deed irrespective of the number of policies concerned. It seems that the full charge that could be made under the Act is rarely enforced if, as some contend, each deed and each policy mentioned in the notice entitles the company to a fee.

The notice should contain the date, names of parties, and effect of the deed, and where the number of the policy is not stated it should be identified beyond the possibility of any dispute.

The forms of acknowledgment vary from the simple "received notice of which above is a duplicate (or copy)" to an elaborate acknowledgment quoting the Act or stating that the acknowledgment is not admission of title.

Informal or verbal notices cannot, it would appear from a careful perusal of the Act, be altogether disregarded. Notwithstanding the requirement of a written notice, it seems that, as the Act distinctly lays down that the *bonâ fide* payments by the company without notice are as valid as they would have been before the passing of the Act, the question of the effect of verbal or informal notice before the passing of the Act has still to be considered. Prior to the Act, verbal and informal notices had to be regarded, and it is therefore so now; but these notices will not confer on the assignee the powers given under this Act of suing the company in his own name. Such a notice would, however, entitle the claimant under the deed to rank with other claimants who had given notice in writing, in accordance with the date of his verbal or informal notice.

The trustee in bankruptcy should give notice to the insurance company of the bankruptcy, as it has been decided (Palmer *v*. Locke, L.R., 18 Ch. Div., 381–386) that bankruptcy is not notice to the world. Insurance companies are thus relieved, in the case of payments protected by this Act, from the necessity to search the bankruptcy registers.

In a legal text-book, it is stated that notice of assignment is not necessary in order to prevent the policy passing to the trustee in bankruptcy, because policies of assurance, being *choses in action,* are excepted from the order and disposition clause of the Bankruptcy Acts, 1869 and 1883. But it may be argued that

as, under s. 3 of the Policies of Assurance Act, the date on which the notice of assignment shall be received shall regulate the priority of all claims under any assignment, the act of bankruptcy operating as an assignment, actual notice of bankruptcy received by the office prior to notice of an assignment, the title of the trustee in bankruptcy would rank first.

Boná fide payments, without notice protected by the Act, include, certainly, payments made at the maturity of the policy, but do they include surrender-values and the grant of fully paid-up policies?

A policy of assurance is, by the construction clause of the Act, defined as "any instrument by which the payment of money "by or out of the funds of an assurance company on the "happening of any contingency depending on the duration of "human life is assured or secured."

The contracts we now propose to consider, are policies which include a guarantee of surrender-value, or of grant of a fully paid-up policy. These contracts are something more than the policies of assurance as referred to in the Act, and, so far as they are outside the definition of the Act, it is assumed that payments in accordance with clauses of the policy outside the Act have not the protection of the Act.

Do we, then, arrive at this, that in accepting the surrender of such a policy we should have the surrender executed by means of an assignment duly stamped, of which notice would be given to the office in order to bar those who might give subsequent notice of prior deeds?

Under "The Policies of Assurance Act, 1867", an assignee, having given notice of the date and purport of his deed, can sue in his own name for the money payable under the policy, and it may surely be regarded as a hardship, even by an assurance company, that it is liable to be sued by a person as to the validity of whose title it is uninformed, and who can only be compelled to prove his title at the hearing of the action. The company has no power whatever to compel a claimant to produce his title deeds beforehand: if he wishes so to do, he can issue a writ, prove his title in court, and if his title be in order, ask for costs against the insurance company. Such a request might, in a flagrant case, be refused, though this is by no means certain. It is difficult to see how, even in a gross case, the successful litigant—in this supposed case the claimant—could be compelled to pay the loser's costs. So, for the litigious holder of a policy who has a grudge against a company, such an action presents a complete case of "heads I

win, tails you lose." To quote from an editorial note in *J.I.A.*, xiv, 47, which, by means of the *Index to the first Twenty Volumes of the Journal*, I am able to attribute to Mr. Sprague: *

> "In short, the offices are in the position of equity judges, in the case of every assignment—a position for which they are not qualified and which it is neither the interest nor the desire of the public that they should exercise."

Since these words were written "The Customs and Inland Revenue Act, 1888", has intensified the objectionable position here referred to.

In the matter of stamps, this state of affairs puts the companies in a most unfavourable and unfair position of disadvantage when dealing with a reasonable claimant who is convinced of the sufficiency of the stamps on his deeds, but declines to undertake, as he fairly may, the trouble of a reference to the stamp authorities at Somerset House for adjudication. The company, with a well-founded doubt as to sufficiency of stamps, must then either give way and accept the risks incident to payment under deeds not duly stamped, or risk an action at law, for the sole purpose, as it seems to me, of protecting a revenue to which the companies, by the incidence of the income tax, already pay more than their fair share.

Is there, however, a crumb of comfort for the insurance company in the following idea? Let all policies, as many now are, be made expressly payable immediately on proof of death and title. If, now, the litigious claimant declines to produce his title, the policy-moneys are not payable, and his action will have been commenced when no payment was due.

It seems that the least that Parliament can do to mitigate the troubles incident to the effect of the two Acts combined—"The Policies of Assurance Act, 1867", and "The Customs and Inland Revenue Act, 1888"—is to confer on insurance companies the following powers:

1. In all cases to require proof of title, and to compel the claimant to produce and verify his title before commencing to sue the company.
2. The right to take deeds produced in support of title to the authorities at Somerset House for adjudication as to stamp, with or without the consent of the owners of the deed.

* [Mr. Sprague has pointed out that the quoted passage was not written by him, but is an extract from the letter of a correspondent (see p. 416).—Ed. *J.I.A.*]

This last may seem a strong demand, but is not unfair, considering that a company is liable to a penalty if payment is made on an unstamped or insufficiently stamped assignment.

Mr. C. J. Higham, in his paper read before the Institute in 1887 (*J.I.A.*, xxvi, 325), made some valuable suggestions, the careful consideration of which leads us to see how greatly the troubles of those responsible for the investigation of titles to policies of life assurance might have been reduced had the laws under which they have to act been framed as a consistent whole, free from many of the incidents affecting other and different classes of property, instead of, as has been the case, being allowed to grow up governed by general principles of law, and tinkered as pressing necessity arose.

There seems no valid reason why the prospective share of a person in the stock and funds of an assurance company should not be in the same legal position as his actual share in the stock and funds of a railway. The position of all parties would, I think, under such circumstances, be more satisfactory than at present, or even than that they would have occupied under Mr. Higham's scheme.

In these days of easy and rapid travel, of policies world-wide and free from all conditions, when the merchant or professional man finds it, perhaps after years of quiet residence here, profitable or advantageous to move his domicile to some colony of the British Crown, or some foreign country, no consideration of the difficulties met with in the legal department of a life office would be in any sense complete that did not take account of the effect of dealings with policies under foreign law. Mr. Stenhouse, in his paper referred to already, discusses the effect of a foreign assignment of a Scotch policy on the life of a Scotchman domiciled at home, but in that instance the foreign country is England.

The case of the Scottish Provident Institution *v.* Walker and others is interesting on several grounds. Mr. Stenhouse quotes it at some length from the *Scottish Law Reporter*, November 1888, but some quotations I shall make were not within the scope of his paper.

The head-note of the case is as follows:

"A domiciled Scotsman borrowed money from a money-lender in England. As security, he delivered a promissory note and a policy of insurance effected with a Scottish insurance company. At his death, in June 1887, the loan was only partially repaid, and in

August the lender notified to the insurance company that the policy of insurance had been assigned to him. In December the estate of deceased was sequestrated, and a trustee appointed thereon. In a multiplepoinding, the Court ranked the lender preferably to the trustee for the debt still due in respect that the loan was negociated in England, by the law of which, as the parties admitted, deposit of the policy operated as an equitable mortgage in favour of the lender, and notification thereof by the lender to the insurance company before the bankruptcy of the borrower conferred on him a preferable right."

In the judgment, it is laid down that—

"The claimants' right under the contract between them and (the borrower) Andrew Allen Walker fall to be determined by the law of England * * * * *."

"It appears to be reasonably clear that the validity of the assignment must be determined by the law of the country within which the assignment was made."

"The right of credit (created by the policy) follows the domicile of the creditor wherever he goes, and is capable of being assigned or dealt with by him in any manner which the law recognizes."

"The validity of the assignment will in general be determined by the *lex loci contractus*—that is, according to the law of the country in which the transference is made or security given."

"Any question of competing right between the trustee and a creditor claiming upon a preferable security must, apparently, be determined by the laws of the country in which the competition arises."

I have taken these extracts as particularly clear and pertinent to the subject under discussion. The law of England on the point being at the same time, in that particular, on all-fours with that of Scotland.

To refer to an English case, we take Lee *v.* Abdy, 17 Q.B.D., 309.

The head-note here reads:

"The plaintiff sued the trustees of an English life insurance company as assignee of a policy of life insurance granted by such company. The assignment of the policy was made in Cape Colony, and at the time of such assignment the assured, the assignor, was, and he remained till his death, domiciled in Cape Colony, and the plaintiff was his wife. By the law of that Colony such an assignment was void, by reason of the alleged assignee being the wife of the assignor.

"*Held* that the law of Cape Colony applied to the assignment of the policy, and therefore that the defendants were entitled to judgment."

Mr. Justice Day, in giving judgment, said:

"* * * The parties to the alleged assignment are domiciled and contracting in Cape Colony, and by the laws of that Colony, as it

seems to me, the validity or invalidity of such contract must be determined."

Mr. Justice Wills, in his judgment on the same case, added:

"Taking into consideration the difficulties that may arise with regard to the validity of assignments according to English law, it does not seem to me that the difference caused by this additional difficulty is appreciable."

Taking, first, the case of the death of the assured under a policy, the sum assured under which is payable to the executors, administrators, or assigns of such assured.

Section 19 of 52 & 53 Vict., c. 42, enacts: "that where a "policy of life assurance has been effected with an assurance "company by a person who shall die domiciled elsewhere than "in the United Kingdom, the production of a grant of repre-"sentation from a Court of the United Kingdom shall not be "necessary to establish the right to receive the money payable "in respect of such policy."

This appears to cover both the case of a person domiciled out of the United Kingdom when the policy was effected and at the time of his death, and of a person domiciled within the United Kingdom when the policy was effected but out of the United Kingdom at the time of his death. The only question, then, that arises in the payment of such a claim, so far as the question of personal representative is concerned, is the domicile of the assured. It would, however, appear that whatever the domicile of the assured, the claim can safely be paid on the grant of representation from a Court in the United Kingdom, provided the company has no notice of any assignment, as the competition between a personal representative and any assignee or mortgagee would arise in the United Kingdom, and be determined by the laws of the United Kingdom.

In the case of assignment of a policy in a foreign country the company has to consider the effect of the law of the country in which the assignment is made, and, if notice of assignment has been given, the effect on such assignment of subsequent dealings with the policy in the United Kingdom.

For example, A assures his life whilst domiciled in the United Kingdom, he proceeds to South Africa, and, whilst domiciled there, assigns his policy, assuming the assignment to be good in accordance with Cape (Anglo-Dutch) law, and the company is asked to pay in the United Kingdom. Here it would seem that if the assignor has power to give a good discharge for the

policy-moneys, the deed of assignment, though valid according to Anglo-Dutch law, must nevertheless be stamped in accordance with our Inland Revenue Acts. If, however, payment is required in the Cape, the insurance company, being the debtor, would have to follow the creditor and pay there, unless the policy contained a clause stating that the sum assured was only payable at the company's office.

If the deed does not give an absolute title to receive the policy-moneys the consent of the personal representative, under a grant from a Court within the United Kingdom, would appear to be necessary also. In the case of voluntary assignment within two years of the death the consent of personal representatives of deceased would appear to be necessary before making payment in the United Kingdom.

The case also arises of an assured, whilst domiciled abroad, assigning his policy and obtaining a re-assignment; he dies subsequently, domiciled here. The validity of the assignment being determined by the *lex loci contractus*, it would appear that stamp duty is not payable here on the assignments, and that this is a case in which deeds, though links in the chain of title, need not be stamped when the claim is paid under a grant of representation from a Court here.

"The Judicature Act, 1873", provided, that in the event of opposing or conflicting claims the debtor or trustee shall be entitled, if he think fit, to call upon the several persons claiming to interplead, or he may pay the same into Court under the provision of the Trustee Relief Act.

Prior to the passing of this Act attempts had been made to obtain for insurance companies the benefits of the Trustee Relief Act, but insurance companies were held, in the case of ordinary policies, to be debtors, and thus not entitled to the relief (Matthew *v.* Northern, 9 Ch.D., 80).

The Judicature Act, however, gives the debtor, in the case of conflicting claims, power to interplead, when the Court will direct issues to be tried, or adopt one of several other courses open. Where, however, the policy-moneys are the subject of a trust, payment into Court can be made under the Trustee Relief Act without interpleader, as the benefit of that Act extends to all having on their hands money belonging to any trust.

There appears, however, still to be one case unprovided for—where a depositee or mortgagee has not power to give a valid discharge owing to any defect in title, and the consent of the

personal representatives to the payment cannot be obtained. Here it seems that the company is subject to an action by the claimant for the policy-moneys, in which the Court, according to Chancery Division, has power to dispense with the personal representative. The Court of Appeal in another case, however, doubted whether the Chancery Division had that power.

Though the interpleader action is represented to be an innocent, short, and easy method of getting an authentic decision, insurance companies are, I think, almost as adverse to them as they are to the ordinary actions at law.

The efforts of the companies to secure in all cases prompt settlement of claims, and the sensitive condition of the public mind on assurance matters, makes it a matter of the first importance that no public appearance of delay, such as is given by an action in the Courts, should, if possible, be allowed. It is probably owing to the tact of those charged with dealing with these matters (officials and legal advisers) that more cases do not come for trial.

The mercantile community resorts largely to arbitration: could not some arrangement be made that the same prompt and inexpensive method of procedure might be open to the companies—insurance companies, when a claim comes up for settlement, being practically in the position of stakeholders, though legally debtors.

The PRESUMPTION OF LIFE, or, from our point of view, the presumption of death, is an interesting subject, but one on which little need here be said, as there exists a very complete and recent paper on the subject.

Mr. W. C. Spens (Advocate and Sheriff-substitute of Lanarkshire) read a paper before the Insurance and Actuarial Society of Glasgow, in February 1888, entitled "The Law as to Presumption of Life in connection with the disappearance of Assured Lives", which was published in the *Transactions* of that body. Mr. Spens dealt fully with the English law on this subject.

Since that was written, a Bill was introduced into Parliament, in July 1888, which proposed practically to re-enact "The Presumption of Life Limitation (Scotland) Act, 1881", and to fix a presumed date of death. In the Act of 1881 and the Bill of 1888, it was specially provided that they should have no application to any claim under a policy of life assurance, and that the person claiming under a policy should be required to prove the death of a person whose life is assured in the same manner as if the Act had not been passed. I notice that the Lord Advocate, in reply

to a question in the House of Commons, on 6th instant, stated that he proposed to bring in another Bill on this subject this session.

Mr. Spens sums up the law of England on this point as follows:

"(1). On disappearance of an assured life for more than seven years, after due enquiry made, the Court, in the ordinary case, will presume the assured is dead, and the policy will become a claim. (2). Subject, however, to this exception that, if the assured have probable or even reasonable motives for concealment of existence from relations or previous acquaintances, the presumption of death will not hold good. (3). There is some slight authority for holding that, if an insurance company pays a claim on presumption of death, founded on the disappearance of the assured for more than seven years, the former is entitled to demand security for repayment in the event of its being afterwards proved that the assured, at the date of payment, was still alive. (4). Though it does not appear to me of any practical importance in this question that while the assured, having disappeared for more than seven years, is presumed to be dead at the expiration of the seven years, there is no presumption of death at any particular period during the whole seven years."

The difficulty, so far as the insurance offices are concerned, in this statement of the law—which appears to me, on investigation, to be correct—is, when did the assured die, and when did the payment of premiums cease to be exigible? Possibly at the end of the seven years.

Mr. Spens, in conclusion, made a suggestion, that offices should insert in their policies a requirement that the Directors, before paying a claim, "shall be entitled to be certiorated of the place and manner of death of life assured."

The Married Women's Property Acts have already more than once formed the subject of papers and discussions at the Institute, and the general principles have been considered in papers by Mr. Sprague, Mr. Bunyon, and Mr. Hughes; but there still remain points to which attention may, I think, be usefully directed.

In the debate on Mr. Hughes' paper, Mr. Saunders said "I have found it most expedient to advise them (*i.e.*, the public) "to have nothing to do with the Married Women's Property "Acts, but if they really wish it to endorse the policies them-"selves, and adopt the wording of the Act of 1882." In the advice to leave the Acts alone I am inclined to agree with Mr. Saunders, for the difficulties that have from time to time arisen, and the expenses incurred by the holders of the policies,

render it doubtful whether the Acts have secured, so far as the life assurance policies are concerned, the benefits claimed for them.

It is, however, difficult to see how any endorsement affecting the company, such as that suggested by Mr. Saunders, made after issue by the holder, can in any way affect a document completed by the company before it reaches the holder.

It has always seemed to me that when the husband has insured his life for the benefit of his wife under the provisions of the Married Women's Property Act, the power of the wife alone, or of the husband and wife jointly, where the policy-moneys are payable to the husband should his wife predecease him, to surrender the policy is doubtful, and it is clear that any surrender-value would be subject to the limits of the policy-moneys.

If the settlement on the wife had been made by a separate trust deed it is clear that no such power would have existed, unless specially provided by the deed of settlement. The policy contains the terms of the settlement, and operates as a deed of settlement, hence, unless special powers are given in the policy or the Act, the parties, it may be contended, have no powers beyond that of receiving the policy-moneys and applying them to carry out the trusts mentioned in the policy. It is, I believe, the general custom to allow surrenders, and no doubt it is in many cases for the benefit of the trust that this should be so, but the question arises, would not these trusts be equally as well if not more completely carried out by the grant of a paid-up policy, subject to the same trusts. With regard to the assignment of such a policy, Porter, in his *Laws of Insurance,* says that probably such a policy is assignable during the husband's life, since the wife, being alone named, would be sole and absolute beneficiary under the policy if she survived her husband.

There is no doubt that the husband can exercise his influence to coerce the wife to join in any surrender, and the creditor can bring potent influence to bear on the wife to surrender or join in the surrender of the policy, and thus, though the wife's general power of contracting perhaps permits the surrender, the object of the Act to provide a settlement free from control of husband or creditor is very largely, if not completely, defeated.

The length to which these notes have already attained deter me from dealing further with the Married Women's Property Acts and some other interesting matters.

Amongst the subjects worthy of full statement and further discussion that remain some may be indicated.

(1). The interest which a person takes in the policy-moneys when he has paid premiums thereon, either as a volunteer, or to keep alive the policy as providing a fund from which he may be paid any debt owing to him.

(2). The position of a creditor who, having proved in bankruptcy for the amount of his debt, less the agreed value of his security (a policy of life insurance), has to bear any loss that may arise in realization, but is debarred from receiving any adventitious profit (by death, for example, of the life assured) in such realization before the close of the bankruptcy proceedings.

(3). The effect of domicile of the insurance company on the power to sue foreign insurance companies, or to enforce payment, especially in the case of war.

(4). The position of an assured where the agent exceeds his authority in giving time for payment of a premium, or gives wrong advice as to the necessity of obtaining license from the company for foreign residence or travel.

Discussion.

The President (Mr. B. Newbatt) having invited discussion,

Mr. A. H. Bailey said he was surprised that nothing was said in the paper about the Life Assurance Companies Acts of 1870 and the following years. If the Act of 1870 were amended, which no doubt was required, it would be desirable to take the opportunity to consolidate all the existing laws relating to life assurance. Mr. Lemon had commenced with the earliest of these, namely, the Act 14 George III, chap. 48, known as the Gambling Act. He should be glad if that Act were repealed altogether. He believed it arose from an idea that policies of insurance might be effected upon the lives of men in public positions, Prime Ministers and the like, and that they might be murdered for the assurance money. That idea was surely out of date, and the way in which the Gambling Act worked was very unsatisfactory. By it, one man was not allowed to effect an assurance on the life of another unless he had a pecuniary interest in the life. It had always been held that a creditor might insure the life of his debtor, and it was clear, in numerous instances, that the interest of the creditor was not

in the life, but in the death of the debtor. He had known cases where policies of insurance had been given as security for a debt, and the insured had expressed some indignation that the life assured would do anything but die. On the other hand, there were numerous instances in which it would be right and proper that such insurance should be effected. As to the Act of 1867, he thought they could get over a great many of Mr. Lemon's difficulties in a summary way, by providing that the form of assignment in the Act of 1867 should not be voluntary but compulsory. Life assurance policies would then be assigned as stocks and shares are, so that instead of registering notices of assignment of policies, they would register the assignments themselves, and just as Consols, railway stock, or other similar property was the subject of trusts, so policies of assurance could be subjects of trust, and be transferred into the names of the trustees. That would do away, as far as assurance companies were concerned, with the multitude of deeds which they now had to examine. The parties into whose names the policies were transferred, and they alone, should have power to surrender a policy or otherwise deal with it, and that would simplify a number of the present difficulties. As to the Stamp Acts, he thought the Act of 1888 gave reason why a Legal and Parliamentary Committee of the Institute was required. By that Act, penalties were imposed upon insurance companies for acting upon unstamped assignments. Unstamped assignments existed by the thousand, and the Act, contrary to the general principles of legislation, was made retrospective. It was very unfair that without any discussion or any notice such penalties should have been imposed. The only other point he should mention was as to the disappearance of insured lives. A man who had not been heard of for seven years was by law presumed to be dead. Two or three cases had occurred in his own experience in the last few years which presented difficulties. In one case a man was a bankrupt, and disappeared from his creditors. There was very strong suspicion that he was alive in some part of the world at the present time, but the company were advised that they had no alternative but to pay the sum assured, and it was paid accordingly. That was rather a hardship upon them, because the object of the Act was to protect trustees, which was a different matter. It was considered that if a man entitled to property did not claim that property within seven years he might reasonably be presumed to be dead. That was a wholly different thing from a case where a man ran away from his creditors, or had brought himself within the pale of the law, and, as in one case in his experience, changed his name.

Mr. Augustus Hendriks differed from the author in regard to the statement that assurance companies could not call upon a claimant under a policy to prove his claim by production of the deeds in support of the title at the office of the insurance company. He was fortified in his opinion to the contrary by the views of the solicitors to his company, whom he consulted on the subject some little while ago, not so much upon the question of taking the deeds into Court as of inspecting them at the office of the solicitor of the claimant. They were frequently asked to attend at the office of the

solicitor to the claimant; very often they did so as a matter of courtesy, but lately it had been demanded of them in many cases as a matter of right. No doubt this arose from the legal axiom that the debtor was bound to follow his creditor. With reference to this view, the solicitors to his company held that the position of debtor and creditor did arise between the company and the original policy-holder, but not between the company and the assignee of the policy until legal evidence of the assignment had been produced to the company at its own office. That was satisfactory, because if—notwithstanding the clause which existed in many policies, that the sum assured was payable at the office itself—they were to be bound to follow the debt and pay the sum assured wherever the deeds of assignment might be, it would be very inconvenient to the company. As to the seven years' disappearance being taken as proof of death, that was one of the greatest hardships insurance companies were enduring in the present generation. He hoped some combination would take place in order, if possible, to alter what was said to be the existing law. With regard to that law, there was a certain amount of misapprehension on the part of some solicitors and also of eminent counsel, and in a very recent case it was held on behalf of the claimant, and advised by counsel, that the production of the probate was sufficient to enable a claimant to force the insurance company to pay his claim. It might well be said that nothing more unjust than that could be conceived. One could well imagine that to get probate for the distribution of property, such as shares and stocks, or even land, which existed before the disappearance was a very just operation, because if the man had disappeared from one cause or another it was only right that if he kept people unadvised of his existence his representatives should proceed to divide his property on the assumption of his death; but when they came to a policy of insurance, where the sum assured was created or said to be created by the act of disappearance, it was very hard that they should be called upon to assume that the man was dead upon the granting of the probate, and it was only less hard that it should be held that seven years' disappearance should be sufficient to constitute death from the point of view of the life policy. He had heard of a case recently where a very young man who had been leading a fast life in England was assisted by his friends to go to Australia. He sent from there three or four letters to his friends, and then left off corresponding. At the expiration of seven years the assignee, who had bought the policy as a speculation, claimed the money of the insurance company, and he believed the sum was paid; but whether paid or not, it was monstrous that such a state of things should be allowed to exist. Then there was another case which would be a *cause celebre*—that of an Irishman, who left his own country and came to Birchington for sea-bathing. He wrote several letters, and then left his clothes on the sea-shore and disappeared. The insurance companies were asked to pay the sums assured, simply because probate had been granted. The judge in Ireland held that the grant of probate was utterly insufficient and inapplicable to the realization of the insurance money, and that they must wait at least seven years before they could again bring forward their claim. He hoped that before the expiration of

that seven years there might be an alteration in the law, if the law really required payment after seven years of disappearance. It might, no doubt, be said, on the other side, that after a man's disappearance it was very hard to have to go on paying premiums for an indefinite period. It would, perhaps, be fair to provide that payment should be made by the company when a period had elapsed, equivalent to the term of the expectation of life at the time the disappearance took place. As regards stamps on assignments and mortgages, he thought that insurance companies were made responsible to an extent quite uncalled for. He agreed with Mr. Bailey as to the advantage of simplicity in assignment of policies. He (Mr. Hendriks) had previously stated that he saw no reason why policies should be so far removed from stocks and shares in this respect. He believed it would become law that deeds of assignment should be dealt with as transfers, in the same way as in the case of stocks and shares, as to which all transfers were registered in the books of the company and from which certificates were given in proof of ownership.

Mr. W. Trower (a Visitor) said that as regarded the insurable interest, he endorsed Mr. Bailey's remarks. He did not see why an insurable interest should be necessary, and he understood the practice of insurance companies was not always to insist upon it; and he thought that the law should be made to follow the practice, instead of, as was usual, the practice following the law. As regarded the assignment and mortgage of policies, from a lawyer's point of view, he could say they would all welcome a proper system of registration of policies like the registration of railway stock by a simple deed of transfer. With reference to that, he suggested that the policy should be treated very much the same as a stock certificate; that no transfer should be registered unless the policy was produced at the same time, because it seemed to him that, while they should wish for a system of registration applicable both to policies and every other kind of property, they must not in any way fetter the public in dealing with policies. For instance, a policy might be taken out by a man who wished to borrow money from his bankers; and he should be able to take it to his bankers, who would be protected, if it was necessary for the policy to be produced before a transfer could be registered. It was mentioned in the paper that Mr. Higham had drawn attention to the subject in 1887, and he thought they had allowed a long time to elapse before applying to Parliament to deal with the question. The insurance companies had only to combine, and apply to Parliament, and the thing would be done. He did not venture to make any remarks upon the question of the proof of death, as that seemed to him to be an actuarial rather than a legal point.

Mr. McKee (a Visitor) referred to a *donatio mortis causâ*, and said that the author in his paper stated, properly enough, that it was subject to legacy duty; it was liable to account duty, which was equivalent to probate duty—an additional legacy duty taking the place of probate duty. Then he spoke of "the failure of the gift on deficiency of assets." That was not a correct expression. The gift did not fail. The donee took it subject to debts, but the gift to the donee was perfectly good, and the donee could give a receipt for the policy in

the same way as an administrator of the deceased could give one. Therefore it was not a failure in the gift, simply that a gift was liable in the hands of the donee to the debts of the assured. The author dealt with the question of the deposit of policy without any written document, and suggested that if the policy was simply deposited without any written document, the depositee or mortgagee could not give a good receipt for policy moneys. That proposition was debateable. By one of the sections of the Conveyancing Act a mortgagee could give a good receipt for policy money. The section was perfectly wide: it did not say "a mortgagee who held under any written document:" and it was perfectly clear that a perfectly good equitable mortgage of a policy could be given by a mere verbal deposit, and he should maintain that an equitable mortgagee by deposit could give a good receipt for policy money. Of course, when they came to practically dealing with what evidence a company should require on deposit it was a different question, but he maintained that an equitable mortgagee by a verbal deposit could give a good discharge for policy moneys. The author suggested that if they took a mortgage in the form of assignment given in the Policies of Assurance Act, the mortgagee would then be able to give a good discharge. That would be so if it was by deed. That was just a mere technical point that required to be borne in mind, because the power of sale which was conferred by the Conveyancing Act was only applicable to mortgages under seal. They might get a perfectly good assignment of a policy under hand only, but it would not confer the power of sale given by the Conveyancing Act. The author referred to marriage settlements. His (Mr. McKee's) experience had been that one of the most difficult tasks that now fell upon the legal advisers to an office was to advise whether a marriage settlement was properly stamped or not—a settlement 30 years old, dealing with all sorts of shares and stocks, and containing a future property clause was perhaps brought before them, and they had to find out what was the value of all the stocks and shares included in the settlement at the date, and if the future property clause included the words "now or at any time shall become entitled to," then they ought to inquire whether the lady was, at the date of the deed of settlement, entitled to any other property not mentioned. Was it possible to get a satisfactory answer? She might have outstanding reversionary interests, and so on, of which nobody could tell. The only safe way was to insist upon the deed being adjudicated. The Inland Revenue authorities were prepared to take a statement that the lady was not entitled at the time, and upon that they would adjudicate; but no solicitor advising an office should be satisfied with a mere statement of that kind. The question of stamps was a most terrible subject to embark in, and one that was of more trouble now than any other. They often heard it suggested that where there was a difficulty in arriving at what the estate consisted of, they should take out probate for a nominal amount. That was very convenient, no doubt, but if they presented the probate to a banker or an insurance company and claimed a large sum under the probate taken out for a nominal amount, the debtor ought to insist on seeing that the stamp was of sufficient amount to cover the sum that they were to pay. Another awkward

question was what was the proper stamp upon documents executed abroad, and governed by the foreign law which dealt with English policies. The author said that they should be stamped in accordance with the Inland Revenue Act. One of the sections of the Stamp Act of 1870 provided that if a deed was executed in England relating to property abroad it must be stamped with English duty, and also that if executed abroad but relating to property in England it must be stamped with English duty. The author gave a case of an assured assigning his policy abroad and obtaining re-assignment, dying subsequently domiciled here. He suggested that stamp duty was not payable here on the assignments. In that he (Mr. McKee) thought the author was incorrect. It had been laid down in one case that stocks and shares were assets at the head office of the company—that a man dying in France possessed of stock in the London and North-Western Railway Company, and having his certificates abroad with him. nevertheless these were assets in London at the head office of the London and North-Western Railway Company; and in the same way with an English policy payable in England, although the owner might be domiciled abroad and assign it by foreign instrument to a foreigner, yet he was dealing with English property, and although the law appertaining to that assignment might be foreign, it ought to be stamped with an English stamp. The author stated "In the case of voluntary assignment within two years of the death, the consent of personal representatives of the deceased would appear necessary before making payment in the United Kingdom." That, he presumed, was with reference to a section of the Bankruptcy Act which provided that a voluntary assignment should be void in case of bankruptcy within two years, and that within 10 years it should be void, unless it could be shown that at the date the assignor was solvent. That was not a correct statement, because there was a case which decided that at the death of the settlor the section of the Bankruptcy Act did not apply to the administration of his estate, and it was only in a case where the assignor or settlor continued to live that the difficulty arose. With reference to the Married Women's Property Act, there was a suggestion that the husband and wife should together not have any power to deal with a policy effected for the wife's benefit under the Act. If they wished to get a document that would inevitably, if kept on foot, produce a fund for the wife, it was very simply done, by adding to the policy the words "without power of anticipation." Those words had been held to apply to capital as well as to interest, and so long as the coverture lasted the policy could not be surrendered or dealt with in any way unless by the Court. The difficulties in advising upon a title to a policy of assurance were not anything like so great as those in advising as to title to land, although precisely the same point might arise, because when they were dealing with land certain conveyancers advised as to title one day, and in a week's time the same title was put before another who took a very different view. But in the case of a policy the advice was given once for all, because the policy was handed over to the office, the money was paid and the business concluded. They had the additional advantage, that in six years the Statute of Limitations ran, and they were completely protected. If they were to codify the law relating to

policies of insurance that would be practically codifying the whole law of England. That would be a very good thing if it could be done, but they would not do it in a piecemeal way by simply dealing with policies.

Mr. C. D. Higham remarked that the author had said: "There seems no valid reason why the prospective share of a person in the stock and funds of an assurance company should not be in the same legal position as his actual share in the stock and funds of a railway." That was the very position that he (Mr. Higham) took when he brought his paper before the Institute in 1887, when he said that "in future a policy should be assigned, as far as possible, as a chattel, much as a share in a joint stock company is, the deeds being lodged at the company's office, and a register of policyholders and mortgagees kept there." He felt strongly that trusts should not be recognized in such a register kept by an assurance company, but he went beyond those two things, and also made provision for a man who desired only to mortgage his policy. This system was understood to work admirably in New Zealand. But, so far as an absolute assignment went, that was the object of the paper he brought forward, that there should only be one holder known to the office. As to the Gambling Act, in accordance with Lord Ellenborough's ruling in Godsall *v.* Boldero, it was held for 50 years that a policy was a contract of indemnity. That was upset by Dalby *v.* the India and London, and yet now, apparently, they were gradually retracing their steps, and several gentlemen had said it would be a good thing if the law as to insurable interest were abolished. In America, opinion seemed to be going just the other way. The Court of Appeal of New York maintained the English law, and the principal New York companies followed it, but the Supreme Court of the United States, and many of the State courts, went dead against that, and many of the companies would not pay under an absolute assignment unless they had proof of interest. The reason given was the old *contra bonus mores*, and in a judgment given by the Supreme Court lately, they said policies without the proper interest had a tendency to create a desire for the event—just the thing which had been mentioned that evening as being out of date. The judges in the Supreme Court said it was absurd to refuse to issue a policy without interest, and yet to pay under an absolute assignment. Mr. Lemon quoted a letter from the Inland Revenue, saying that an endorsed receipt for surrender-value did not want a penny stamp. The Inland Revenue did write that letter, but afterwards wrote another to an exactly contrary purpose. Two things had not been mentioned about stamps: one, that an endorsement on a policy had been adjudicated to require a stamp—an endorsement, that was, with both parties concurring, as in cases of reduction of the sum assured and premiums. Such an endorsement required a 10*s*. stamp if under seal, or a 6*d*. stamp if under hand only, while the policy stamp might be 1*s*. or less. He had been advised also that an undertaking for production of deeds required a 6*d*. agreement stamp. The consolidation of the law, which Mr. Bailey spoke of, was a very desirable thing. Australasian companies had, he thought, made a mistake in mixing up laws as to assignments

with ordinary Life Assurance Acts. Laws as to assignments should be by a separate Act, because, for one person who wanted to see the Life Assurance Act, there were hundreds who wished to see as to assignments.

Mr. A. G. Mackenzie said that certain recent frauds showed that the baser sort of agents were making a business of gambling insurances, and although in some cases the policy was taken out in the name of the proposed life, and was afterwards assigned, yet in some cases it was taken out by a person falsely stating he had an insurable interest in a life proposed. In those cases it was certainly an advantage to the office, on repudiating the contract, to find the proposer had infringed the provisions of the Gambling Act, and that protection should not be taken away from insurance offices. He sympathized with the observations made by Mr. Hendriks with regard to a case recently decided in the Irish courts, where the insurance offices had good ground for believing that the assured life had only disappeared; and, although the Court found there was no proof of death, the premiums were being paid under protest, and the claim would, no doubt, be presented at the expiration of seven years from his disappearance. His own view was very similar to that which Mr. Hendriks had stated, that in a case where a man disappeared, especially where it was a young life, seven years was far too small a limit, and it should approximate to the expectation of life. Even in that case it would be in favour of the representatives of the assignee of the assured, because in the event of his death being proved within the expectation of life, the claim would be paid, so that the office would have to pay either at the end of the expectation of life or the earlier period, whereas the life might live considerably beyond the expectation.

Mr. F. Bell said he understood one speaker to say that companies were bound, before paying a claim under probate, to see that duty had been paid on an amount greater than the sum assured, and bonuses if any. He wished to ask under what Act was this so? He had himself an experience of a case in which this question arose, but the officials at Somerset House seemed to admit that there was no law which compelled a company to see what duty had been paid on the probate.

The President said that, with the permission of the members, he would now read the major part of the long letter which had been addressed to him by Mr. T. B. Sprague. Mr. Sprague opened by discussing the question of "insurance" and "assurance", referring to a paper of his own, and giving a long extract from that paper. That was already on record in the *Journal* (*J.I.A.*, xvi, 77) and need not be repeated. Then, addressing himself more closely to the particular matter before them, he said:

"As regards the assignments of life policies. I think that Mr. Lemon uses the word 'assignment' in much too wide a sense. 'The Customs and Inland Revenue Act, 1888', makes a distinction between an assignment of a life policy and an equitable mortgage, which distinction (I think) Mr. Lemon has lost sight of. It is quite clear that a mere memorandum of deposit of a policy is not an 'assignment.' Similarly, a gift made in anticipation of death, *donatio mortis causâ*, is not an 'assignment.' As

regards the deposit of a policy, if this is accompanied by a written memorandum explaining the transaction, it will (I believe), provided notice is given to the office, give a creditor a good charge over the policy. If there is no written memorandum, but notice is given to the office, the creditor will (I imagine) still have a good charge if he can clearly prove his claim, and that the deposit was made to secure it. Mr. Lemon says that a mere deposit, without notice to the company, would not be of any benefit as regards the company; but I think this is going a little too far. If a company is asked to pay a claim without production of the policy, it should make strict enquiry as to what has become of the policy, and, at all events, decline to pay the claim under discount; and if it fails to do this, it might be held answerable to the depositee for any loss he sustained in consequence of premature payment of the claim. As regards re-assignments, I am satisfied that when a policy has been assigned by way of mortgage, no re-assignment is necessary. A simple discharge, endorsed upon the mortgage, cancels it, so that it does not form a link in the chain of title. Such a discharge is not a 're-assignment', and does not require any stamp whatever. The same rule, of course, applies to loans by insurance companies on their own policies; and I am surprised to learn that there is a custom for insurance offices, on repayment of such a loan, to give a stamped receipt. Even if a penny receipt stamp, or a sixpenny agreement stamp, is considered necessary in such a case, it is quite certain that the document need not be stamped as a re-assignment. The above remarks apply to the case where A assigns his policy to B by way of mortgage, and A subsequently pays off the mortgage. It is here quite immaterial to the office whether the original assignment was duly stamped or not. In any case the mortgage is cancelled by repayment of the loan, and drops out of the chain of title. In such a case A does not claim under the assignment and re-assignment, but as original grantee of the policy, and therefore his title is not struck at by the Act. Mr. Lemon states that there seems to be no method by which the defect of an unstamped deed can be remedied, except that of stamping it. This, I believe, is true of absolute assignments executed after 16 May 1888, but it is otherwise with assignments by way of mortgage. When A has so assigned his policy to B, and B claims payment after A's death, his mortgage deed being insufficiently stamped, the difficulty may be got over by paying the amount of claim to A's legal personal representative, with the consent of B. I cannot agree with the statement that the old practice, which may, for anything I know, still exist, for banks to accept deposits of deeds by unstamped instruments, with an undertaking from the depositor to pay the stamp duty and penalty if called upon, is in any way illegal, much less an approach to a conspiracy to defraud the revenue. The law of the land distinctly states the consequences that will follow if documents are not duly stamped at the time, and if banks or other persons are willing to take those consequences, it seems to me nobody has any right to object. In fact, I consider that they are strictly within their legal and moral rights in leaving their documents unstamped, and running the risk of having to pay the penalty in consequence. Coming now to 'The Policies of Assurance Act, 1867', this provides that written notice must be given to the office of the date and purport of an assignment. I see Mr. Lemon says that the notice should contain the date, names of parties, and 'effect' of the deed. The question might, perhaps, be raised in some case whether the particulars given in a notice are the 'purport' of the deed within the meaning of the Act. But this question cannot arise if, as is sometimes the case in Scotland, a certified copy of the assignment is lodged with the company as the notice. I observe that Mr. Lemon says that prior to the passing of the Act, verbal notices of assignment had to be regarded; and as this is quite a new idea to me, I should like to know what is the authority for it. I believe that Mr. Lemon's statement of the law as regards bankruptcy is correct, although it differs from the view formerly

held. It seems to be now established that an assignee in bankruptcy must give notice to the office, like any other assignee, and that his claim will rank according to the date of his notice. I notice that Mr. Lemon expresses a doubt whether payment of a surrender-value is valid as against an assignee who has not given notice to the office, but I think there can be no doubt that it is perfectly valid, as it is a payment made 'in respect of' a policy which comes within the operation of the Act. The policy, in fact, is one and indivisible, and cannot be held to be within the Act for one purpose and outside of it for another purpose. As regards the grant of a fully paid-up policy, no question can arise if there is no new policy issued, but the convenient course is followed of allowing the original policy to lapse, and then reviving it as a paid-up policy for a reduced amount. Mr. Lemon is not correct in attributing to me the passage he quotes from the editorial article in *J.I.A.*, xix, 47. The article itself was written by me, but the quoted passage was an extract, as I clearly stated, from the letter of a correspondent; and, although I thought it desirable to give publicity to my friend's views, I am not to be held as endorsing them. Lastly, as regards the settlement policies under the Married Women's Property Acts, further experience has not led me to modify in any way the opinion I formerly expressed, namely, that these policies are beneficial, as enabling a man to make a provision for his family which will not be affected by any business reverses. I also believe that almost all the difficulties that have arisen in connection with these policies may be avoided if they are properly worded. I am surprised to hear that it is the general custom to allow these settlement policies to be surrendered, the practice in my own office being only to pay the surrender-value when it is of very small amount, and in other cases to convert the policy into a paid-up policy after it has lapsed, this being, I believe, in most cases the most beneficial course for the family."

The President, continuing, said he had heard with surprise the opinion of Mr. Bailey that they could afford to dispense with the Gambling Act. If there was one subject in connection with the administration of insurance companies upon which the judges had of late years been more stringent and severe in their language, it was with regard to this very question. It was true that the question had usually been in relation to industrial policies, but surely the greater the sum at stake the greater the temptation, and the greater the necessity for caution and protection. Although it was perfectly true that they had long ceased to follow the question of insurable interest after the policy had been granted, he was not aware that they had abandoned the practice of requiring to be satisfied at the time the policy was granted that there existed the proper insurable interest, and he hoped they would never cease to require evidence on that point. The question of probate duty had been referred to by Mr. Bell. The Board of Inland Revenue issued, a year or two ago, a circular to insurance companies upon that very question. He, himself, was in communication with the Secretary to the Board on that point, because they originally contended that insurance companies were bound to see that the amount upon which duty had been paid was larger than the sum that they had to pay. He pointed out that that was not correct, and they at last practically confessed that his view was right, and that it was the gross assets to which they ought to look. As to the question of stamps, he possibly had greater opportunities than any other member of the Institute of knowing what was about to happen with regard to the Customs and Inland Revenue Act

of 1888. That Act had received a great deal of denunciation in that room. To some extent he was in harmony with the opinions expressed about it. What he had to complain of was not so much the spirit and the apparent intention of that Act as its administration. The great injury which the Act had done was making the matter retrospective. If the Act had been made prospective only, he thought the enactment a righteous one. Mr. Sprague had pronounced a very emphatic opinion that it was both legal and moral to do that which, with all respect to Mr. Sprague, and those who thought with him, he (the President) would say was an intentional evasion of the law. Stamps were not imposed to be accepted or not as they pleased: they were imposed for the purpose of revenue. It was the duty of every honest citizen to pay the duty which the law said he should pay. The law, for its own protection, had invented a great variety of penalties. Take, for example, the very simple case of the penny receipt stamp, where a penalty of £10 could be recovered in a court of law on the motion of a common informer. That was a penalty out of all proportion to the amount of tax which had been evaded. With regard to policies of insurance, the penalty followed the general penalty imposed with regard to all deeds, and with deeds dealing with land or real property the penalty was usually sufficient to protect the revenue. An unstamped deed, unless stamped at the request of the purchaser, would void a title, and that was a penalty which no man would be likely to incur. This penalty, in connection with policies of insurance, had been proved to be utterly inadequate. And why? For reasons altogether apart from the question of stamps, or the justice or injustice of the amounts of the particular stamps imposed, the offices did not or dared not enforce the penalty. As the case stood before the passing of the Act of 1888, a man with an unstamped deed could come to the offices and say, "Pay me my money." They might reply, "Your deed must be stamped, and till it is stamped we will not pay." Without discussing the matter further, the man served the company with a writ. They, if they pleased, followed him into court, he would pay the stamp and penalty across the table in open court, he would recover a verdict, and the company would be mulcted in costs. Apart from the obloquy of the transaction, the penal consequences were of a nature to deter insurance companies from attempting to enforce the penalty. What was the consequence? That it was the custom of men who ought to have known better—of bankers, for example—to have these unstamped deeds by the thousand, and to refuse to pay the duties which the law had said they should pay. Was not that a condition of things that needed to be remedied; and in what way could it be remedied? The only way was to create a protection for insurance companies, and the object of that clause was as much for their protection as for that of the revenue. It enabled the companies to say they could not pay without the stamp, and a clause of that kind was a perfectly right and proper clause. A great deal had been said of the difficulties which existed with regard to these stamps, but he thought those difficulties were exaggerated. In the first place, with regard to all deeds executed prior to 16 May 1888, they might safely take an indemnity from the persons presenting those deeds against any penal

consequences. In the second place, he doubted whether it was necessary that so many of the deeds which so often were presented to insurance companies in connection with a claim, need be stamped. Only that day he had had brought to his notice a letter received from the Board of Inland Revenue, which stated that A having mortgaged his policy to B, and having taken a re-assignment from B, one or other or both of the deeds being unstamped need not prevent the insurance company paying the sum insured without calling for those deeds to be stamped; they were no part of the title, and in that expression—deeds which formed the necessary title—was the solution of a large portion of the difficulty. If there was link after link running through the deeds, every one of them would have to be stamped, but when they were mere excrescences on the title it was very doubtful indeed whether those deeds required to be stamped. That letter might be contradicted to-morrow, but, making that allowance, it brought before them a position which, to a certain extent, would alleviate the stress of the difficulties they found with regard to the Stamp Act. Was it possible that they could do anything to make the assignment of policies simpler, and thus get rid of all question of stamps? They never heard of any difficulties with regard to stamps on transfer of railway stock; and if their policies could be transferred in the same way by entries in their own registers, all questions of stamp duty would be avoided. He had been pleased to hear what Mr. Trower had said upon that question. He had wondered why the actuaries had been asleep for four years. It was because they had been afraid of the lawyers. If Mr. Trower could assure them that the lawyers would welcome the change in system the thing could be done. He believed the Institute of Actuaries would probably take up this question with more spirit and enthusiasm than of late. There was one difficulty which would have to be considered, namely, the case of an assured who wanted to create three or four charges on his policy, how could the second mortgagee be protected if only one person was to be on the register? This was, however, a difficulty of a kind which, no doubt, the ingenuity of the actuary and the lawyer would easily solve.

Mr. Lemon, in reply, said that the various speakers had answered one another on several subjects, leaving to him only the more difficult points, and such that could not be replied to off-hand. He was surprised that anyone wished to get rid of the Gambling Act, which, to his mind, contained very necessary provisions. With reference to the production of deeds, he should be glad to find an authority, such as Mr. Hendriks had referred to, that would give insurance companies the power to require the production of deeds. Mr. McKee had pointed out one or two matters in which, perhaps, he was in error, but they were mostly capable of being defended. He referred to the question of a stamp where the assignment and re-assignment were not both executed abroad. He (Mr. Lemon) was particularly careful not to deal with the case where an assignment was executed abroad and a re-assignment here, because that was one which presented difficulties to his mind. Probably both deeds would have to be stamped. But where both were executed abroad, the policy of assurance being a *chose in action*, the effect of dealing with it would

seem, from the decision in "Lee *v.* Abdy", to be determined by the law of the country in which the contract was made, and it appeared that the assignment and re-assignment both being in a foreign country would in no way be affected by the English revenue law. Further, if the statement in the letter which the President had quoted was accepted, assignment and re-assignment to original holder were no longer to be regarded as links in the chain of title. Mr. Sprague referred to deposits and payment under discount as, perhaps, affecting the position of the depositee. He had not considered the subject with reference to payment under discount, as that was now so rarely done. Then he referred to an unstamped deed or an improperly-stamped deed, and payment with the consent of the assignee. But if that consent was given with a view to defrauding the Customs and Inland Revenue Act, the whole receipt was void. As to verbal notice of assignment, the Text-Books certainly laid down that prior to the Act of 1867 such notice had to be recorded. He, in conclusion, begged to apologize to Mr. Sprague for having attributed to him words which he did not himself write, and to thank the meeting for the kind reception accorded to his paper.

On a Method frequently adopted of treating Under-average Lives for Assurance Purposes, by making Temporary Deductions from the Sums Assured. By A. W. Sunderland, M.A., F.I.A.

[Read before the Institute, 23 March 1891.]

My first impressions on considering the magnitude and pretensions of the investigation which I now present to the Institute were that it would perhaps be more suitable for a communication to the *Journal,* or that at any rate it would scarcely provide sufficient matter to occupy the attention of the meeting for a whole evening. This impression was strengthened when I considered the importance and wide scope of the researches which have been presented as food for our reflection at many of the meetings held in recent years. But, on further consideration, it appeared the subject I had chosen was one likely to give rise to useful discussion. Also I feel that, from a good number of the members of the Institute, at any rate, on whose time and attention the cares of business are continually making greater and greater demands, I may hope for forgiveness if I place before them a somewhat lighter paper than usual.

In these days of severe competition continually greater and greater efforts are made to meet the requirements of the public. One consequence of this is that novel modes of insurance and

refinements of practice, undreamt of in the middle of this century, are devised so that all tastes, even the most fastidious, may be suited. Whether these inventions have materially helped the cause of life assurance it would perhaps be a difficult task to determine. One of the methods by which it is sought to popularise life assurance is that which forms the subject of this paper. There is a natural objection on the part of a man who proposes to insure his life to be requested to pay a premium at a rate higher than that which corresponds to his actual age. In addition to the pecuniary loss to him arising from the surcharge, there is also the stigma of inferior vitality stamped upon him as it were by the office. In very numerous cases he resents the imposition of an extra premium and the reproach which the surcharge seems to convey, and a method of soothing his wounded feelings and relieving him from pecuniary damage if he should turn out a good life has been adopted by certain offices. It consists in issuing the policy at the ordinary rate of premium, but making a temporary deduction from the sum assured, so that, in the language of friendly societies, the man does not enter upon full benefit until after the lapse of a specified period, reckoned from the issue of the policy. The deduction from the sum assured may remain fixed until the period expires, or it may fall by equal decrements within the period until it is extinguished at the end of it. Other modifications are occasionally introduced, as, for instance, that no bonuses vest until the expiration of the period. The plan most frequently adopted is to make a deduction from the sum assured, which decreases uniformly throughout the expectation of life, at the end of which it is extinguished, and this is the method I propose to discuss this evening. The policies considered will be those for the whole term of life. The problem, then, before us is as follows:

A life having been rated up a certain number of years on a whole-term policy, the proposer pays the ordinary premium, and the defective vitality of the life assured is allowed for by a deduction from the sum assured, this deduction decreasing annually by uniform amounts throughout the expectation of life, at the end of which period it is extinguished. Find a formula for calculating these deductions.

Let x be the rated-up age, and m the expectation of life at the real age, m being taken to the nearest integer. The assurance consists of an immediate assurance and a temporary annual increase. Let X denote the annual increase, the ultimate sum

assured being unity. Thus the sum assured in the first year is $1-mX$, and the increasing assurance is

X	in the	second	year.
2X	,,	third	,,
. . .	. . .	. . .	. . .
mX	,,	$(m+1)$th and subsequent years.	

Considering, in the first instance, pure premiums only, the annual premium for the immediate assurance is

$$\pi_x(1-mX),$$

and for the increasing assurance

$$X\frac{R_{x+1}-R_{x+m+1}}{N_{x-1}}.$$

If r is the addition made to the age, π_{x-r} is the pure premium paid, and we therefore have

$$\pi_{x-r}=\pi_x(1-mX)+X\frac{R_{x+1}-R_{x+m+1}}{N_{x-1}} \quad . \quad . \quad . \quad (1)$$

an equation from which X may be found.

Let us now consider the question when office premiums are taken account of.

Suppose a policy whose ultimate amount is unity to be effected on a life aged $x-r$, x being the rated-up age, and of the office premium paid upon it let e denote the portion applicable for expenses and shareholders' profits, and b that for bonuses. If p denote the remaining portion of the premium, p must be that which provides the sum assured, and therefore

$$p=(1-mX)\pi_x+X\frac{R_{x+1}-R_{x+m+1}}{N_{x-1}},$$

X and m having the same meanings as before.

Since the premium paid is the office premium for unit policy at age $x-r$, a suitable symbol for it is P_{x-r}. Accordingly, we have

$$P_{x-r}=(1-mX)\pi_x+X\frac{R_{x+1}-R_{x+m+1}}{N_{x-1}}+e+b.$$

It will, I think, be considered reasonable to assume that e, the annual contribution which the policy makes to expenses and shareholders' profits, is the same as the contribution made by unit policy effected at age $x-r$. If, for this latter policy, δ_{x-r}

denotes the portion of the premium which is supposed to provide the bonuses, we have, on the foregoing assumption as to expenses and shareholders' profits,

$$P_{x-r}=e+\delta_{x-r}+\pi_{x-r},$$

whence we obtain

$$\pi_{x-r}=(1-m\mathrm{X})\pi_x+\mathrm{X}\frac{\mathrm{R}_{x+1}-\mathrm{R}_{x+m+1}}{\mathrm{N}_{x-1}}+b-\delta_{x-r} \quad . \quad (2)$$

The value to be given to b in this equation will depend on the manner in which the increasing assurance is treated in the matter of bonus.

In the first place, let us suppose the policy gets the same cash allotments (not the same reversionary bonuses), as unit policy effected at age $x-r$ on an ordinary life. Then $b=\delta_{x-r}$ and we obtain equation (1) over again. In this case, then, the reduction in the sum assured is the same, whether the office premium or the pure premium for the policy is taken account of. In par-cular, formula (1) is applicable to non-participating policies.

As a second case, suppose the policy takes the same cash allotments as unit policy effected at age x. This will be the case, for instance, where what is called the uniform reversionary bonus system is adopted, on the assumption that for distribution purposes the policy is regarded as a unit policy. Here we have

$$\pi_{x-r}=(1-m\mathrm{X})\pi_x+\mathrm{X}\frac{\mathrm{R}_{x+1}-\mathrm{R}_{x+m+1}}{\mathrm{N}_{x-1}}+\delta_x-\delta_{x-r}.$$

Formula No. (2) is more general than formula No. (1), but, after consideration, I have come to the conclusion that in ordinary cases it would be a useless refinement to consider values of b differing from δ_{x-r}. On making enquiries I am informed at one office that "the bonuses on policies thus rated up are allotted precisely the same as if the lives were accepted as first-class", and at another that "the bonus is allotted at each division on the "actual amount which would then be payable in the event of a "claim." For practical purposes I shall disregard these differences in the mode of allotting bonuses, and assume that the policies get the same cash allotments as ordinary policies effected at the same actual ages, that is to say, formula (1) will be the one made use of. Although, however, in the tables no use will be made of formula No. (2), yet both this formula itself and the analysis by which it has been obtained are, I venture to think, instructive. They show us under what circumstances and from what cause the deductions from the sums assured obtained when

office premiums are taken account of in the calculations differ from the deductions obtained when pure premiums are assumed.

The equation (1) from which X is to be found may be written in the form

$$X\left(m\pi_x - \frac{R_{x+1} - R_{x+m+1}}{N_{x-1}}\right) = \pi_x - \pi_{x-r},$$

and from it the value of X, and the initial deduction from the sum assured, namely mX may be found without difficulty for any mortality table and rate of interest for which π and R are tabulated. In the annexed tables, columns (3) and (4) give the values of X and mX respectively. They are based upon the H^M Table, and two rates of interest, namely, $3\frac{1}{2}$ and 4 per-cent, have been taken. The first and second tables apply to lives rated up five years, the third and fourth to lives rated up 10 years.

From the nature of the case it was scarcely to be expected that the tables of the values of X and mX would proceed with great regularity. One disturbing element is the expectation of life, which in these calculations is not a continuous function, since it has to be taken to the nearest integer.

A method commonly employed for calculating the initial deduction from the sum assured is to take the difference between the office premiums for the full amount of the policy at the actual age and the rated-up age, and to multiply this difference by the expectation of life. Thus, if the actual age be 30 next birthday and 35 the rated-up age, and the office rates for these ages be £2. 10*s.* and £2. 17*s.* 6*d.* per £100 assured, the expectation of life by the H^M Table would be 35, and the initial deduction from the full amount of the policy would be £13. 2*s.* 6*d.* for each £100 ultimately assured. A policy on which a premium of £2. 10*s.* was paid would begin by assuring £86. 17*s.* 6*d.*, and the sum assured would increase by 7*s.* 6*d.* each year until, after 35 years had elapsed, the ultimate amount of £100 would be reached. This method may be described as empirical, since it has no pretensions to being founded on any scientific basis. It by no means follows that it should on that account be rejected without investigation, to see whether it produces results roughly in accordance with those which theory would dictate. The usual method of rating-up is itself a somewhat empirical method of dealing with under-average lives. Not only are important differences of opinion frequently found to exist as to the eligibility of a life for assurance, but the method of treating an inferior life by making an addition to the age is far from perfect. The practical business

of life assurance, however, compels us frequently to make a compromise and modify the results of theory (which may be impossible to carry out strictly in practice), so as to establish a convenient *modus operandi*. Theory, however, cannot be neglected. It is necessary, in order that life assurance may be carried on as a business with safety, that the dictates of theory should underlie any practical methods which may be adopted, and that, speaking generally, the terms granted to persons who effect policies should, in the main, conform to those which theory would prescribe.

Let us apply these considerations to the subject of the present paper, and endeavour to form a comparison between the deductions from the sum assured obtained by theory, and by the practical method already noticed. To form a table of the deductions which the practical method leads to, recourse may be had to the published rates of an insurance office, but a more useful table—and one independent of the rates quoted by a particular office—may be obtained in the following manner.

The tables of office rates usually quoted may, with fair exactness, be obtained from the H^M pure premiums by an addition consisting of two parts, one constant throughout the table, the other a constant percentage of the pure premium. Thus, we may put

$$P_{x-r}=\mu+\lambda\pi_{x-r}$$

$$P_x\ =\mu+\lambda\pi_x$$

Subtracting, we have

$$P_x-P_{x-r}=\lambda(\pi_x-\pi_{x-r}),$$

so that the difference between the office premiums for the real and rated up ages is proportional to the difference between the corresponding pure premiums. This, for any particular table of office premiums, will not be exactly the case, but near enough for the purposes of the comparison I have in view. The above equation, $P=\mu+\lambda\pi$, would for any office table probably be found to hold fairly well whether π be taken at $3\frac{1}{2}$ or 4 per-cent, though, of course, different values of μ and λ would apply to the two different cases. I shall take π at 4 per-cent, and assume $\lambda=1{\cdot}25$. With these values, the empirical deductions, as I will call them, will not, I think, be under-estimated for an ordinary with-profit table of rates. Dividing the values of $1{\cdot}25\pi$ by the corresponding values of X, we obtain the ratios of the empirical to the theoretic deductions, and so are enabled to judge whether the

empirical deductions substantially meet the requirements of the case.

Referring again to the tables appended to this paper, column (5), appearing only in Tables I and III, shows the values of $\pi_x - \pi_{x-r}$ by the H^M Table at 4 per-cent interest. The column (5) for Table II would be the same as for Table I, and for Table IV the same as for Table III. It will be convenient to use the symbol Y to denote the function tabulated in column (5), so that $m\lambda Y$ will denote the empirical deduction from the sum assured as opposed to mX, the theoretic deduction. Dividing the numbers of column (5) by the corresponding numbers of column (3), we get the ratios of the empirical to the theoretic deductions for $\lambda = 1$. In order to obtain the ratio of the empirical to the theoretic deductions for any particular table of rates, we have only to multiply the numbers of column (6) by the appropriate value of λ. The numbers in column (7) have been obtained from column (6) by multiplying by the value of λ mentioned above, namely, 1·25. This value will, I think, apply fairly well to an ordinary table of with-profit rates, and will probably not under-estimate the effect of the empirical deductions.

It is important to notice that column (6) has been formed merely for the purpose of calculating approximate values of the empirical deductions as found from any actual table of office premiums, and that, accordingly, though it has been obtained from the table of H^M 4 per-cent pure premiums, it is appropriate to use it whether the theoretic deductions are estimated at $3\frac{1}{2}$ per-cent or 4 per-cent interest.

An inspection of the appended tables show us that, generally speaking, the empirical deductions fall far short of those required by theory. This may also be seen at a glance from the following specimen table, which relates to a £1,000 policy. In it the theoretic deductions are those given by the H^M Table, with 4 per-cent interest, and the empirical deductions have the values given by $1{\cdot}25mY \times 1{,}000$, Y having the value given above.

Real Age	Deduction for $r=5$		Deduction for $r=10$	
	Theoretical	Empirical	Theoretical	Empirical
	£	£	£	£
20	311	96	580	223
30	381	131	663	299
40	464	173	755	402
50	514	229	793	543
60	527	300	782	726

The differences, it will be observed, between the theoretic deductions and the empirical are very great, except at the higher ages for $r=10$, far too great, as I think, to be disregarded in practice. For lives rated up five years the empirical deductions appear to allow for a rating up of less than two years. I would not, however, be thought to lay too great stress on the above figures and those of the appended tables. For real age 30, for example, there appears, for a £1,000 policy, to be a great difference between a deduction of £131 and a deduction of £381, but the deduction, it is to be remembered, is largest in the early years of life, and of the policy, when the risk of a claim arising is not so great as in the later years. I may here draw attention, by way of parenthesis, to the fact that I have taken no account of selection. Even had Mr. Sprague's "Select Mortality Tables", as published in the *Journal,* included values of the function R_x, it is doubtful whether they would have been at all suitable for dealing with under-average lives. Assuming the theoretic deductions are those which should be made, one mode of estimating the benefit granted to the person effecting the policy by making only the empirical deductions, is to find the difference between the premium which should be paid and that which actually is paid. Taking the illustration given above for example, the deduction £381 corresponds to a deduction of about 10 times 7*s.* 6*d.* in the office premium, and the deduction £131 may correspond to a deduction of about one-third of this amount, say 10 times 2*s.* 6*d.* The difference between the two, namely, 5*s.* per £100 assured, would be about 10 per-cent of the office premium charged.

In defence of the empirical method there is probably much to be said. In the first place, the adoption on the part of the person who effects the policy of a temporary deduction from the sum assured instead of the usual surcharge, if, as is usually the case, the assured is also the life assured, may be taken as an indication that he thinks favourably of his prospects of living, and the superior vitality of the lives assured under endowment-assurance policies affords ground for the opinion held by some actuaries, that weight should be attached to a man's own feeling as to his prospects of living. Again, as pointed out to me by Mr. Rothery, the office is placed, to some extent, in a position of advantage by this temporary reduction in the sum assured to meet a failure to come up to the normal standard. The surcharge is not spread uniformly over the policy's existence, but is heavier at the beginning, so that there is less inducement for a rated-up life to

endeavour, after the policy has been some years on foot, to try to get the surcharge reduced or removed.

The insufficiency of the empirical deduction might, perhaps, be met by some special provision in regard to the amounts of the bonuses to be allotted to these policies; but such special provision would probably have to depend on the circumstances of particular offices, and I have not entered upon the discussion of this part of the subject.

I desire here to express my thanks to Mr. H. C. Thiselton and Mr. R. Todhunter for the assistance they have kindly rendered me in the preparation of the tables appended to this paper. Of these tables I give below an explanation for convenience of reference.

Explanation of Tables.

Column (1).—$x-r$, the actual age at entry.

Column (2).—m, the expectation of life at the actual age at entry by the H^M Table, taken to the nearest integer.

Column (3).—X, the theoretic value of the amount by which the policy increases each year until the full sum assured is attained.

Column (4).—mX, the theoretic value of the total initial deduction from the sum assured.

Column (5).—$\pi_x - \pi_{x-r}$, by H^M 4 per-cent. This function is represented by the symbol Y. The column is given for the purpose of forming a table of empirical deductions, applicable to any given table of office rates.

Column (6).—This gives the ratio of the numbers of column (5) to the corresponding numbers of column (3). Multiplying the numbers of this column by the value of λ appropriate to any table of office rates, we get the ratios of the empirical to the theoretic deductions.

Column (7).—The numbers in this column are those of column (6) multiplied by 1·25, and may be considered as showing approximately the ratios of the empirical to the theoretic deductions for an ordinary table of with-profit rates.

TABLE I.

Lives rated up 5 Years. X by H^M $3\frac{1}{2}$ *per-cent.*

Age at Entry	m	X	mX	Y	Y : X	1·25Y : X
(1)	(2)	(3)	(4)	(5)	(6)	(7)
15	46	·008010	·3685	·001928	·2407	·3009
16	45	·007867	·3540	·001869	·2376	·2970
17	44	·007719	·3396	·001805	·2339	·2923
18	44	·007334	·3227	·001765	·2407	·3008
19	43	·007442	·3200	·001771	·2380	·2975
20	42	·007746	·3253	·001834	·2368	·2960
21	41	·008183	·3355	·001933	·2362	·2953
22	41	·008312	·3408	·002058	·2476	·3095
23	40	·008835	·3534	·002185	·2473	·3091
24	39	·009340	·3643	·002305	·2468	·3085
25	38	·009820	·3731	·002414	·2458	·3073
26	38	·009768	·3712	·002515	·2575	·3219
27	37	·010212	·3778	·002616	·2562	·3202
28	36	·010702	·3853	·002729	·2550	·3188
29	35	·011251	·3938	·002856	·2538	·3173
30	35	·011197	·3919	·002996	·2676	·3345
31	34	·011812	·4016	·003150	·2667	·3333
32	33	·012467	·4114	·003309	·2654	·3318
33	32	·013161	·4211	·003473	·2639	·3299
34	32	·013046	·4175	·003645	·2794	·3493
35	31	·013772	·4269	·003828	·2780	·3475
36	30	·014574	·4372	·004032	·2767	·3458
37	30	·014463	·4339	·004267	·2950	·3688
38	29	·015380	·4460	·004533	·2947	·3684
39	28	·016381	·4587	·004822	·2914	·3680
40	27	·017473	·4718	·005133	·2938	·3672
41	27	·017275	·4664	·005452	·3156	·3945
42	26	·018336	·4767	·005769	·3146	·3933
43	25	·019449	·4862	·006086	·3129	·3912
44	25	·019053	·4763	·006417	·3368	·4210
45	24	·020197	·4847	·006765	·3350	·4187
46	23	·021489	·4942	·007149	·3327	·4159
47	22	·022970	·5053	·007584	·3302	·4127
48	22	·022515	·4953	·008069	·3584	·4480
49	21	·024153	·5072	·008593	·3558	·4447
50	20	·025996	·5199	·009162	·3524	·4406
51	20	·025414	·5083	·009765	·3842	·4803
52	19	·027358	·5198	·010400	·3801	·4752
53	18	·029536	·5317	·011073	·3749	·4686
54	18	·028745	·5174	·011801	·4106	·5132
55	17	·031113	·5289	·012573	·4041	·5051
56	16	·033833	·5413	·013393	·3959	·4948
57	16	·032804	·5249	·014258	·4346	·5433
58	15	·035791	·5369	·015168	·4238	·5298
59	14	·039265	·5497	·016113	·4104	·5130
60	14	·037922	·5309	·017124	·4516	·5645

Table II.

Lives rated up 5 Years. X by H^M 4 per-cent.

Age at Entry	m	X	mX	Y : X	1·25Y : X
(1)	(2)	(3)	(4)	(6)	(7)
15	46	·007790	·3583	·2475	·3094
16	45	·007605	·3422	·2458	·3072
17	44	·007419	·3264	·2433	·3041
18	44	·007024	·3090	·2513	·3141
19	43	·007102	·3054	·2494	·3117
20	42	·007396	·3106	·2480	·3100
21	41	·007830	·3210	·2469	·3086
22	41	·007987	·3275	·2577	·3221
23	40	·008508	·3403	·2568	·3210
24	39	·009014	·3515	·2557	·3197
25	38	·009489	·3606	·2544	·3180
26	38	·009455	·3593	·2660	·3325
27	37	·009886	·3658	·2646	·3308
28	36	·010368	·3733	·2632	·3290
29	35	·010910	·3818	·2618	·3272
30	35	·010882	·3809	·2753	·3441
31	34	·011493	·3907	·2741	·3426
32	33	·012140	·4006	·2726	·3407
33	32	·012824	·4104	·2708	·3385
34	32	·012740	·4077	·2861	·3576
35	31	·013454	·4171	·2845	·3557
36	30	·014246	·4274	·2830	·3538
37	30	·014175	·4252	·3010	·3763
38	29	·015087	·4375	·3005	·3756
39	28	·016085	·4504	·2998	·3747
40	27	·017169	·4636	·2990	·3737
41	27	·017015	·4594	·3204	·4005
42	26	·018069	·4698	·3193	·3991
43	25	·019167	·4792	·3175	·3969
44	25	·018806	·4701	·3412	·4265
45	24	·019938	·4785	·3393	·4241
46	23	·021213	·4879	·3370	·4213
47	22	·022678	·4989	·3344	·4180
48	22	·022271	·4900	·3623	·4529
49	21	·023894	·5018	·3596	·4495
50	20	·025721	·5144	·3562	·4453
51	20	·025185	·5037	·3877	·4847
52	19	·027109	·5151	·3836	·4795
53	18	·029263	·5267	·3784	·4730
54	18	·028518	·5133	·4138	·5173
55	17	·030864	·5247	·4074	·5092
56	16	·033553	·5369	·3992	·4989
57	16	·032570	·5211	·4378	·5472
58	15	·035523	·5328	·4270	·5337
59	14	·038956	·5454	·4136	·5170
60	14	·037661	·5273	·4547	·5684

TABLE III.

Lives rated up 10 *Years.* X *by* H^{M} $3\frac{1}{2}$ *per-cent.*

Age at Entry	m	X	mX	Y	Y ÷ X	1·25Y ÷ X
(1)	(2)	(3)	(4)	(5)	(6)	(7)
15	46	·013186	·6066	·003762	·2853	·3566
16	45	·013356	·6010	·003802	·2847	·3558
17	44	·013580	·5975	·003863	·2845	·3556
18	44	·013288	·5847	·003950	·2973	·3716
19	43	·013707	·5894	·004076	·2974	·3717
20	42	·014270	·5993	·004248	·2977	·3721
21	41	·014938	·6124	·004448	·2978	·3722
22	41	·014944	·6127	·004674	·3128	·3910
23	40	·015682	·6273	·004914	·3134	·3917
24	39	·016436	·6410	·005161	·3140	·3925
25	38	·017201	·6536	·005410	·3145	·3932
26	38	·017064	·6484	·005665	·3320	·4150
27	37	·017816	·6592	·005925	·3326	·4157
28	36	·018623	·6704	·006202	·3330	·4163
29	35	·019499	·6825	·006501	·3334	·4168
30	35	·019303	·6756	·006824	·3535	·4419
31	34	·020248	·6884	·007182	·3547	·4434
32	33	·021269	·7019	·007576	·3562	·4453
33	32	·022367	·7158	·008006	·3579	·4474
34	32	·022104	·7073	·008467	·3831	·4788
35	31	·023243	·7205	·008961	·3855	·4819
36	30	·024460	·7338	·009484	·3877	·4847
37	30	·024114	·7234	·010036	·4162	·5202
38	29	·025389	·7363	·010619	·4183	·5228
39	28	·026763	·7494	·011239	·4200	·5249
40	27	·028245	·7626	·011898	·4212	·5266
41	27	·027739	·7489	·012601	·4543	·5678
42	26	·029256	·7607	·013353	·4564	·5705
43	25	·030887	·7722	·014155	·4583	·5729
44	25	·030225	·7556	·015010	·4966	·6208
45	24	·031925	·7662	·015927	·4989	·6236
46	23	·033803	·7775	·016914	·5004	·6255
47	22	·035893	·7897	·017984	·5010	·6263
48	22	·035070	·7715	·019142	·5458	·6823
49	21	·037321	·7837	·020394	·5465	·6831
50	20	·039828	·7966	·021735	·5457	·6822
51	20	·038844	·7769	·023158	·5962	·7452
52	19	·041502	·7885	·024658	·5941	·7427
53	18	·044483	·8007	·026241	·5899	·7374
54	18	·043257	·7786	·027914	·6453	·8066
55	17	·046489	·7903	·029697	·6388	·7985
56	16	·050203	·8033	·031623	·6299	·7874
57	16	·048746	·7799	·033725	·6919	·8648
58	15	·052882	·7932	·036040	·6815	·8519
59	14	·057729	·8082	·038633	·6692	·8365
60	14	·056010	·7841	·041507	·7411	·9263

Table IV.

Lives rated up 10 Years. X *by* H^M 4 *per-cent.*

Age at Entry	m	X	mX	Y : X	1·25 Y : X
(1)	(2)	(3)	(4)	(6)	(7)
15	46	·012797	·5886	·2940	·3675
16	45	·012931	·5819	·2940	·3675
17	44	·013126	·5775	·2943	·3679
18	44	·012847	·5653	·3075	·3843
19	43	·013247	·5696	·3077	·3846
20	42	·013804	·5798	·3077	·3847
21	41	·014466	·5931	·3075	·3844
22	41	·014514	·5951	·3220	·4025
23	40	·015251	·6101	·3222	·4028
24	39	·016007	·6243	·3224	·4030
25	38	·016768	·6372	·3226	·4033
26	38	·016671	·6335	·3398	·4218
27	37	·017416	·6444	·3402	·4253
28	36	·018218	·6558	·3404	·4256
29	35	·019088	·6681	·3406	·4257
30	35	·018936	·6627	·3604	·4505
31	34	·019880	·6759	·3613	·4516
32	33	·020900	·6897	·3625	·4531
33	32	·021995	·7038	·3640	·4550
34	32	·021785	·6971	·3887	·4858
35	31	·022920	·7105	·3910	·4887
36	30	·024135	·7240	·3930	·4912
37	30	·023844	·7153	·4209	·5261
38	29	·025116	·7283	·4228	·5285
39	28	·026485	·7416	·4243	·5304
40	27	·027963	·7550	·4255	·5319
41	27	·027511	·7428	·4580	·5725
42	26	·029023	·7546	·4601	·5751
43	25	·030646	·7662	·4619	·5774
44	25	·030033	·7508	·4998	·6247
45	24	·031725	·7614	·5020	·6275
46	23	·033591	·7726	·5035	·6294
47	22	·035669	·7847	·5042	·6302
48	22	·034899	·7678	·5485	·6856
49	21	·037138	·7799	·5491	·6864
50	20	·039632	·7926	·5484	·6855
51	20	·038698	·7740	·5984	·7480
52	19	·041339	·7854	·5965	·7456
53	18	·044298	·7974	·5924	·7405
54	18	·043121	·7762	·6473	·8091
55	17	·046331	·7876	·6110	·8012
56	16	·050020	·8003	·6322	·7903
57	16	·048611	·7778	·6938	·8672
58	15	·052721	·7908	·6836	·8545
59	14	·057537	·8055	·6714	·8393
60	14	·055870	·7822	·7129	·9287

Discussion.

The President (Mr. B. Newbatt) having invited discussion,

Mr. H. J. Rothery said that the subject of the paper was one of practical importance, and it should afford a good opportunity for discussion by the younger members as well as the seniors of the profession. The question of the methods by which proposers who, naturally objecting to the payment of extra premiums, might be satisfied was always one of difficulty, and he could say from his own experience that the plan of charging the ordinary rate of premium and imposing a surcharge in the form of a temporary deduction from the sum assured, either fixed or decreasing, had worked satisfactorily. He thought it was unfortunate that the author had confined himself only to the plan of decreasing deduction, and considered only one empirical method among the many that were probably in practical use. There was nothing to be said against the formula Mr. Sunderland had obtained, except that probably few actuaries would be willing to take into account the refinement introduced by dividing the premium into its constituent parts. The tables given certainly showed that the empirical method referred to by Mr. Sunderland was not suitable and did not give sufficiently large deductions. It would be interesting to know Mr. Sunderland's own opinion as to what plan should be adopted. Mr. Sunderland had not made any reference to former writers on the subject. He thought Mr. Morrice Black, whose name was attached to other improvements in life assurance practice, had been the first to mention the system of a fixed deduction, and in 1861 he published a pamphlet in which he gave formulas. Reference was made to that pamphlet by Mr. Younger in the 10th volume of the *Journal*, where he made the curious remark that he proposed "to *assume as a fundamental truth* that the fact of a person whose life was considered an extra-hazardous one living to the end of the expectation period, is complete evidence that medical opinion was wrong, and that no additional premium was necessary." It seemed to him (Mr. Rothery) that they could not follow him in that conclusion. He practically assumed that on the average the medical estimate was always too strict, and sought to find a plan of reducing the extra rate. Mr. Younger was replied to by Mr. Brabrook, who went to the other extreme, saying that "indeed any debt whatever on a policy was unsound and subversive of the principles of assurance." Other notes on the subject had appeared in volumes of the *Journal*, but nothing need be mentioned except Mr. Makeham's paper in the 17th volume, where he referred to the very important question of the *incidence* of the extra risk, which, after all, must be an important factor in assessing the surcharge, whether in one form or another. Mr. Makeham had given some convenient rules for ascertaining the deductions, but in his calculations he was compelled to do what Mr. Sunderland had done, namely, assume that if a life was rated-up a certain number of years, it must be treated for every purpose as a healthy life of the rated-up age—an assumption very wide of the truth. It seemed a matter of great difficulty to get any experience from assurance companies which would be of use to them in discussing the question of the incidence of the extra mortality.

The classification of risks would be very difficult, and the policies would have to be subdivided into so many different groups that stability in the results could not be expected. He would mention as an addition to Mr. Sunderland's paper that the theoretical formula for the amount of the deduction to be fixed during the expectation period could be written

$$\frac{P_x - P_{x-r}}{P_x - P_{m|}A_x}.$$

In making practical use of that plan, they might modify somewhat the theoretical results obtained. In theory, the extra premium was to be imposed during the whole future lifetime, but in practice that was not always the case, as applications for the removal of the surcharge were sometimes successfully made. He believed their President thought that such a concession should never be granted; but others held the opposite view. Again, if a life were rated-up ten years, and a few years afterwards was so improved in health that he could be assured in another office for an addition of three or five years, or at table rates, the surcharge must be removed or the assurance would no doubt be transferred. The deductions were not so likely to be questioned, as they did not immediately affect the proposer's pocket. On the other hand, if it were assumed that the surcharge was only to be imposed during the expectation period, the formula became

$$\frac{P_x - P_{x-r}}{P^1_{x.\overline{m}}};$$

but there was only a slight difference between the financial results of this formula and that previously given. The plan adopted in the office with which he was connected was one that answered well practically, although it was not theoretically sound. It was to calculate what amount of assurance the normal premium would purchase at the rated-up age, and to treat that as the sum assured during the expectation period. The formula (which really assumed that the deduction would operate throughout the whole of life) was

$$\frac{P_x - P_{x-r}}{P_x},$$

so that the assured was given the benefit of a deferred assurance without payment. He might mention as an interesting point that the value of the future extra premiums on a policy at the time of its issue was exactly the same as the value of a policy effected at age $x-r$, which had endured for r years. Thus:—$(P_x - P_{x-r})(1+a_x)$ $={}_rV_{x-r}$. The surcharged proposers to his office were always given the option of the fixed or decreasing debt, but they nearly always chose the former. In considering the plans of deduction in lieu of extra premium, it might be thought of advantage that under them the amount at risk was reduced at that period of the assurance where the difference between the amount assured and the reserve value was greatest.

Mr. W. O. Nash said that from one point of view it might be considered that, in the case under discussion, the office took the

assurance at the rated-up premium and re-assured with the policy-holder himself a certain part of the risk during the earlier years. Mr. Sunderland's formula

$$X\left(m\pi_x - \frac{R_{x+1}-R_{x+m+1}}{N_{x-1}}\right) = \pi_x - \pi_{x-r}$$

was the formula wanted to demonstrate the re-assurance point of view. The right-hand side of the equation $\pi_x - \pi_{x-r}$ was the re-assurance premium that the office gave up on condition of being relieved of a certain amount of its risk. The left-hand side of the equation as it stood was not altogether intelligible. If, however, a slight alteration were made in it by taking the m outside the brackets—and for shortness he would put

$$f(R) \text{ for } \frac{R_{x+1}-R_{x+m+1}}{N_{x-1}} —$$

they would then have

$$mX\left(\pi_x - \frac{f(R)}{m}\right)$$

which was the annual premium for the decreasing temporary re-assurance that the office obtained in return for giving up $\pi_x - \pi_{x-r}$. They had an equation of benefits between the premium given up and the amount of cover obtained for it. The formula for the fixed portion of the assurance was as follows:

$$(1-m) = \frac{\pi_{x-r} - \dfrac{f(R)}{m}}{\pi_x - \dfrac{f(R)}{m}}$$

That was a neat and symmetrical formula, and intelligible to everyone. There was no reason why m should coincide with the expectation of life—which, in his opinion, was not the best term to take, and if they took m infinitely large the right-hand side of the equation became

$$\frac{\pi_{x-r}}{\pi_x},$$

which was clearly what it ought to be. It would be seen that in formula 1, π was used to represent the pure premium, but the formula would be equally correct and more instructive if they considered that the equation held good whether π represented the pure premium, the without-profit premium, or the with-profit premium. If π was the with-profit rate, then clearly the assured was entitled to a bonus on the fixed amount of the insurance, but not on the increasing amount, unless a loaded value of $f(R)$ was taken. The author had not referred to the question of the proper valuation reserve for these policies, and he thought that if the expectation of life, or whatever the term might be, had expired, it would in some cases be desirable to make a special reserve, because it might be said that the life was not a first-class one, but that no further consideration was receivable on that account. That,

of course, would not apply to cases in which the extra risk practically expired after a certain lapse of years.

Mr. F. W. Frankland said that he had just heard that the New Zealand Government Life Insurance Department had, since he left a few months ago, discontinued altogether the practice of imposing contingent debts in lieu of extra premium. He was not altogether surprised at that, because when he first became actuary of that office he found two alternative methods in existence of imposing contingent debts in lieu of extra premium, and both of those methods were so little made use of by the outside public that he decided to abolish one of them, and that was the one which formed the subject matter of the present paper. He found that in the case of every proposer on whom the board of advice thought it desirable to impose an extra premium a three-fold option was offered, (1) to accept the imposition of that extra premium; or (2) he was allowed to pay the premium for his true age, and, in consideration of that, to have a fixed debt imposed on his policy for the period of the expectation of life at his true age—the reason, of course, for selecting the true age in preference to the office age was that it constituted a better *argumentum ad hominem.* (3) The third option was a contingent debt decreasing as described in the paper—the debt diminishing by equal amounts annually, until it reached zero, on survival of the expectation at the true age. He had found that the immense majority of persons to whom these options were offered accepted the first alternative (though not always very quietly and contentedly) and paid the extra premium. A minority accepted the second alternative, and took the fixed contingent debt; while a quite insignificant minority accepted the third option, and as that was also more troublesome to calculate, he very soon abolished it altogether. He was not sorry that the fixed debt system had now also been abolished. He recognized from the outset that the fixed and decreasing debts arrived at by the formulas were more than the real conditions of the case required. In reaching that conclusion he had not sufficiently taken into account the reasons which the author had adduced in his paper, but the reason which had weighed most with him was that pointed out by Mr. Rothery, namely, that the formula assumed not merely the correct assessment of the extra premium, but also that if we took a body of rated-up lives of a certain age, say 30, which had to pay a premium equal to that paid by a body of healthy lives aged 35, the two bodies of lives were exactly similar in regard to the decrements of life taking place year by year until all had died. Now that was a very different assumption indeed from the mere assumption that the two groups ought to pay the same premiums. Suppose that two life tables were formed, one of under-average lives commencing at age 30—lives impaired to such an extent that an annual payment equal to that payable by a healthy life aged 35 was correctly charged; the other table beginning with the same number of healthy lives at the age of 35. If those two groups were subjected to the same decrements of life year by year, it would follow that the value of the policy for each duration would be the same for the two groups, in other words, that the reserve needed for an under-average life was the reserve at office age. He had always understood from researches made on the

incidence of extra mortality in under-average lives that it was probable, not merely that the value of a policy on an under-average life (taking one sort of under-average life with another) was less than the value at office age, but that it was even less than the value at true age. That meant that the incidence of the extra mortality in the case of under-average lives made their life-table present some of the characteristics of the Northampton Table. Let them construct a mortality curve for healthy lives, beginning at age 35, and another for healthy lives at age 30, and super-impose the initial ordinates. Further, let the same thing be done for the body of under-average lives aged 30, and let the initial ordinate for these also be super-imposed on the initial ordinate, for healthy lives aged 30. Then they would have constructed two diagrams, each containing two mortality curves; and in each case the difference of ordinate between the two curves represented extra mortality experienced by a group of lives over and above that experienced by healthy lives commencing at age 30. Now, on comparing the two diagrams, he believed it would be found that the extra mortality was greater at starting among the diseased lives, and greater at the finish among the healthy lives of higher initial age. The consequence of that would be that for the under-average lives the present value of a given contingent debt would be greater than the present value of an equal contingent debt imposed on healthy lives of the higher initial age, and therefore the contingent debt required for the under-average lives, as the true equivalent for the extra premium, would be less than that given by the formula. The argument was really a little more complicated than this through the fact that the extra premium was annual instead of single, but he believed the conclusion was correct. He had, however, in spite of this, never seen his way to adopt the empirical methods which had been referred to; and he had always felt that the method condemned by the author must necessarily give a debt falling far short of that which the real facts of the case warranted.

Mr. W. P. Pulley thought that the reason why the deductions given in the table were so inadequate was that the difference between the premium at the true age and the rated-up age was simply multiplied by the value of *m*, the expectation of life, instead of being accumulated for that number of years. He had accumulated the differences at 4 per-cent, and got the following figures in lieu of the empirical values in column 3—250, 287, 314, 355, and 407, which were very much nearer the author's theoretical values. And in lieu of those in column 5, the figures 579, 654, 729, 842, and 985, which were still nearer the theoretical values. In the first case the values still fall short of the theoretical values, but not nearly so much as the empirical ones: in the latter case they approximate very closely at first, and at the last two ages, 50 and 60, even exceeded the theoretical values. In the tables at the end of the paper, the values of Y, which were mentioned as being calculated at 4 per-cent, were put in Tables I and III, where the values of X were calculated by H^{M} 3½ per-cent. He thought, perhaps, it would be simpler for reference if they were put in Tables II and IV, where X was calculated by H^{M} 4 per-cent.

Mr. G. H. Ryan thought the proper tone and direction had been

given to the debate by Mr. Rothery when he said that the great service Mr. Sunderland had done was to introduce a practical matter which stood in great need of discussion. Nowadays there was little doubt that many more people inquired after some alternative method of charging extra premium than used to be the case. He found that to be so in the office with which he was connected. It was difficult to fix the amount of deduction which should theoretically be made, and to keep one's-self at all within the bounds of competition. The empirical method which the author had brought up for discussion, and had justly condemned, was unfortunately adopted by several companies, and that certainly increased the difficulties with which offices had to contend in framing any plan whereby an alternative might be offered in lieu of extra premium. He considered one of the chief difficulties of the situation lay in the fact that they could not in the first instance feel assured that their mode of rating lives was either scientific or sound. No doubt great strides have been made in actuarial science, and members of the Institute were now fully acquainted with the points that should be borne in mind, such as the true incidence of mortality. Unfortunately, however, it did not entirely rest with members of the Institute—there were the medical officers of their companies to be considered. Their doctors could not be expected to enter into the refinements of actuarial science—it was beyond their functions, and in many cases they frankly confessed that they did not attempt in any way to trench upon the actuaries' duties, but merely endeavoured to assess the risk broadly, on the basis of making an addition to the age in order that the life should stand upon a level with healthy persons. Often they might feel that the medical officer, when advising a simple addition to the age, was leaving out of account some important characteristic of a risk—in dealing with consumptive lives, for example, and other lives where the incidence of the extra mortality came in the early years of insurance. But what were they to do? They could scarcely hope to induce the medical gentlemen to give the necessary time to study the matter in detail from the actuarial point of view, and, on the other hand, it was neither expedient nor agreeable to be continually differing from their colleagues. The article which Mr. Makeham had contributed to the *Journal* (*J.I.A.*, xvii, 153) was full of suggestion. On examining Mr. Makeham's results, which were merely rough and ready, and comparing them with those now brought out by Mr. Sunderland, a remarkable similarity would be found to exist. A previous writer on the subject was Mr. Macfadyen, who in the same volume had made an interesting contribution to that important matter, and it was worth while referring to his paper because it illustrated very clearly the point which Mr. Frankland had raised. Mr. Macfadyen took the case of mariners' lives who were charged a certain extra premium, and showed that that extra premium was equivalent to a certain addition to the normal age. Then, comparing the experience of such lives with the experience of healthy persons at the advanced ages, he proved that the two mortality curves were essentially distinct from beginning to end. From that fact he deduced the conclusion which he (Mr. Ryan) believed was now universally accepted, that rated-up policies did not strictly require

that the reserve should be made at the assumed or rated-up ages. The question of debts upon endowment-assurance policies presented a field for investigation which the author had not touched, showing that the subject had by no means been exhausted.

Mr. A. H. Bailey said that no one could take exception to the author's deductions, but he thought that something might be said with regard to the foundations upon which they were built, because they appeared to him to be at variance with the essential principles of life assurance. In his opinion, the term "expectation of life" was an unfortunate one, as it was commonly supposed to mean the term which an individual man might be expected to live. But it meant nothing of the sort. A much better term was "average duration of life." Surely if a healthy man, taken as an ordinary life, died within the average duration, that was no proof that he was not a good life in the first instance; and neither, because he had lived beyond that average duration, was it a proof to the contrary. The same reasoning applied to under-average lives, and therefore, instead of the unfortunate term "rated-up age", it would be better to rate up the premium. He had come to the conclusion that, for the purposes of valuation, those extra risks should be treated in the same way as extra risks of climate, &c., namely, by regarding the extra premium income as an annual fund out of which the increased mortality was to be annually met, and making no further reserve for the purpose. The notion that those risks should be valued according to the so-called rated-up age abounded with fallacies. Supposing they took the case of a young man of 25, who was rated-up 10 years because of undoubted instances of consumption in his family. He lived to be 65, and according to the prevailing ideas he would be valued as at 75, which was an absurdity. A man might be rated-up for gout, which might probably affect his prospects of longevity throughout life. But gout was complicated with all sorts of other diseases. He therefore held that those methods of making a deduction if the life died within the expectation were at variance with the first principles of life assurance, and ought to be altogether discontinued. Mr. Ryan had referred to the case of mariners, and all would agree that the risks of mariners were greater than those of landsmen. But it happened that two years ago he had paid a claim on the death of the oldest man that had come within his experience, who, had he lived two or three months longer, would have attained the age of 100. He was a captain in the navy, and he supposed in the ordinary way his rated-up age would have been 110 or 120. He hoped that the Institute might get more statistical information upon the subject of the rated-up lives, as at present their information on the subject was scanty.

Mr. H. E. Nightingale said that, in treating under-average lives for assurance, it occurred to him that considerable advantage was derived by making each transaction support its own *extra* risk, and a corresponding disadvantage was attached to the method of throwing them into one class and treating them in the aggregate. It was difficult to convince an individual proposer of the equity of the company's decision, whereby he was placed among a class of inferior lives. If, by living to his expectation, the assured should, in his own opinion, justify his claim to be treated as an average life, he would

feel much aggrieved by a contract under which he would have paid extra premiums for many years, with little or no compensation in the way of extra bonus. Hence, to the majority of under-average lives (except those who knew themselves to be bad risks) the method of reducing the sum assured in the event of premature death would appear just to each individual. Every transaction would thus support its own extra risk, and the assured could not complain of a deduction equitably computed and which was exigible only if death should occur in the early years of assurance; whereas, if the life attained his expectation no extra charge whatever would have been made. He scarcely imagined that rated-up lives would approve of a decreasing deduction, for this method (as shown by the tables of Mr. Sunderland and Mr. Chisholm) gave very large deductions from the outset. A fixed deduction throughout the term of expectation would seem to be the more popular and practicable course. Imposing an equal annual extra payment was in practice only strictly applicable when the extra mortality was either a constant or an increasing quantity. By adding a constant of, say, 1 per-cent to the select q_x column, the *ratio* of increase decreased rapidly with each year of assurance. In the absence of statistics it remained to be shown whether rates arrived at by adding a constant did not fairly represent the extra risk of a body of lives whose family history was tainted with, say, consumption.

Mr. F. B. Wyatt said he had hoped that the discussion would take a wider scope than had been the case. Many vexed questions, such as surrender-values, commutations of premiums, and bonus allocations in cases where the life has been rated-up, might have been discussed with advantage. He had not had much experience of under-average lives, but one case had come before him in which the life was rated up very heavily. It was a case of an advance on a reversionary life interest, necessitating a large insurance, and the terms were very onerous. He thought that a plan similar to that discussed by Mr. Sunderland was exactly the one wanted in such a case, a smaller insurance only being required at first, with an ultimate increase up to the maximum. He had made a calculation in that particular case, and he found the saving to the borrower would be considerable. There were many instances, on the other hand, in which it could not be used, such as in cases of marriage settlement or advances on life interests.

Mr. A. G. Mackenzie said that he was recently surprised to find that the plan discussed by the author appeared in the prospectuses of several assurance companies. He was also surprised in reading the report of a very active and well-conducted life assurance company that the proportion of the proposals which were either declined or not taken up only amounted to between 3 and 4 per-cent of the total proposals. That company did business on the principle described in the paper. He thought it was of public interest that it should be shown that that method was a risky one for any office to undertake. He should rather join issue with the author with regard to the advantage which an office adopting the method of deduction as described would get from the opinion of the proposer himself as to his chances of longevity. He thought men, especially when they were young, were mistaken as to their own constitutions. The man

who had the fullest faith in his constitution usually exposed it to the greatest risks: and men who were suffering from constitutional diseases were often the most sanguine with regard to their prospects of life, and most confident that the doctor was mistaken. If there was a selection he thought it would be strongly exercised against the office, because proposers who believed most in their chances of longevity would not consent to any deduction from the sum assured, and would not pay any extra premium.

Mr. F. Schooling said that a source of information which had not been touched upon was the "diseased life" experience of the Institute. Some time ago there had been prepared in his office a complete table from this source, graduated by the graphic method, and he found that, taking the age of 21, the net premium was equal to that for age 29, H^{M} 3 per-cent. a rating-up of eight years. At age 30 the premium was equivalent to that for a healthy life aged 36, at age 40 to that for age 44, at age 50 to that for age 53, and at age 60 to that for age 62. These figures were derived from the aggregate table, and in the cases of diseased lives they knew that the particular form of disease from which lives were suffering would affect the rating-up. The figures showed the average rating-up required according to D^{mf} mortality.

Mr. T. J. Searle wished to point out that the way in which the author had solved the question was not the most general way of dealing with it. If they capitalized the difference between the normal and rated-up premiums they knew what amount they were giving up, and they could convert that amount into an assurance deduction, either as a fixed, increasing, or decreasing assurance. Instead of using the "expectation" they could use any other term of years. They could also take into account another matter, which they could not do by a fixed formula—the incidence of the extra risk. He did not think that they ought to treat it as a question of the general incidence of extra risk, but that they ought to apply it to the particular case which they had under consideration.

Mr. A. W. Tarn thought there was a grave objection to the method described by Mr. Sunderland, as it made the amount at risk lowest where selection was greatest. One other method had not been mentioned, namely, that of a combination of extra premium and reduced sum assured. Where the life had been rated-up a good many years, the premium could be rated-up, and the sum assured could be reduced at the same time, and this process would meet many of the objections urged against the method.

Mr. J. Chisholm said there was a practice among offices of making a deduction from the sum assured in lieu of extra premium, but there was not always a distinction drawn as to the class of cases which it was proper to treat in that way. He had recently come across the prospectus of one company explaining its practice, and he would read an extract from it which would show the way that that principle was applied: "To meet this class of cases this company has prepared special tables, which enable it to insure at ordinary rates of premium those who suffer from asthma, rheumatism, rupture, or similar complaints, and others who may not be considered as first class." He wished to point out that that was the very class of cases

to which the system mentioned in the paper was not applicable. He was quite willing to admit that it was applicable in many cases where the risk was at the commencement of the insurance, such as those in which there was a family history of consumption; but in cases of gout, or where there was a rheumatic tendency, or rupture, which was a constant risk, he could not say that the system met the risk at all. A year or two ago he had looked into the matter, and had expressed his views in a work published by Dr. Pollock and himself (*Medical Handbook of Life Assurance*). Having estimated the value of the extra risk, they ought to see that the exact equivalent of that extra was imposed by way of deduction from the sum assured. He had compared the deductions brought out by Mr. Sunderland in his paper with those in the work he had mentioned, and he found that they agreed very well. He was surprised that in actual practice many offices, some of which were advised by eminent actuaries, made use of deductions which were only a third or a fourth of what they ought to be.

Mr. S. G. Warner said it was most important that they should do full justice to rated-up lives, but they must be very careful lest in the attempt to do that they did injustice to the bulk of their policy-holders who were healthy average lives. The assumption underlying the new method was that the life assured would die within the ordinary expectation, and from the particular cause for which his life had been rated-up. If a man lived to within one year of his expectation, and then died from a cause entirely extraneous to that for which his life was rated-up, it might be questioned whether it was fair to make the deduction. He would suggest whether something more like even justice might not be done with regard to the majority of rated-up cases, where the incidence of mortality which they desired to provide against rested upon the earlier years of the assurance, by deferment of a considerable portion of the assurance for a comparatively short term of years. It seemed that in the majority of cases the incidence of mortality against which they desired to protect themselves by extra rating fell upon the earlier years of assured life.

Mr. G. Humphreys said that after a long investigation at his own office, which embraced a period of over 40 years, he found that the curve from which he deduced the expectation for the rated-up lives was almost identical with that of the expectation for the rated-up ages upon the basis of the mortality table from which the premiums were formed. Therefore if any of those lives had been relegated back to the sound class, the office would have lost most undoubtedly. Mr. Rothery had referred to one or two papers, but no one had mentioned that written many years ago by Mr. Pinckard, which was a useful and practical paper with regard to the experience of his own office. Mr. Meikle had also compiled a work of great value, published in 1872, in which the treatment of unsound lives was carefully considered.

The President said they might at least congratulate the author on the attainment of one of the purposes he had in view in writing his paper. They had had a very instructive discussion, which had been taken part in by many new voices. He had also demonstrated the delusion that the particular system of assessing under-average

lives, with which he had been dealing, was a right and safe one. It was certain that no office dared propose to a man whom they considered as an under-average life the true amount of initial deduction which alone would justify that system. He referred to the paper which had been written a few years ago by Messrs. White and Whittall, in which the authors pointed out that essential difference, which some members had recognized during the discussion, between the classes of cases to which the rating-up should be applied, those in which the risk was a decreasing risk, those in which the risk was practically a uniform risk, and those in which the risk was an ascending or increasing risk towards the end of life. After reading that paper, he had been very desirous, if possible, to institute a plan which would enable a term to be put to the penalty which had been imposed upon the extra-rated life. He had attacked the problem from another point of view, and that was with the desire that the extra premium which had been charged should gradually decrease, and wholly cease at the end of a given time. He did not quarrel with Mr. Bailey as to the use of the word "expectation." It might not, perhaps, have been a happy expression, but some term must be fixed upon for the purpose of the calculation, and there was no doubt that, speaking popularly, there was an idea that if a man had attained what they called the average duration of life, but which he would probably call the "expectation of life", that should be the particular term fixed upon for bringing the matter to an issue. The result of the investigation which he made at that time went to show that the initial premium would be so large that, to use the expression of Mr. Ryan, it would be altogether out of the range of competition. He should like to suggest that there was possibly, from the point of view from which the matter had been looked at that evening, another mode of dealing with the problem. Instead of a debt being, as it were, created upon a policy at starting, and that debt being gradually diminished until at a given moment it was wholly extinguished, they might grant a policy for a given amount (at the increased premium they desired), but for every year of existence an increasing amount of insurance should be given. That was a crude suggestion, but it got rid of the question of "expectation", and it had also an advantage with regard to the matter of valuation. He did not agree with Mr. Bailey that they should regard rated-up policies for the purpose of valuation as ordinary policies effected at the true age, and that the extra rating was merely in the nature of a liberty premium, working itself out year by year. Apart altogether from what he regarded as the unsoundness of that plan in regard to the office itself, it had the great disadvantage that it practically prevented them from doing their assured any justice if he should prove by long life that they had in his case over-estimated the risk they had to bear. He could not but think that the other plan, which took into account the increased age of the life assured, which gave a bonus on the increased premium paid, and in that way undoubtedly ameliorated, and as his life progressed would more and more ameliorate the original surcharge to which the assured had been subjected, was on the whole the truest and the most equitable system. Mr. Rothery had attributed to him a feeling, and he supposed he must also have

meant an expression of opinion, of which he (the President) was not altogether conscious, but which he would not repudiate, that he did not like the reduction of ratings: and his reason for that was that if they re-assessed part of their risks, to be fair they should re-assess the whole. In the practical case which had been referred to, where if they would not make some difference in the charge another office would, he would advise the assured to discontinue the existing policy and take out a new one on such better basis as the improved condition of his health might warrant. He said that because he held the doctrine very strongly, and in agreement with Mr. Sprague, that rated-up lives ought not to have the surrender-value that would attach to the policy if it had been effected at a true age corresponding to the rated-up age in that particular case. That was the one point in regard to which he quarrelled with the expression which had been used that evening that a rated-up life should for all purposes be treated as though it was of the real age which it was assumed to be. He was quite conscious of that which had been said as to the theoretical defects of the system. He had received a communication from Mr. Manly, who had pointed out that Mr. Sunderland had used well-known symbols in his paper for purposes different from those in which they were usually employed, and he would hand the letter to Mr. Sunderland.

Mr. Sunderland, in reply, explained the circumstances under which it had occurred to him to write the paper. A re-assurance was offered to his office; the life was rated-up five years, the condition being that they were to make a corresponding deduction from the sum assured. They accepted the offer, but when the deduction from the sum assured came to be ascertained, it appeared that his calculations resulted in a deduction which was three times the amount of the deduction proposed by the other office, so that they were obliged to withdraw. It was that which led him to consider the subject. He had not gone into all the different methods, because he did not think it necessary or appropriate to do so. Mr. Rothery and other members had pointed out that somewhat similar problems had been discussed by various writers who had preceded him, but under the circumstances he did not think it desirable to follow out such an investigation unless it had an immediate practical object. The object of his paper had really been to condemn a method now existing. In his own office he had calculated temporary deductions in a great variety of ways, he had made temporary deductions which remained at a fixed amount for a certain number of years and then ceased altogether, or he had made it a condition that no bonus should be added until the expiration of a certain number of years, and so on, the exigencies of competition compelling him to endeavour to meet the public in every possible way. Regarding certain objections which Mr. Rothery has raised to the paper, he had not gone into other formulas and other matters, but had kept himself strictly to the point in hand, which was to condemn an existing method, and he hoped he had succeeded. He had been much interested in Mr. Frankland's remarks, and was by no means sorry to hear that the two methods had been abandoned by the New Zealand Government Department. His remarks on the mortality prevailing among under-average lives

were very interesting, but he did not think he (the author) could have dealt with them in a paper like that under discussion. It was intended to consider a very practical point, and he thought that had to be done in a fairly rough and ready way. Mr. Ryan had put the matter in somewhat different words by saying what he (the author) had in fact already said in the paper, that the practice of adding a number of years was far from perfect. Mr. Nash had made various criticisms and suggestions as to the mode in which the formula had been derived. He (the author) did not pretend to remember the formulas which occurred in the Text-Books, and had worked them out afresh, in more than one way, and had put them in a manner that he thought would be most intelligible to the members of the Institute. Mr. Bailey had objected to the terms "expectation of life" and "rated-up age": but, of course, he (the author) was not responsible for those expressions. He was dealing with methods now in use, and had adopted the terms in which they were described in the prospectuses of assurance companies.

On the Theory of Inverse Probabilities. *By* William Matthew Makeham.

(Concluded from p. 251.)

Section 4.

In his great work, the *Théorie Analytique des Probabilités* (which may be supposed to embody the author's latest and most matured views respecting that abstruse and difficult subject), Laplace reduces the fundamental principles of the whole doctrine of probabilities (both direct and inverse) to four comprehensive propositions. The four "Principes" of Laplace are extremely well adapted to show how important is the part, in that doctrine, which belongs to the indirect or inverse branch of it; and I know of no means better calculated to throw light upon the subject of this article than the course which I propose in the present section to follow, namely, to illustrate each "Principe" by an example relating to a coin falling head a certain number of times in succession.

It will be observed that, in enunciating his four fundamental principles, Laplace makes no reference whatever to the ordinary distinction of "dependent" and "independent" events. Following the usual definition, events may be said to be dependent or independent, according as the occurrence of one does, or does not, affect the *occurrence* of the others. When a coin is tossed twice

in succession, the occurrence of head in the first throw has no direct effect whatever upon the *occurrence* of head in the second; and therefore these two events, according to the preceding definition, are evidently "independent." But let us suppose an urn to contain two balls which are known (*a priori*) to be either (1) *both* white, or (2) *one* white and the *other* black. A black ball has been drawn and not replaced, which we will term the *first* event, the second being the drawing a black ball in the *second* trial. Now, as the black ball drawn in the first trial is not replaced, the occurrence of the first event has obviously rendered the second impossible—the two events in this case being therefore clearly "dependent." On the other hand, if the ball first drawn had been replaced, it is equally clear that, as in the coin-tossing experiment, the two events would then be "independent"; that is to say, the fact of the first event having occurred would not in the least affect the *occurrence* of the second—for the original ratio of white and black balls would have been restored by the replacement of the ball drawn in the first trial. Nevertheless, in the latter case, it is evident that the observer's *knowledge* of the result of the first trial will have affected very greatly his estimate of the *probability* of the occurrence of the second event; for by means of it the precise contents of the urn—namely, one white and one black ball—will have been definitively ascertained.

Instead of merely asking "Does the occurrence of one given "event affect in any way the *occurrence* of another given event, "or, in other words, are the two events *dependent* or *independent*"? the question we have more especially to do with is the much wider one, "Should the occurrence of one given event affect (and "to what extent) *our estimate of the probability* of another given "event"? This latter question constitutes the problem of "inverse" probability; and we shall find that, without exception, the application of each one of Laplace's four fundamental principles (covering, as they do, the whole field of the doctrine of probabilities) involves an application of the *inverse* theory. This fact alone is sufficient evidence of the great importance of the subject which I have undertaken to discuss in the pages of the *Journal of the Institute.* With these few preliminary observations, I proceed to introduce the four "Principes généraux de l'analyse des Probabilités" as finally laid down by Laplace.

I^{ER} PRINCIPE.

The probability of an event compounded of two simple events is the product of the probability of one of the latter events by the probability that, the one in question having occurred, the other will also happen.

Let the required probability be that of a coin falling head twice in succession—an event which we may suppose to be compounded of the following two simple events, namely, (1) head in the first throw, and (2) head in the second throw. If p denote the probability of head in the first throw (or, the antecedent probability of head), and p_1 the probability that, *head having occurred once* (in the first throw), it will occur again in the second, then, by this I[er] Principe, the required probability of the compound event is $p.p_1$, and generally the probability of head m times in succession is $p.p_1.p_2 \ldots p_{m-1}$, where any given factor, p_x, denotes the probability that, *head having occurred x times in succession*, it will again occur in the $(x+1)$th trial. The form of the function p_x will vary according to *the observer's knowledge* of the "actual ratio of chances", that is, of the inherent tendency of the coin to fall head or tail. To illustrate this, I will take the three following cases:

Case 1.—Suppose, first, that the observer knows the actual chances for and against head to be as $1+w$ to $1-w$. The probability of head in the first throw—namely, p—is then $\frac{1+w}{2}$. And as the knowledge which the observer is supposed to possess, *a priori*, comprises all that there is to be known, the *result* of the first throw (be it what it may) should evidently not affect his estimate of the probability of the second. Therefore, in this case, $p_1 = \frac{1+w}{2}$ also, and the required probability of the compound event, $p.p_1 = \left(\frac{1+w}{2}\right)^2$. And generally, by the same reasoning, the probability of head m times in succession is $\left(\frac{1+w}{2}\right)^m$.

[*Note.*—This result is generally given in the text-books as the probability of the compound event "when the two simple events are independent." But it will be seen that in the present case the result in question is not due exclusively to the fact of the events being independent—the completeness of the observer's

a priori knowledge respecting the actual ratio of the chances being an equally indispensable condition.]

Case 2.—Suppose, next, that the observer has no *knowledge* of the actual ratio of chances due to the construction of the coin, in which case he will have to make the best estimate which the circumstances allow of the antecedent probability, p. The value of p_x in this case will be expressed by the generalized formula $\frac{x+rp}{x+r}$ (see Section 2), so that $p.p_1 = p \cdot \frac{1+rp}{1+r}$. And generally the probability of head m times in succession will be

$$p \cdot \frac{1+rp}{1+r} \cdot \frac{2+rp}{2+r} \cdot \frac{3+rp}{3+r} \cdots \frac{(m-1)+rp}{(m-1)+r}.$$

The actual ratio of the chances being, say, as $1+w$ to $1-w$, if we suppose that (numerically) w may be anything between 0 and 1, and, so far as the observer knows, is just as likely, within those limits, to have any one value as any other (which suppositions are identical with the assumptions of Laplace's formula), we have, for the values of r and p, respectively, 2 and $\frac{1}{2}$; so that, in the case supposed, we get the following very simple expression for the probability of head m times in succession, namely,

$$p.p_1.p_2 \ldots p_{m-1} = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdots \frac{m-1}{m} \cdot \frac{m}{m+1} = \frac{1}{m+1}.$$

Case 3.—Lastly, let us suppose that the observer knows the actual chances to be as $1+w$ to $1-w$, but does not know on which side the preponderance lies. Supposing it to be *in favour* of head, and w positive, the probability of head in the first throw is $\frac{1+w}{2}$; if *against* head, the probability is $\frac{1-w}{2}$. And as the two suppositions are *a priori* equally likely, the true value of p is $\frac{1}{2}\left\{\frac{1+w}{2} + \frac{1-w}{2}\right\}$, or $\frac{1}{2}$. Also, under the first supposition, the probability of head *twice* in succession is $\left(\frac{1+w}{2}\right)^2$, and under the second $\left(\frac{1-w}{2}\right)^2$. Hence the true probability of head twice in succession, or $p.p_1 = \frac{1}{2}\left\{\left(\frac{1+w}{2}\right)^2 + \left(\frac{1-w}{2}\right)^2\right\} = \frac{1}{2} \cdot \frac{1+w^2}{2}$, and therefore $p_1 = \frac{1+w^2}{2}$. In the example which I propose to give in

illustration of the IV^e Principe, it will be shown generally (by a direct demonstration) that in the present case p_x is of the form $\frac{1}{2}\cdot\frac{(1+w)^{x+1}+(1-w)^{x+1}}{(1+w)^x+(1-w)^x}$. Consequently the probability of head m times in succession, or $p.p_1.p_2 \ldots p_{m-1}$, is in this case:

$$\frac{1}{2^m}\cdot\frac{2}{2}\cdot\frac{(1+w)^2+(1-w)^2}{2}\cdot\frac{(1+w)^3+(1-w)^3}{(1+w)^2+(1-w)^2}\cdots\frac{(1+w)^m+(1-w)^m}{(1+w)^{m-1}+(1-w)^{m-1}}$$

$$=\frac{1}{2^m}\cdot\frac{(1+w)^m+(1-w)^m}{2}=\frac{1}{2}\left\{\left(\frac{1+w}{2}\right)^m+\left(\frac{1-w}{2}\right)^m\right\}.$$

To recapitulate the results obtained in these three several cases—each of which, it is to be observed, has reference to *independent* events only—it appears that in the first, where the observer is supposed to know, *a priori*, all that there is to be known (namely, the actual ratio of chances inherent in the coin), the falling of head in the *first* trial has no effect whatever upon the probability of head falling in the *second*, and therefore, in this case, the probability of head twice in succession is the square of the antecedent probability of head. In case 2, where the observer is supposed to be in complete ignorance of the *actual* ratio, and can only estimate the *probable* ratio, p, the fact of head turning up in the first trial has increased the probability of head in the second from p to $\frac{1+rp}{1+r}$, or from $\frac{1}{2}$ to $\frac{2}{3}$ if we suppose the assumptions of Laplace's formula to be applicable. And in case 3, where the *extent* of the preponderance (w) is supposed to be known, but not *the side on which it lies*, the falling of head in the first trial has increased the probability of head in the second from $\frac{1}{2}$ to $\frac{1+w^2}{2}$.

But, although the results are thus found to differ in the three cases according to the extent of the observer's supposed knowledge of the actual ratio of the chances inherent in the coin, they are nevertheless all deduced from, and governed by, one and the same fundamental principle. There can be no possible reason, therefore, for considering any one of them as an "aberration of the ordinary theory", which would not apply equally to the other two. This expression, which Laplace, upon one occasion only, makes use of with reference to the result of case 3, occurs in a passage which is quoted by Todhunter from one of the great author's earliest works, published in the year 1774 (see *History of the Theory of Probabilities*, p. 472). Now, Laplace's I^{er}

Principe is usually enunciated by means of two distinct propositions—the one relating to "independent" and the other to "dependent" events; but Laplace, in his *Théorie Analytique,* more properly includes them both under a single comprehensive general principle, which applies equally to both cases. It is not impossible, however, that in his earlier and less matured view of the subject, he also may have supposed the result of case 1 alone to embody the true fundamental principle governing independent events generally. Be this as it may, although he again reverts to the subject in his great work, and gives an explanation of the seeming paradox referred to in Section 1,* he does not repeat on the latter occasion his earlier observation respecting the supposed "aberration of the ordinary theory."

II^{E} PRINCIPE.

The probability of a future event, derived from an observed event, is the quotient found by dividing the probability of the event compounded of the two events in question, and determined a priori, *by the probability of the observed event, similarly determined* a priori.

Let the compounded event be that of a coin falling head $m+m'$ times in succession; which we may suppose to be compounded of the two following events, namely, (1) of the *observed* event of a coin having already fallen head m times in succession, and (2) of the *future* event of its again falling head m' times further in succession. Now the probability of the compounded event, *determined a priori,* we have seen is

$$p \cdot p_1 \cdot p_2 \cdots p_m \cdot p_{m+1} \cdots p_{m+m'-1} \quad . \quad . \quad . \quad [1]$$

and, similarly, the probability of the *observed* event, also *determined a priori,* is

$$p \cdot p_1 \cdot p_2 \cdots p_{m-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [2]$$

Hence, dividing the first product by the second, we get for the probability of the *future* event, derived from the *observed* event,

$$p_m \cdot p_{m+1} \cdots p_{m+m'-1},$$

* That is to say, of a coin supposed to be dynamically true being to the observer less likely to fall head (or tail) *twice in succession* than one known to be unevenly balanced, when (in the latter case) the side on which the preponderance lies is unknown.

which, in terms of the generalized formula, becomes

$$\frac{m+rp}{m+r} \cdot \frac{m+1+rp}{m+1+r} \cdots \frac{m+m'-1+rp}{m+m'-1+r};$$

or, adopting the assumptions of Laplace's formula,

$$\frac{m+1}{m+2} \cdot \frac{m+2}{m+3} \cdots \frac{m+m'}{m+m'+1} = \frac{m+1}{m+m'+1}.$$

If we put $m'=1$ in these two results they reduce to $\frac{m+rp}{m+r}$ and $\frac{m+1}{m+2}$ respectively, which evidently ought to be the case.

III^E Principe.

If an observed event may be the result of one of n *different causes; their probabilities are, respectively, as the probabilities of the event derived from their existence; and the probability of each of them is a fraction of which the numerator is the probability of the event upon the supposition of the existence of the cause, and of which the denominator is the sum of the several similar probabilities with respect to all the causes.*

Suppose that it is known to the observer that the ratio of chances due to the construction of the coin are as $1+w$ to $1-w$, *the preponderating side being unknown,* and suppose further that head has fallen m times in succession. There will then be only two possible causes, either of which may be (and, *a priori,* is equally likely to be) the true cause; for the inherent chances of the coin for and against head must be either (1) as $1+w$ to $1-w$, or (2) as $1-w$ to $1+w$. The probability of the observed event, as derived from assuming the existence of the first-mentioned cause, is $\left(\frac{1+w}{2}\right)^m$; and, as derived from assuming the existence of the second cause, it is $\left(\frac{1-w}{2}\right)^m$. Hence, the probability of the existence of the first-mentioned cause is to the probability of the existence of the second as $\left(\frac{1+w}{2}\right)^m$ to $\left(\frac{1-w}{2}\right)^m$, or as $(1+w)^m$ to $(1-w)^m$.

And the probability of the existence of the first-mentioned cause is $\left(\frac{1+w}{2}\right)^m$ divided by $\left(\frac{1+w}{2}\right)^m + \left(\frac{1-w}{2}\right)^m$, namely, $\frac{(1+w)^m}{(1+w)^m+(1-w)^m}$. And similarly the probability of the existence of the other-mentioned cause is $\frac{(1-w)^m}{(1+w)^m+(1-w)^m}$.

IV^E PRINCIPE.

The probability of a future event is the sum of the products of the probability of each cause, derived from the observed event, by the probability that, supposing the cause to exist, the future event will happen.

Suppose the future event in question to be head falling in the next trial, after having already fallen m times successively,—the knowledge of the observer (*a priori*) being as stated in the last case. The probability of the existence of the first cause, derived from the observed event, as we have seen, is $\frac{(1+w)^m}{(1+w)^m+(1-w)^m}$, and the probability that, supposing this cause to exist, the future event will happen, is $\frac{1+w}{2}$, the product of the two being $\frac{1}{2}\cdot\frac{(1+w)^{m+1}}{(1+w)^m+(1-w)^m}$. Similarly, the probability of the existence of the second cause, derived from the observed event, is $\frac{(1-w)^m}{(1+w)^m+(1-w)^m}$, and the probability that, supposing the second cause to exist, the future event will happen, is $\frac{1-w}{2}$, the product of the two being $\frac{1}{2}\cdot\frac{(1-w)^{m+1}}{(1+w)^m+(1-w)^m}$. Hence, the probability of the future event (*i.e.*, head falling in the next trial, having already fallen m times in succession) is the sum of these two products, namely, $\frac{1}{2}\cdot\frac{(1+w)^{m+1}+(1-w)^{m+1}}{(1+w)^m+(1-w)^m}$. This is the result referred to, and assumed in the third example illustrating the I^{er} Principe.

Although, as we have seen, each of the four preceding fundamental principles affords an illustration of the wide application and great importance of the *inverse* theory in the doctrine of probabilities, yet the III^e Principe has, perhaps, the most direct bearing upon the subject of the present article. One of the problems discussed by Laplace, as an example of the principe in question, is exceedingly well adapted for that purpose; but, unfortunately, by an obvious oversight (somewhat remarkable in this unrivalled master of the subject), the result deduced by Laplace is, I think, unquestionably erroneous. The example referred to is stated in the *Théorie Analytique*, p. 183, as follows: "Suppose an urn contains three balls, each of which must be "either white or black; that after a ball has been drawn it is

" replaced in the urn before another drawing is made; and that " after m drawings white balls only have been drawn. It is " evident that, *a priori*, only four hypotheses can be made, for the " three balls must be either all white, or two white and one black, " or two black and one white, or, lastly, all black."

Laplace then proceeds to consider these four hypotheses upon the *tacit* assumption that, *a priori*, they are all equally probable. But it may be easily shown that the *a priori* probabilities of the four hypotheses, in the order stated above, are really in the proportions of 1, 3, 3, and 1 respectively. For if we distinguish the three balls by the symbols (1), (2), and (3), we have evidently the *eight* equally probable hypotheses which I have set forth in the following table, with their several *a priori* and *a posteriori* probabilities,—the latter determined in strict accordance with Laplace's IIIᵉ Principe,—from which it is seen that while Laplace's first and fourth hypotheses have only *one* case each, the second and third have as many as *three* each in their favour.

Table of Elementary Cases, or Hypotheses of equal Antecedent Probability.

1	2			3	4		5
Hypotheses	Colours of the several Balls in the Urn			Ratio of Chances, or Probability of drawing a White Ball, on each Hypothesis	Relative Probability of each Hypothesis		Absolute Probability (*a posteriori*) of each Hypothesis
	(1)	(2)	(3)		*A priori*	*A posteriori*	
a	W	W	W	$\frac{3}{3}$	1	3^m	$\frac{3^m}{3^m+3.2^m+3.1^m+0^m}$
b	W	W	B	$\frac{2}{3}$	1	2^m	$\frac{2^m}{3^m+3.2^m+3.1^m+0^m}$
c	W	B	W	$\frac{2}{3}$	1	2^m	$\frac{2^m}{3^m+3.2^m+3.1^m+0^m}$
d	W	B	B	$\frac{1}{3}$	1	1^m	$\frac{1^m}{3^m+3.2^m+3.1^m+0^m}$
e	B	B	B	$\frac{0}{3}$	1	0^m	$\frac{0^m}{3^m+3.2^m+3.1^m+0^m}$
f	B	B	W	$\frac{1}{3}$	1	1^m	$\frac{1^m}{3^m+3.2^m+3.1^m+0^m}$
g	B	W	B	$\frac{1}{3}$	1	1^m	$\frac{1^m}{3^m+3.2^m+3.1^m+0^m}$
h	B	W	W	$\frac{2}{3}$	1	2^m	$\frac{2^m}{3^m+3.2^m+3.1^m+0}$

We might, of course, omit the index of 1^m in the last two columns, for (if we except ∞) $1^m = 1$ for all values of m. But not in the case of 0^m, as the latter, when $m=0$, assumes the "singular" form 0^0, and changes its value from 0 to 1. A proof of this will be given later on.

The results stated in the preceding table have reference only to the case where the observer's knowledge is limited to the two facts, (1) that each of the three balls is either white or black, and (2) is just as likely to be of the one colour as the other. But other suppositions may be made respecting the extent of the observer's knowledge of the contents of the urn; and the corresponding probabilities may be found by properly combining the tabulated elementary results. Thus, suppose that in addition to the above-mentioned two facts the observer also knows that the balls are *not all white*. In this case the first hypothesis (a) must be excluded, and the absolute probability of each of the seven remaining hypotheses would be found by simply omitting the first term of the common denominator in column 5, namely, 3^m. Or the observer might be supposed to know still more, namely, that the balls are *not all of one colour*, in which case he would have to exclude also the fifth hypothesis (e), and to omit the corresponding term 0^m of the common denominator. In the latter case the probabilities of the remaining six possible hypotheses would be as under:

Hypotheses	Absolute Probability (*a posteriori*) of each Hypothesis	Hypotheses	Absolute Probability (*a posteriori*) of each Hypothesis
b	$\dfrac{2^m}{3.2^m + 3.1^m}$	f	$\dfrac{1^m}{3.2^m + 3.1^m}$
c	$\dfrac{2^m}{3.2^m + 3.1^m}$	g	$\dfrac{1^m}{3.2^m + 3.1^m}$
d	$\dfrac{1^m}{3.2^m + 3.1^m}$	h	$\dfrac{2^m}{3.2^m + 3.1^m}$

Although in the heading of the second and fourth columns of this table I have used the expression "*a posteriori*" in designating the probabilities therein referred to, there was no absolute necessity for this, as the same formulas, in fact, serve also to express the "*a priori*" probabilities by giving to m the value 0. The same remark applies also to the "*a posteriori*" probabilities (both relative and absolute) contained in the

elementary table; but, in the latter case, in order to substantiate this assertion, I must give a satisfactory demonstration of the previously stated fact, that when, by taking $m=0$ in 0^m, we get the singular form 0^0, the value of the expression 0^m becomes 1.

De Morgan, in his *Differential and Integral Calculus*, p. 172, gives various examples of the more obvious singular forms. In his process demonstrating the ordinary rule for evaluating these forms, he proceeds upon the assumption that they *necessarily* arise from assigning a certain value to x, considered as the variable of two given functions, which functions are consequently thus supposed to vary *dependently;* and, so far as I have had an opportunity of observing, the same course is adopted generally by writers on the subject. The singular form which we have had to deal with, however, has reference to quantities which are supposed to vary *independently* of each other—a fact which induces me to venture upon the following brief attempt to generalize De Morgan's demonstration, so as to cover the case of independent as well as dependent variation.

Let $\phi(x)$ denote any given function, and $\phi'(x)$ its differential coefficient. Now, De Morgan shows (*Differential and Integral Calculus*, p. 67) that the equation $\dfrac{\phi(x+h)-\phi(x)}{h}=\phi'(x+\theta h)$ is necessarily true for some value of θ less than unity. Therefore, denoting by $\psi(y)$ and $\psi'(y)$ any other given function, with its corresponding differential coefficient, we must have $\dfrac{\psi(y+h)-\psi(y)}{\phi(x+h)-\phi(x)}$ $=\dfrac{\psi'(y+\theta' h)}{\phi'(x+\theta h)}$, whether the variations of x and y be *dependent* or *independent* of each other. By giving to x and y, respectively, the values necessary to cause $\psi(y)$ and $\phi(x)$ to vanish, the last equation becomes $\dfrac{\psi(y+h)}{\phi(x+h)}=\dfrac{\psi'(y+\theta' h}{\phi'(x+\theta h)}$, and, θ' and θ being each less than unity, by taking $h=0$ we have finally

$$\frac{\psi(y)}{\phi(x)}=\frac{\psi'(y)}{\phi'(x)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (A)$$

Such being the case when $\psi(y)$ and $\phi(x)$ are each $=0$,—that is when $\dfrac{\psi(y)}{\phi(x)}$ takes the singular form $\dfrac{0}{0}$,—it follows that it is also true when they are each $=\infty$, or when $\dfrac{\psi(y)}{\phi(x)}$ takes

the singular form $\frac{\infty}{\infty}$. For, in the latter case, $\frac{1}{\psi(y)} \div \frac{1}{\phi(x)}$ $= \frac{d}{dy} \cdot \frac{1}{\psi(y)} \div \frac{d}{dx} \cdot \frac{1}{\phi(x)}$ (by A) or $\frac{\phi(x)}{\psi(y)} = \left(\frac{\phi(x)}{\psi(y)}\right)^2 \cdot \frac{\psi'(y)}{\phi'(x)}$, and therefore $\frac{\psi(y)}{\phi(x)} = \frac{\psi'(y)}{\phi'(x)}$, as in the preceding case. Further, it also follows that the relation expressed by (A) having been proved to be true generally, it is true when x and y are identical, that is, if $\frac{\psi(x)}{\phi(x)}$ take the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, its value in either case is $\frac{\psi'(x)}{\phi'(x)}$.

The fundamental rule embodied in (A) furnishes a key to the evaluation of most of the singular forms enumerated by De Morgan; but I must resist the temptation to extend the length of this digression by giving instances of its various applications, and confine myself strictly to the particular case which has necessitated the digression. Suppose x^y to be the general form of the expression to be evaluated for $x=0$, $y=0$. Now $x^y = \epsilon^{\log x . y}$, so that, putting the index $\log x . y$ in the equivalent form $\frac{\log x}{y^{-1}}$ $\left(\text{which becomes } -\frac{\infty}{\infty}\right)$ and applying the rule, we have $x^y = \epsilon^{-\frac{y^2}{x}}$. Here, the index $-\frac{y^2}{x}$ becomes $-\frac{0}{0}$; which necessitates another application of the rule, giving $-\frac{y^2}{x} = -\frac{2y}{1}$. Hence we have finally $x^y = \epsilon^{-2y} = \epsilon^{-0} = 1$.

It is to be observed that Laplace's error in the example of the urn supposed to contain three balls, each of which is known to be either white or black, is one of detail only, and in no way affects the principle which the example was intended to illustrate. Instead of the *eight* equally probable hypotheses enumerated in the elementary table, Laplace appears to have assumed that there were only *four* possible and equally probable cases, for he says: "It is evident that we can make, *a priori*, but four hypotheses, "for the three balls must be either *all white*, or *two white and one* "*black*, or *two black and one white*, or, lastly, *all black*." These four "compounded" hypotheses (as they may be termed) can be formed by properly combining the several elementary cases contained in the first table. Thus:

1	2	3	4		5
Hypotheses	Colours of the several Balls in the Urn (1), (2), (3)	Ratio of Chances, or Probability of drawing a White Ball, on each Hypothesis	Relative Probability of each Hypothesis *A priori*	*A posteriori*	Absolute Probability (*a posteriori*) of each Hypothesis
a	All white	$\frac{3}{3}$	1	3^m	$\frac{3^m}{3^m+3.2^m+3.1^m+0^m}$
b, c, h	2 white, 1 black	$\frac{2}{3}$	3	3.2^m	$\frac{2^m \times 3}{3^m+3.2^m+3.1^m+0^m}$
d, f, g	2 black, 1 white	$\frac{1}{3}$	3	3.1^m	$\frac{1^m \times 3}{3^m+3.2^m+3.1^m+0^m}$
e	All black	$\frac{0}{3}$	1	0^m	$\frac{0^m}{3^m+3.2^m+3.1^m+0^m}$

The "relative" and "absolute" probabilities in this table are simply the sums of the corresponding probabilities of the elementary table for the several hypotheses denoted in the first column. It will be seen, therefore, that for the correct solution of these inverse problems, everything depends upon the proper determination of the elementary values, that is, upon the correct analysis of the elementary hypotheses of equal antecedent probability, and, this being attained, it is scarcely possible to fail in arriving at a correct conclusion.

The simplest application of the theory of inverse probabilities consists in the determination, from the recorded results of a large number of previous trials, of the probability that a marksman will in a future trial strike the target at a specified distance from the centre. This is a problem in the theory of "errors of observation"; and upon commencing the present article I purposed making use of that theory, and of the application above referred to, in an attempt to obtain a more accurate approximation to the true value of r in the generalized formula $\frac{x+rp}{x+r}$, introduced in the example to the I^{er} Principe. But the unexpected length to which the present section has reached, compels me to relinquish that intention, so that I may not incur the risk of exhausting the reader's patience in these somewhat neglected paths of actuarial science. I will therefore conclude the present article by a brief application, in the next section, of certain results previously arrived at, to the practical question of the best method of utilizing

observations on the rate of mortality experienced by life assurance offices.

Section 5.

In the application of the approximate generalized formula $\frac{m+2p}{m+n+2}$ to the determination of the true "unadjusted" probability of living a year at any given age, the quantity denoted by p is identical with what is generally understood as the "adjusted" probability: and it is therefore evident that we cannot determine directly the true *unadjusted* probabilities of life (which are to be subsequently used as the bases for deducing the *adjusted* probabilities) until we are actually in possession of those adjusted probabilities. Such being the case, our only course is to proceed by way of successive approximations.

We must commence, then, by assuming the observed ratio $\frac{m}{m+n}$ for a first approximation to the value of p. Substituting, accordingly, $\frac{m}{m+n}$ for p in the formula $\frac{m+rp}{m+n+r}$, the latter likewise becomes $\frac{m}{m-n}$, which is therefore the first approximation also to the true *unadjusted* probability of which we are supposed to be in search. Taking out these approximate unadjusted probabilities for successive ages, and adjusting them by Woolhouse's or some equally suitable process, we get our *second* approximate value of p, and recalculating $\frac{m+rp}{m+n+r}$ with these improved values, we obtain a second approximation for the required unadjusted probabilities. These latter would then have to be adjusted in their turn, in the same way as the first series; and the results thus obtained could either be adopted as the final adjusted probabilities, or, if not considered sufficiently accurate, could be used as the bases for a third approximation to the true unadjusted series.

From this it appears that Bernouilli's formula $\frac{m}{m+n}$ is the first in the legitimate series of successive approximations to the true value of the unadjusted probability. I use the word "legitimate" because the observed ratio $\frac{m}{m+n}$ (as well as the rest of the series) has the all-important requisite that it is just as likely to be *above*

as *below* the true value; and thus, in a series of observations at successive ages, when combined for the purpose of forming the series representing the number living, the errors of the several individual observations practically neutralize each other, and thus the true aggregate mortality throughout the table is substantially preserved. As Mr. Hardy has pointed out, this is not the case with Laplace's formula $\frac{m+1}{m+n+2}$, inasmuch as the latter formula tends necessarily to exaggerate the observed mortality, (with the exception only of a few extremely high ages) in consequence of its inherent false assumption of $p=\frac{1}{2}$. Even if we suppose Laplace's assumptions were actually true in any given case, that is, say, assuming that we were totally ignorant respecting the true antecedent probabilities, the formula $\frac{m+1}{m+n+2}$ would still be quite unsuitable for the required purpose; for the exaggeration of the aggregate mortality resulting from the use of the formula would not be neutralized by our ignorance. We cannot prevent the operation of a given cause by merely shutting our eyes to, or ignoring the fact of, its existence; and the case is in no wise altered when our ignorance is the result not of choice but of necessity.

Having endeavoured to point out what appears to me the true way of procedure, I must leave it to each one to judge for himself how far in any particular case the several steps in the process of approximation should be carried. Considering the somewhat troublesome nature of the process, and the very small differences (so far as I have tested them) between the first and second approximations, I should think it sufficient generally to stop at the first approximation: that is to say, to adhere to the present practice of adopting for the unadjusted probability the values deduced from Bernouilli's formula $\frac{m}{m+n}$. For supposing the approximations to be carried to an extent sufficient to ensure (practically) *perfect* accuracy in the calculation, still, the errors necessarily inherent in the unadjusted series (irrespective of the errors of the process used in their calculation) are so great that our labour would be very suggestive of the proceeding known as "cutting blocks with a razor." At the same time it would be an improvement, I think, if instead of designating our unadjusted results as the "probability of living a year" (as in the tables of the *Mortality Experience*, published by the Institute) we were to

distinguish them by some such heading as *the ratio of "Survivors" to "Exposed to Risk"*, or, the *approximate unadjusted probability of living a year*. The practice of calling things by wrong names is always objectionable, as it is often the source of a confusion of ideas, if not, indeed, of absolute error.

REVIEW.

*The Fluctuations in the Rate of Mortality in England and Wales.**

AMID the many pressing questions arising from the growth of the population of this country during the past half-century, those which relate to the changes in the rates of mortality command a foremost place in the consideration of the actuary. When it is remembered that between the years 1831 and 1891 the population of England and Wales increased from 13,896,797 to 29,001,018—that is, by upwards of 108 per-cent—alterations in the death-rates of an important character might be expected. The returns of the Registrar-General furnish ample proof that such is the case. In popular opinion the view is often expressed that an increase of no inconsiderable amount has occurred in the duration of life—in short, that people nowadays live longer than they used to do.

Vital statistics are a treacherous basis for sweeping assertions; and statements of this nature, which are more usually the fruit of general impressions and hasty conclusions than the results of careful study, are apt to be misleading when applied to so intricate a question as the mortality of the population, which is one requiring special caution, and on which a tentative opinion, framed on a careful survey of original materials, is more suitable than brilliant and dogmatic expressions.

That the progress of medical science, more general vaccination, and improved sanitation, have been fruitful in checking epidemics and maintaining life, are facts capable of strict demonstration; but when we come to analyze the death-rates and the causes

* *Studies in Statistics*—Social, Political, and Medical. By George Blundell Longstaff, M.A., M.B., &c. London: Edward Stanford, 1891.

of death, with the object of ascertaining the periods of life at which the improvements have occurred, and their effect on the prolongation of life, we reach a more difficult branch of the subject.

Civil registration has been in force in England for upwards of 50 years, having been established in 1837; and a glance at the tables of birth and death-rates published in the annual reports of the Registrar-General will be a useful preliminary step. The birth-rate recorded in 1838 was 30·3 per 1,000 persons living, a figure probably below the truth, owing to defective registration. The rate increased in succeeding years by irregular steps, interrupted at intervals by falling rates, until it reached a maximum, in 1876, of 36·3. Since that year there has been a continuous fall; and in 1889 the rate stood at 30·5, a figure almost identical with that for 1838.

The changes in the death-rates have been of a more fluctuating character: and perusal of the figures in the following table (page 461) will disclose features of considerable interest.

Many sharp fluctuations, caused for the most part by epidemics, are found in the death-rates of successive years; changes of over 1 per 1,000 are not uncommon, changes of over 2 per 1,000 have occurred three times, while on one occasion a fall of 4·3 was registered. But the feature which stands out most prominently is the gradual and important decline effected during the past 15 years. In 1870 the rate stood at 22·9 per 1,000, and in 1875 at 22·7, while in 1880 it had fallen to 20·5; since that date it has been steadily falling, until in 1888 and 1889 the minimum rates of 17·8 and 17·9 were recorded.

There is good reason for supposing, as will be shown later on, that much of the credit for these reduced rates is due to the effect of legislation on sanitary matters. The passing of the Sanitary Act of 1866 was said by Sir John Simon to mark the beginning of a new era in the progress of sanitary reform; and this Act, after having been amended in 1868 and considerably strengthened in 1872, was consolidated with the other sanitary Acts by the comprehensive Act of 1875.

The general death-rate of a population, expressed in a round figure, fails to assist us in determining the changes which may have occurred in the average duration of life, since it ignores the effects of age-distribution and the disturbing element of infant mortality; and for this purpose we require to consider detailed

Annual Birth and Death-rates in England and Wales per 1,000 *persons living*, 1838–1889.

Year	Births to 1,000 living	Deaths to 1,000 living		
		Persons	Males	Females
1838	30·3	...	...	...
1839	31·7	21·9	22·8	21
1840	31·8	22·8	23·7	22
1841	32·2	21·6	22·4	20·8
1842	32·1	21·7	22·4	21
1843	32·3	21·2	22	20·5
1844	32·6	21·5	22·3	20·8
1845	32·5	20·9	21·7	20·1
1846	33·8	23	23·9	22·2
1847	31·5	24·7	25·5	23·9
1848	32·4	23	23·8	22·2
1849	32·9	25·1	25·8	24·4
1850	33·4	20·8	21·4	20·1
1851	34·2	22	22·8	21·2
1852	34·2	22·3	23·2	21·5
1853	33·3	22·9	23·8	22
1854	34·1	23·5	24·4	22·7
1855	33·7	22·6	23·5	21·7
1856	34·4	20·5	21·3	19·6
1857	34·4	21·8	22·6	21·1
1858	33·7	23·1	23·9	22·3
1859	35	22·4	23·3	21·5
1860	34·3	21·2	22·1	20·3
1861	34·6	21·6	22·7	20·6
1862	35	21·4	22·4	20·5
1863	35·3	23	24·1	21·9
1864	34·4	23·7	24·9	22·5
1865	35·4	23·2	24·5	22
1866	35·2	23·4	24·6	22·2
1867	35·4	21·7	23	20·5
1868	35·8	21·8	23·1	20·7
1869	34·8	22·3	23·6	21
1870	35·2	22·9	24·2	21·6
1871	35	22·6	23·9	21·3
1872	35·6	21·3	22·6	19·9
1873	35·4	21	22·4	19·8
1874	36	22·2	23·6	20·9
1875	35·4	22·7	24·1	21·4
1876	36·3	20·9	22·3	19·6
1877	36	20·3	21·7	18·9
1878	35·6	21·6	22·9	20·3
1879	34·7	20·7	22	19·6
1880	34·2	20·5	21·8	19·3
1881	33·9	18·9	20	17·8
1882	33·7	19·6	20·7	18·5
1883	33·3	19·5	20·7	18·5
1884	33·3	19·5	20·7	18·4
1885	32·5	19	20	18
1886	32·4	19·3	20·3	18·3
1887	31·4	18·8	19·8	17·8
1888	30·6	17·8	18·8	16·8
1889	30·5	17·9	18·8	16·9

statements such as are to be found in the Returns of the Registrar-General.

It is, however, time, before proceeding further with the subject, to draw attention to an important work on vital statistics recently published by Dr. G. B. Longstaff, entitled *Studies in Statistics*, which we welcome as a valuable guide, not only to the matter more immediately in hand, but also to other cognate questions connected with our complex social system. A glance through the clearly-printed pages is sufficient to show that in disturbing the dust upon blue books and analyzing their contents, Dr. Longstaff has found a congenial task.

Registration Returns, Census Reports, both home and foreign, and other documents equally formidable in character, have been studied and digested, with the result of disclosing an enormous amount of valuable information, given in an interesting and most readable form, and illustrated by a free use of diagrams. The opening chapters deal with the Birth, Death, and Marriage-Rates in England and Wales for 50 years from 1838; these are followed by papers on such subjects as The Growth of Population, Migrations in the Nineteenth Century, The Growth of New Nations, Food Supply. Many of these papers were, in the first instance, read before the Royal Statistical Society, while the papers on medical questions, of which there are several of great importance, were submitted to various medical societies.

The diagrams already mentioned, which have been executed by the publisher with considerable skill, form an attractive portion of the work, but we doubt whether the arrangement is entirely satisfactory by which they have been made to do duty for tabular statements of original facts. The author seems to have dreaded overloading the letterpress with formidable tables, but the value of a book of this kind for standard reference is greatly enhanced if it contain the original materials from which deductions may be drawn; and if these had been added in an appendix, the readable character of the work would not have been impaired. Taking the book, however, as it stands, as the production of an experienced physician and statistician, it will be found to yield an unusual amount of pleasure and profit, and an ample justification of its motto, "He apprehends a world of figures here."

In addressing ourselves to the subject of Dr. Longstaff's article on "The Recent Decline in the English Death-Rate,

considered in connection with the Causes of Death", we have selected that portion of the volume which will prove of the greatest interest to our profession. The question is one which directly affects insurance societies, since, although it must be affirmed that statistics of the general population, taken in bulk, are not suitable materials for forming definite conclusions respecting assured lives, yet it is most important to ascertain the character and incidence of any marked changes which may have appeared in the death-rates.

By means of a minute and careful analysis of the causes of death, Dr. Longstaff has arrived at certain conclusions regarding the alterations in the death-rates of both sexes at different ages. Before alluding to the opinions formed by the author, we propose to make a brief separate investigation, based upon the ages at death of the population, and then proceed to show how far the results thus obtained harmonize with Dr. Longstaff's conclusions. With this object in view, we have extracted the following table from the *Supplement to Forty-fifth Annual Report of the Registrar-General.*

Annual Mortality per 1,000 *of Males and Females in successive Decennial Periods.*

	Males				Females			
	1841-50	1851-60	1861-70	1871-80	1841-50	1851-60	1861-70	1871-80
All Ages	23·11	23·05	23·61	22·61	21·58	21·32	21·28	20·00
0– 5	71·20	72·43	73·16	68·14	61·09	62·74	63·43	58·10
5–10	9·16	8·51	8·15	6·67	8·89	8·42	7·76	6·20
10–15	5·12	4·88	4·46	3·69	5·42	5·06	4·48	3·70
15–20	7·05	6·69	6·16	5·23	7·88	7·38	6·62	5·43
20–25	9·50	8·83	8·45	7·32	9·08	8·53	7·96	6·78
25–35	9·94	9·57	9·90	9·30	10·55	9·92	9·68	8·58
35–45	12·85	12·48	13·46	13·74	12·91	12·15	12·03	11·58
45–55	18·22	17·96	19·16	20·05	16·04	15·20	15·55	15·59
55–65	31·81	30·85	33·00	34·76	28·44	27·01	27·77	28·54
65–75	67·51	65·33	66·69	69·57	60·97	58·66	58·80	60·82
75–	168·56	165·40	164·64	169·08	157·89	155·45	154·28	155·83

It appears from this table that the mean annual death-rates of both males and females were lower in the years 1871-80 than in either of the preceding periods. The improvement which produced this general result occurred entirely at the younger ages, for it

will be observed that an increase was registered in the case of males above the age of 35 and females above 55. These changes are more clearly shown in the following figures, which furnish the increase or decrease per-cent of the death-rates in the years 1871-80, compared with the mean of those for the three preceding decennial periods.

Increase or Decrease per-cent in the Death-rate for 1871–80 *compared with the mean of the rates for* 1841–50, 1851–60, *and* 1861–70.

	Males	Females
All Ages	− 2·79	− 6·58
0– 5	− 5·70	− 6·92
5–10	−22·06	−25·80
10–15	−23·45	−25·80
15–20	−21·11	−25·50
20–25	−18·02	− 8·68
25–35	− 5·10	−14·62
35–45	+ 6·46	− 6·31
45–55	+ 8·67	...
55–65	+ 8·99	+ 2·88
65–75	+ 4·60	+ 2·24
75–	+ 1·73	...

The figures in this table show that changes of a very marked character occurred in the death-rates during the later period. The general average rate for males decreased 2·8 per-cent, and for females 6·6 per-cent. A lower rate is observable for males from birth up to age 35, the greatest improvement being at ages 5 to 25, while above 45 the mortality increased. In the case of females the decreased mortality endured from birth to 45, the rates for ages 5 to 20 being especially low; at ages 45–55 they coincided with those of the earlier period; and at ages 55–75 slightly exceeded them.

We obtain an important clue to the incidence of the mortality by examining the causes of death, which are summarized in the following table, extracted from the *Supplement to the Registrar-General's Forty-fifth Report.*

Annual Deaths per Million Persons living.

	1861-70	1871-80	Annual Decrease in 1871-80	Annual Increase in 1871-80
Small Pox	163	236	...	73
Other Zymotic Diseases . . .	4,085	3,146	939	...
Cancer	387	473	...	86
Phthisis	2,475	2,116	359	...
Hydrocephalus	347	317	30	...
Other Tubercular Diseases . . .	437	445	...	8
Diseases of the Nervous System . .	2,785	2,770	15	...
Diseases of the Circulatory System and Dropsy	1,349	1,477	...	128
Diseases of the Respiratory System .	3,364	3,760	...	396
" " Digestive " .	981	978	3	...
" " Urinary " .	298	392	...	94
All other and unstated Causes . .	5,745	5,162	583	...
	22,416	21,272	1,929	785
Balance of Decrease	...	...	1,144	...

Of the various causes which claimed a lessened death-roll, the zymotic diseases were most significant; since out of a total annual decrease of 1,144 deaths per million persons living, no less than 866, or 76 per-cent, occurred in the zymotic class.

If we trace the ages at which the zymotic diseases are fatal, we find that upwards of 80 per-cent of deaths from these causes occur at ages below 10.

It follows, therefore, that a very large percentage of the diminished mortality of the country must be expressed not as increased longevity, but rather as saving of life in infancy and childhood.

Among the causes producing this important reduction in the death-roll of recent years, a foremost place must be given to those legislative measures for sanitary reform which the enormous increase of population rendered imperative, and to which reference has been already made.

The limits of this article forbid our entering in detail into the incidence of the various diseases mentioned in the table, a subject to which several chapters in Dr. Longstaff's book are devoted. But, regarding the matter from the point of view of life assurance, some features stand out prominently for notice. It has been seen that the zymotic diseases attack infant life and childhood, so that the effect of the diminished death-rate from these causes has been that children, in increasing numbers, have lived to attain the assuring ages.

Of the children thus saved from zymotic diseases some will

develop sound constitutions and attain old age; while others, lacking in stamina, will dwindle off and swell the death-rates at adolescence. But the improvement, which commenced early in the decade 1871-80, is too recent to enable us to judge at present what its effect on the general death-rate will be. The evidence points in the direction of a general levelling-up of vitality during the productive ages; and although against this must be set off the increased mortality in manhood, yet, as Dr. Longstaff shows, whatever may have produced this result, it cannot have been caused by the subsequent deaths of children saved from zymotic diseases since 1870, as sufficient time had not elapsed for them to have reached the ages at which the higher rates are apparent.

The mortality from phthisis, which showed the remarkable diminution of 359 deaths per million persons living, is more than counter-balanced by an increased loss of 396 per million from diseases of the respiratory organs. The other diseases which caused an increased death-roll are cancer, diseases of the circulatory system and dropsy, and of the urinary system; and it will be found that all these complaints operate chiefly at adult ages.

It will be interesting to note in this connection the incidence of various diseases in a new country, and this we are able to do by means of the comprehensive statistics contained in the *Victorian Year Book*, from the current number of which the following figures have been extracted.

Annual Death-rate from each Group of Causes and Certain Diseases in Victoria and England and Wales.

Class	Sub-Class	Causes of Death	Number of Annual Deaths per 100,000 of Mean Population			
			Victoria			England and Wales
			Year 1889	Average of 5 years, 1881-85	Average of 10 years, 1871-80	Average of 5 years, 1876-80
...	...	All Causes	1,756·04	1,464·31	1,538·01	2,081·74
		CLASSES.				
I	...	Specific Febrile or Zymotic Diseases	296·30	228·97	356·74	332
II	...	Parasitic Diseases	7·33	8·57	9·21	5·74
III	...	Dietic Diseases	25·63	24·64	24·83	6·72
IV	...	Constitutional Diseases . . .	258·35	232·70	215·89	358·26
V	...	Developmental Diseases . . .	122·43	97·11	75·82	163·14
VI	...	Local Diseases	790·18	670·12	642·57	1,004·80
VII	...	Violent Deaths	107·40	92·99	108	72
VIII	...	Ill-defined and not Specified Causes.	148·42	109·21	104·95	139·08

Annual Death-rate from each Group of Causes and Certain Diseases in Victoria and England and Wales—(continued).

Class	Sub-Class	Causes of Death.	Number of Annual Deaths per 100,000 of Mean Population			
			Victoria			England and Wales
			Year 1889	Average of 5 years, 1881-85	Average of 10 years, 1871-80	Average of 5 years, 1876-80
		SUB-CLASSES AND DISEASES.				
I	1	Miasmatic Diseases	159·01	100·56	193·98	219·26
		Measles	1·72	8·47	26·10	38·54
		Scarlet Fever	3·71	6·18	51·46	68·04
		Diphtheria*	·48	14·93	38·15	12·18
		Whooping Cough	21·19	15·36	24·77	52·76
		Typhoid or Enteric Fever	82·40	51·81	49·85	34·66
	2	Diarrhœal Diseases	120·14	107·09	138·48	85·40
		Diarrhœa	94·36	87·54	96·21	83·36
		Dysentery	12·77	13·79	35·71	
	3	Malarial Diseases	·45	·85	2·67	·74
	4	Zoogenous Diseases	...	...	·01	·36
	5	Venereal Diseases	3·26	4·78	4·73	9·56
	6	Septic Diseases	13·04	15·69	16·87	16·68
		Puerperal Fever (see also Childbirth and Puerperal *infra*)	5·62	6·88	5·78	6·20
IV	...	Constitutional Diseases:				
		Cancer	56·96	45·26	37·10	49·60
		Phthisis	142·26	140·88	127·42	204·24
		Other Tubercular and Scrofulous Diseases	40·03	32·96	36·35	77·87
V	...	Developmental Diseases:				
		Of Children (Premature Birth or Malformation)	55·87	43·33	41·60	55·80
		Of Old Age	66·47	53·78	34·22	107·34
VI	...	Diseases of the Systems:				
	1	Nervous (Brain Diseases, &c.)	165·81	162·50	169·80	277·80
	2	Organs of Special Sense (Nose, Ear, and Eye Diseases)	1·45	?	?	1·34
	3	Circulatory (Heart Diseases, &c.)	125·15	97·59	85·37	141·70
	4	Respiratory (Lung Diseases, &c.)*	221·86	218·43	194·92	395·50
	5	Digestive (Stomach, Bowel, Liver Diseases, &c.)	200·39	134·46	141·04	117·60
	6	Lymphatic and Ductless Glands	·81	·50	·55	1·26
	7	Urinary (Kidney, Bladder Diseases, &c.)	49·62	33·12	23·53	37·50
	8	Generative (Diseases of Ovary, Uterus, and Vagina, &c.)	4·71	2·63	3·70	5·88
	9	Childbirth (see also Childbirth and Puerperal Fever *infra*)	9·15	13·68	15·90	7·96
	10	Locomotive (arthritis, ostitis, &c.)	5·34	3·00	3·64	10·78
	11	Integumentary (carbuncle, phlegmon, ulcer, &c.)	5·89	4·21	4·12	7·48
VII	1	Accident or Negligence	90·55	80·26	93·45	63·12
	2	Homicide	3·62	2·41	2·38	1·40
	3	Suicide	12·95	10·15	11·93	7·38
	4	Execution	·27	·17	·24	·10
I VI	6 9	Childbirth and Puerperal Fever	14·77	20·55	21·68	14·16

* Deaths from diphtheritic croup are not included with those from diphtheria, but under the head of croup, which is classed as a disease of the respiratory system, Class VI, Sub-Class 4.

Glancing at the main classes of disease enumerated in the foregoing tables, we find that the zymotic class, during the years prior to 1880, caused a higher rate in Victoria than in England; and that, while a considerable decrease occurred in the colonial rate during the five years 1881–85, a large increase, amounting to 30 per-cent above the average of recent years, was noted in the single year 1889. This sudden rise was occasioned by an increased number of deaths from typhoid fever, diphtheria, whooping cough, and diarrhœal diseases.

Constitutional diseases are at present much less fatal in Victoria than in England; but this class—which includes rheumatism, gout, cancer, and phthisis—appears to be slowly gaining in intensity.

The developmental diseases, both of children and of old age, are also much lower in the colony, but the increasing average age of the population is shown by the ratio of deaths from old age in 1889 being practically double that of the average for 1871–80. The proportion of old people (ages 75 and upwards) in the population has increased since 1871–80 by 88 per-cent, whilst the average age of such persons has increased also.

Local diseases, or diseases of the systems, are, as a class, some 50 per-cent lower in Victoria than in England; but, in the case of the digestive system, the home rate is lower than the colonial.

It may be stated that the main part of our investigation was made before the publication of Dr. Longstaff's book; and, in view of his exhaustive treatise, the result of this independent enquiry is chiefly valuable as serving to corroborate many of the conclusions at which the author has arrived.

Our observations on the fluctuations in the death-rates may be suitably closed by an extract from Dr. Longstaff's valuable article in which the author's conclusions on the subject are summarized. The chapter from which these extracts are made is worthy of careful study—indeed, the entire volume may be warmly commended to the attention of all persons interested in the question of social statistics.

"(1). The decline is to be attributed to a diminished number of deaths assigned to fever or phthisis, and, to a somewhat less degree, to scarlet fever, diarrhœal diseases, small pox, diphtheria, and measles.

"(2). There is also a great diminution under the heading, 'All other Causes', but this is largely to be explained by improved diagnosis, and greater precision in filling in certificates of the cause of death.

"(3). There has been, coincidently with a fall in the general death-rate, a considerable increase under certain causes of death, notably diseases of the lungs, heart and kidneys, and cancer.

"(4). The figures prove that only a *portion* of the increase noted under the above headings can be explained by improvement in the

returns. There has been a real, not merely an apparent, increase. More especially, the increase under lung diseases will not explain the fall under phthisis.

"(5). The gain under phthisis is more important than the loss under cancer, kidney diseases, and some others, since the lives saved are more valuable than those lost; also, in the case of phthisis, parents beget children when far advanced in disease, a fact greatly to be regretted.

"(6). Preventive medicine should be especially directed against those diseases which kill the largest numbers, such as 'local diseases' generally, the ill-defined diseases of infancy and phthisis; while among the zymotic diseases diarrhœa and scarlet fever, whooping cough and measles, require especial attention at the present time (1884).

"(7). Since the falling causes of death can only fall to zero, but the rising causes may rise indefinitely, the present changes may conceivably, in course of time, lead to a rise in the general death-rate.

"(8). The tendency appears to be for useful working life to be increased, but for old age to be slowly shortened.

"(9). The increased mortality in the latter years of life, and such phenomena as the increase of defective sight and rapid deterioration of the teeth, probably result from the increasing proportion of the population living under the artificial conditions of town life.

"(10). The present increased mortality at the higher ages cannot possibly be explained by the *recent* diminished mortality at the beginning of life, even on the supposition that the survivors are more delicate.

"(11). The changes that are taking place in the incidence of mortality almost all aid the other causes which contribute to the great superabundance of women in England and Wales.

"(12). They also tend to increase population, both by diminution of deaths, and increase, positively and relatively, of births."

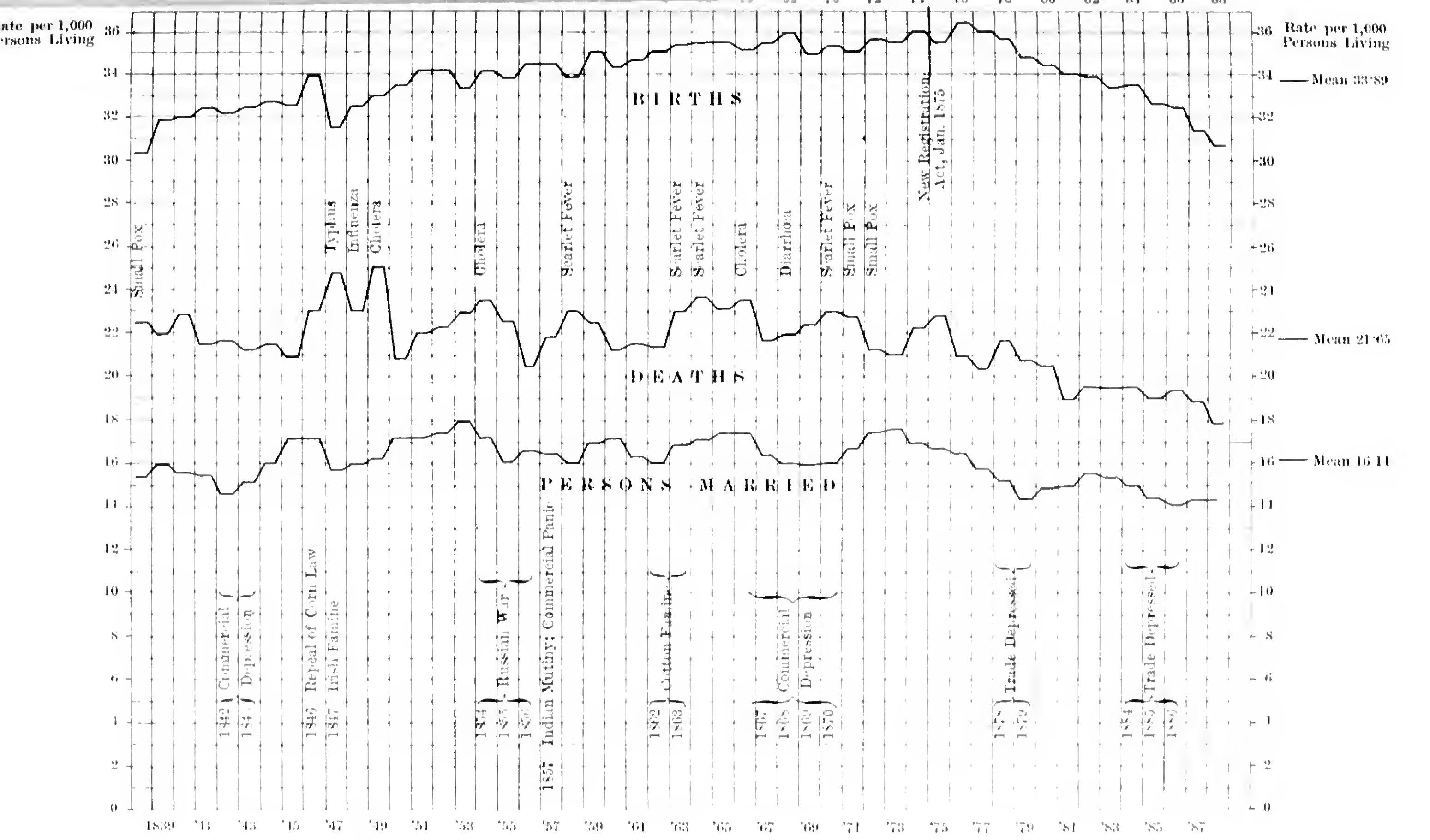
To the foregoing results, which were arrived at by Dr. Longstaff in 1884, he appends a postscript, written in 1890, pointing out that, since the earlier date, the death-rate has continued extremely low. He gives the following table (page 470), in which the death-rates from all causes, at various groups of ages, during the five years 1881–85, are compared with the figures for 1876–80, showing that the mortality has decreased at every age-period in each sex. The fall has been but slight in males aged 65–75, and females aged 45–55. In both sexes the diminution has been greatest at the two extremes of life.

The diagram accompanying this article has been taken from one of those in Dr. Longstaff's book, and we have to acknowledge the author's courteous consent to its reproduction. It shows, in graphic form, the birth, death, and marriage-rates for 50 years in England and Wales, indicating also any special events—such as epidemics and commercial depression—which were likely to have affected the results.

A. F. BURRIDGE.

England and Wales.—Death-rates of Males and of Females from all Causes per 1,000 living, at various Groups of Ages.—The Quinquennium 1881–85 compared with the Quinquennium 1876–80.

	All Ages	0–	5–	10–	15–	20–	25–	35–	45–	55–	65–	75–	85–
MALES:													
1876–80 . .	22·14	67·04	6·34	3·44	4·86	6·72	8·74	13·38	19·82	34·92	69·40	152·2	331·6
1881–85 . .	20·42	59·60	5·78	3·16	4·56	6·00	8·18	12·74	19·42	33·64	68·78	144·6	296·4
Difference . .	1·72	7·44	·56	·28	·30	·72	·56	·64	·40	1·28	·62	7·6	35·2
Fall, *per-cent* .	**7·8**	**11·1**	**8·8**	**8·1**	**6·2**	**10·7**	**6·4**	**4·8**	**2·0**	**3·7**	**·9**	**5·0**	**10·6**
FEMALES:													
1876–80 . .	19·54	56·82	5·88	3·50	5·04	6·24	8·02	11·24	15·36	28·60	60·76	135·5	299·0
1881–85 . .	18·24	50·48	5·62	3·30	4·72	5·94	7·90	10·90	15·24	27·82	59·46	129·4	267·8
Difference . .	1·30	6·34	·26	·20	·32	·30	·12	·34	·12	·78	1·30	6·1	31·2
Fall *per-cent* .	**6·7**	**11·2**	**4·4**	**5·7**	**6·3**	**4·8**	**1·5**	**3·0**	**·8**	**2·7**	**2·1**	**4·5**	**10·4**



Rate per 1,000 Persons Living
Mean 33·89
Mean 21·65
Mean 16·11
BIRTHS
DEATHS
PERSONS MARRIED
Small Pox
Typhus
Influenza
Cholera
Cholera
Scarlet Fever
Scarlet Fever
Scarlet Fever
Cholera
Diarrhœa
Scarlet Fever
Small Pox
Small Pox
New Registration Act, Jan. 1875
1842 Commercial
1843 Depression
1846 Repeal of Corn Law
1847 Irish Famine
1854 1855 1856 Russian War
1857 Indian Mutiny; Commercial Panic
1862 1863 Cotton Famine
1867 1868 1869 1870 Commercial Depression
1878 1879 Trade Depressed
1884 1885 1886 Trade Depressed
1839 '41 '43 '45 '47 '49 '51 '53 '55 '57 '59 '61 '63 '65 '67 '69 '71 '73 '75 '77 '79 '81 '83 '85 '87

THE LIFE ASSURANCE COMPANIES OF THE UNITED KINGDOM.

Summary of the Life Assurance and Annuity Revenue Accounts.

[Extracted from the Parliamentary Return for 1890, published in 1891.]

INCOME	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Balance at the beginning of the Year	160,172,605	7,167,869	167,340,474
Adjustments for two Ordinary (£39,913 and £505,860) and two Industrial (£2,562 and £21,885) Returns discontinued, and for four Balances transferred from the Ordinary Branch, namely, three (£2,800, £15,704, and £121,577) to the Industrial Branch, and one (£20) to General Revenue Account	−685,874	+115,634	−570,240
	159,486,731	7,283,503	166,770,234
Premiums	14,213,386	4,853,735	19,067,121
Consideration for Annuities	1,275,665	...	1,275,665
Interest and Dividends (less Tax)	6,527,543	250,836	6,778,379
Increase in value of Investments	291,675	779	292,454
Fines, Fees, &c.	6,889	395	7,284
Capital Paid-up	35,719	5,481	41,200
Customs Timber Measuring, &c.	3,696	...	3,696
Donations (Itinerant Methodists)	3,612	...	3,612
Special Recoveries (Briton Medical and General)	17,000	...	17,000
Transfers from other Accounts	17,310	3,289	20,599
Miscellaneous	1,734	1,451	3,185
	181,880,960	12,399,469	194,280,429

OUTGO	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Claims	10,627,096	1,928,406	12,555,502
Cash Bonuses and Reduction of Premiums	998,693	7	998,700
Annuities	797,427	992	798,419
Surrenders	819,458	13,807	833,265
Commission	678,324	1,304,529	1,982,853
Expenses of Management	1,361,913	838,110	2,200,023
Bad Debts	906	591	1,497
Decrease in value of Investments	116,845	...	116,845
Interest on Capital and Dividends and Bonuses to Shareholders	593,403	53,899	647,302
Transfers to other Accounts	43,484	70	43,554
Capital withdrawn from Life Account (Sc. Metropolitan)	21,000	...	21,000
Miscellaneous	10,277	...	10,277
Balance* at the end of the Year	165,809,134	8,259,058	174,068,192
	181,880,960	12,399,469	194,280,429

* This Balance includes the whole of the Life and Annuity Funds (£168,511,507), and, in addition, the Capital of Companies whose business is limited to Life Assurance only.

Summary of the Balance Sheets (1890).

LIABILITIES	Ordinary Companies	Industrial Companies	Total
	£	£	£
Paid-up Capital (including sundry Shareholders' Balances)	11,327,149	469,754	11,796,903
Life and Annuity Funds	160,485,372	8,025,323	168,510,695
Fire Funds of Companies transacting Life Business	9,823,360	...	9,823,360
Marine Funds of Companies transacting Life Business	557,000	...	557,000
Reserve Funds	3,712,585	...	3,712,585
Other Funds	451,781	197,292	649,073
Profit and Loss Balances	3,085,244	...	3,085,244
Depreciation and Investment Balances	731,723	5,292	737,015
Globe Annuitants (Liverpool and London)	1,102,800	...	1,102,800
Outstanding Claims	3,148,507	16,546	3,165,053
Outstanding Accounts	430,835	19,691	450,526
Temporary Loans	15,750	4,038	19,788
	194,872,106	8,737,936	203,610,042

ASSETS	Ordinary Companies	Industrial Companies	Total
	£	£	£
Mortgages	78,806,268	279,902	79,086,170
Loans on Policies	8,914,314	25,449	8,939,763
,, Rates (and Rent-charges)	21,391,793	2,431,236	23,823,029
British Government Securities	5,703,832	722,664	6,426,496
Indian and Colonial Government Securities	12,757,092	105, 681	12,862.773
Foreign Government Securities	3,539,651	...	3,539,651
Debentures	18,934,268	1,415,701	20,349,969
Shares and Stocks	12.965,377	173,687	13,139,064
Companies' own Shares	528,846	...	528,846
Land and House Property and Ground Rents	12,091,861	2,715,168	14,807,029
Life Interests and Reversions	3,533,449	720	3,534,169
Loans on Personal Security	1,246,277	8,875	1,255,152
Agents' Balances and Outstanding Premiums	4,096,640	404,920	4,499,560
Outstanding Interest	1,824,650	81,096	1,905.746
Cash, Deposits, Stamps, &c.	8,363,853	84,440	8,450,293
Customs Timber Measuring Balances, &c.	2,536	...	2,536
Book-Room Grant (Itinerant Methodists)	75,000	...	75,000
Deficiencies, Preliminary Expenses, &c.	96,399	288,397	384,796
	194,872,106	8,737,936	203,610,042

INCREASE (+) *or* DECREASE (−) *in the Chief Items of this Year's* SUMMARY *compared with the corresponding Items for the previous Year.*

	Ordinary Companies	Industrial Companies
INCOME.	£	£
Premiums	+ 285,385	+ 493,297
Consideration for Annuities	+ 167,878	− 175
Interest and Dividends (less Tax)	+ 201,865	+ 31,417
Net Increase in Value of Investments	+ 3,502	+ 970
OUTGO.		
Claims	− 374,485	+ 264,745
Annuities	+ 54,101	+ 936
Surrenders	− 69,270	+ 4,312
Commission	− 30,923	+ 125,474
Expenses of Management	− 62,592	+ 133,869
LIABILITIES.		
Paid-up Capital (including sundry Shareholders' Balances)	− 16,185	+ 178,838
Life and Annuity Funds	+5,542,813	+1,146,804
ASSETS.		
Mortgages (including Loans on Rates and Rent-charges)	+ 418,385	+ 641,184
Life Interests and Reversions	+ 265,418	+ 720
Loans on Policies	+ 109,557	+ 14,902
British Government Securities	+ 177,163	+ 22,796
Indian and Colonial Government Securities	+ 229,352	+ 37,185
Foreign Government Securities	− 48,597	...
Debentures	+2,124,775	+ 88,322
Shares and Stocks	+ 495,332	+ 159,037
Companies' own Shares	+ 67,713	...
Land and House Property and Ground Rents	+ 261,092	+ 226,824
Loans on Personal Security	+ 37,318	− 3,334

One Company has been transferred from the Ordinary to the Industrial Summary, and in two other Companies the Ordinary and Industrial portions of the business have been for the first time separately classified.

NUMBER OF COMPANIES.

The total number of Companies appearing in the above Summary is 100, of which 87 have been classed as Ordinary, 9 as Industrial, and 4 appear in both Classes, the Returns of these Companies showing the Ordinary and Industrial business separately.

During the year five names have been removed from the official List of Companies, namely, the Argus, Sovereign, United Kingdom Assurance Corporation (Limited), Western Counties and London, which have either ceased to do or have transferred their business, and the United Brothers (Limited) which is being wound up. And one new name has been added, namely, the Universal Insurance (Limited).

SUMMARY OF THE ASSURANCES IN FORCE, *as shown by the last Returns of the Compani*

ORDINARY BUSINESS.

	With Profits		Without Profits		Total		Re-assurances	Net Amount
	No.	Amount	No.	Amount	No.	Amount	Amount	
Assurances.		£		£		£	£	£
Whole Term of Life	668,317	339,549,585	79,576	54,807,845	747,893	394,357,430	18,365,058	375,992,37
Limited number of Premiums . .	26,150	14,872,298	4,139	1,927,080	30,289	16,799,378	576,108	16,223,27
	694,467	354,421,883	83,715	56,734,925	778,182	411,156,808	18,941,166	392,215,64
Endowments . .	2,934	526,559	5,897	960,286	8,831	1,486,845	4,768	1,482,07
Endowment Assurances . . .	134,882	28,980,185	25,667	7,465,104	160,549	36,445,289	578,422	35,866,86
Joint Lives . .	7,645	2,027,557	1,982	896,208	9,627	2,923,765	303,920	2,619,84
Last Survivor . .	1,210	928,109	1,170	1,199,764	2,380	2,127,873	224,190	1,903,68
Contingent . .	47	48,489	2,847	4,597,568	2,894	4,646,057	1,007,121	3,638,93
Issue . . .	6	7,150	694	2,918,082	700	2,925,232	825,549	2,099,68
Miscellaneous . .	151	82,132	3,791	4,454,129	3,942	4,536,261	1,000,766	3,535,49
	841,342	387,022,064	125,763	79,226,066	967,105	466,248,130	22,885,902	443,362,22
Annuities.								
Immediate . .	...	...	...	...	16,727	750,407	5,889	744,51
Deferred . .	...	...	...	...	4,406	140,001	5,322	134,67
	...	...	...	...	21,133	890,408	11,211	879,19

INDUSTRIAL BUSINESS—(Sickness and Friendly Society Contracts not included).

	With Profits		Without Profits		Total		Re-assurances	Net Amount
	No.	Amount	No.	Amount	No.	Amount	Amount	
Assurances.						£		£
Whole Term of Life	...	...	...	...	9,234,565	83,257,478	...	83,257,478
Limited number of Premiums . .	...	...	...	...	3	7	...	7
	...	...	...	...	9,234,568	83,257,485	...	83,257,485
Endowments . .	...	...	...	...	25,873	348,232	...	348,232
Endowment Assurances . . .	...	...	...	...	71,842	1,129,745	...	1,129,745
Joint Lives . .	...	...	...	...	100,495	1,468,411	...	1,468,411
	...	...	...	...	9,432,778	86,203,873	...	86,203,873
Annuities.								
Immediate . .	...	...	...	...	1	15	...	15

The above figures are based on Returns deposited for the most part during the past five years, an are, therefore, merely an approximation to the amount of contracts in force at the present time. In th case of five Companies, namely, the Co-operative, Customs' Fund, Industrial of Great Britain, Northern and Provincial, the amount of business at a more recent date has been included.

ACTUARIAL NOTE.

On a Problem in Probabilities.

THERE are four urns which are known to contain 3 balls each. It is also known that in one of them the 3 balls are white; in another, 2 are white and 1 black; in a third, 1 white and 2 black; and in the fourth, the 3 balls are black. From one of the urns, *chosen at random*, m drawings have been made (each ball drawn being replaced) with the result that white balls only have been drawn. Required the *a posteriori* probability (after the drawing of the m white balls in succession) of each of the following hypotheses, namely, that the contents of the urn chosen consist of

1. Three white balls,
2. Two white and one black,
3. One white and two black, or
4. Three black.

As the urn chosen may be any one of the four, it is evident that, *a priori*, there is precisely the same chance in favour of each of the four hypotheses in question. We have here, then, *necessarily*, the identical condition *gratuitously* assumed by Laplace in the solution of his well-known problem (see *Théorie Analytique des Probabilités*, p. 183). Consequently, the solution arrived at by Laplace in that case, by the application of his III^{e} Principe, will be the true solution of the present problem, namely,

Hypotheses		Absolute Probability (*a posteriori*) of each Hypothesis
White Balls	Black Balls	
3	0	$\frac{3^m}{3^m+2^m+1^m+0^m}$
2	1	$\frac{2^m}{3^m+2^m+1^m+0^m}$
1	2	$\frac{1^m}{3^m+2^m+1^m+0^m}$
0	3	$\frac{0^m}{3^m+2^m+1^m+0^m}$

There can be no doubt, I think, that the absence from Laplace's problem, as stated by him, of the preliminary condition (that the urn from which the drawings are made is first chosen at random from among a number of urns with specified contents) changes fundamentally the nature of the problem. And it is evident that it may be still further varied by making other assumptions as to the number of the urns and of their supposed contents. I have given elsewhere (see p. 451) what I conceive to be the correct solution of the problem as stated by Laplace.

W. M. MAKEHAM.

9 *September* 1891.

CORRESPONDENCE.

THE TWENTY-THREE GERMAN OFFICES' EXPERIENCE.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—A reference of Mr. Chatham's (*J.I.A.*, xxix, 91) to the method adopted by the compilers of the mortality experience of the twenty-three German offices, immediately following a notice of Mr. Meikle's Paper "On the Official Publications of the Mortality of Assured Lives" (T. A. S. Edin., i, 334), with a mention of his method of computing the exposed to risk by taking the lives from birthday to birthday, has prompted me to offer a description of the exact process employed by the German actuaries, which I trust will be the more interesting, inasmuch as the only previous reference to it occurs in Mr. Meikle's pamphlet.

The difference between the German method and that of other experiences lies fundamentally in the manner of treating the age. In the experiences anterior to this, the observations grouped under age x had been made up really of persons who might have been half a year older or half a year younger than x, and it had been assumed that the average age would not differ materially, in a sufficient number of observations, from age x. In dealing with such an experience the rate of mortality of a group of lives of all ages from $x-\frac{1}{2}$ to $x+\frac{1}{2}$ being determined, the result is presumed to represent with sufficient accuracy the rate among a body of lives who are all of the exact age x. The observations are divided into yearly periods, determined either by the falling due of an annual premium or the end of a calendar year, and the rate of mortality deduced for each year.

The method adopted by the Germans dealt with the age exactly; the entrants were grouped according to age, and the periods of observation were determined to be the time from birthday to birthday —always one year, with the exception of the first and last periods. As the insured do not enter on their respective birthdays, the first period runs from the date of entry to the next birthday—on the average, half a year, but varying from a year to a day; the second period, and every other except the last, runs from birthday to birthday, while the last period runs from the birthday to 31 December 1875, the close of the observations. The last period on the average is, like the first, half a year.

There is no difficulty in dealing with any of the intervening years, but a little further consideration may be given to the first and last periods.

The ages in the first period vary from an exact age to the next exact age, less one day, and the average duration is half a year. Following, in principle, the method adopted by the Institute in dealing with year 0, the rate of mortality has been found by working with half the entrants to get the exposed for a full year, and the age has been taken to be the age at the previous birthday, thus obviously understating the average age at entry by half a year. For the last period the time of observation varies from a year to a day, and the age is the age of the last birthday under observation, while half the number exposed to risk has been adopted as in the first period. This explains the difference noticed by Mr. Meikle between the making up of the experience of the Institute and American tables, and that of the German, which amounted to half the existing at the close of the observations on 31 December 1875.

Considering, further, the differences between this method and that of the Institute, at the commencement of the second period we have a number of lives all of exactly the same age, but the time which has elapsed since entry is different, varying from a year to a day; on the average, half a year has elapsed, and it is assumed that the rate of mortality deduced correctly represents the rate among a body of insured lives who have all entered a society exactly half a year ago. Similarly for other periods of observation—at the commencement of the third period the average time elapsed since the date of entry is $1\frac{1}{2}$ years, at the commencement of the fourth, $2\frac{1}{2}$ years, and so on, the experience being made up in years similar to the calendar years of the Institute and American experiences, and these years may be called years 0, 1, 2, 3 . . . . as if they were truly calendar years of observation.

In conclusion, I may say that the foregoing explanation has been

written after a perusal of a paper in the German Assurance Year Book for 1886,* "On the Methods for ascertaining the Mortality from the Experience of Life Insurance Offices", by the late Mr. W. Lazarus, in which the author not only expounds the method followed by the Germans most carefully and fully, but also examines that which has been adopted by the Institute, by the Gotha, and by the Americans respectively.

I am, Sir,

Your obedient Servant,

York, PHILIP L. NEWMAN.

3 *June* 1891.

"CURRENT AGE."

To the Editor of the Journal of the Institute of Actuaries.

Sir,—In your report of the discussion on Mr. Chatham's prize essay you state that I objected to the phrase *current age*, as incorrect; and I shall be obliged if you will now allow me to state the reasons why I consider it to be incorrect. We can speak of the current week, or the current year, or the current century: because each of these is a definite period of time, which is in progress—in fact, current; but a man's age is a period of time which is not current, but is completed. This is obvious when we have the age stated exactly as so many years, months, and days. In ordinary language, however, if a man is between 29 and 30 years of age, we say that he is 29, taking into account only the completed years; and it may be argued that, as the 30th year of his age is current, it is permissible to speak of his current age being 30 years, as is done in the *Institute of Actuaries' Mortality Experience*. To me this seems as incorrect as it would be to say that the now current century is 1,900 years. The following illustrations, however, may perhaps place the matter in a still clearer light. If the distance between one place and another is between 29 and 30 miles, we may say in general that the distance is 29 miles, neglecting the fraction of a mile; but we could not say the current distance is 30 miles. So, again, if the time is between 11 and 12, we could not say that the current time is 12 hours; but this would be exactly analogous to saying that the current age is 30 years. I trust, therefore, that the objectionable phrase will be dropt, and that the

* *Assecuranz Jahrbuch.* Herausgegeben von A. Ehrenzweig, vii Jahrgang Wien, 1886. Vol. vii, Part ii, pp. 216–239.

old, unmistakeable, and in every way unobjectionable term, *age next birthday*, will in future be always employed.

I am, Sir,

Your obedient Servant,

Edinbro', T. B. SPRAGUE.

17 *June* 1891.

THE INSTITUTE OF ACTUARIES.

SYLLABUS OF SAMUEL BROWN PRIZES.

The Council of the Institute of Actuaries have resolved to offer Two Prizes, of the value of Fifty Guineas and Twenty-five Guineas respectively, for the best two Essays, to be written by Members of the Institute, on the following subject:

> "*The Enfranchisement of Leaseholds, and the Taxation of Ground Rents, Chief Rents, and kindred charges on Land in England and Wales.*"

While allowing the Competitors perfect freedom in their treatment of the subject, it is desired that the following (among other) points should be considered:

1. The origin of ground rents, and the different forms of leasehold tenure in England and Wales; and the advantages and disadvantages of the leasehold system.

2. The terms and conditions upon which enfranchisement, if thought desirable, should be based, with illustrations; and the need, in such event, of appointing any and what controlling authority, and with what powers.

3. The distinction, if any, to be drawn between ground rents and other forms of property; and the economical, social, and political effects of taxing ground rents.

4. The form which taxation of ground rents, if thought desirable, should take; and the consequential treatment of freeholds.

5. The order of procedure which should be followed in the event of both measures being deemed desirable,—*i.e.*, whether enfranchisement should precede taxation, or taxation precede enfranchisement, or both be passed concurrently.

N.B.—Due consideration should be given to suggestions publicly made in regard to any of the subjects dealt with.

CONDITIONS OF THE COMPETITION.

1. That the Essays shall be sent in to the Honorary Secretaries of the Institute of Actuaries on or before 31 March 1892.
2. That the names of the Competitors shall be sent in, under seal, with a Motto corresponding to one to be affixed to the head of the Essay—such Motto and Essay not to be in the handwriting of the Competitor.
3. The Essays to which the Prizes shall be awarded to become the property of the Institute.
4. The unsuccessful Essays to be returned, with the corresponding envelopes unopened.
5. Neither of the Prizes shall be awarded unless the Adjudicators shall consider some Essay worthy of the distinction.
6. The Adjudicators, who must not be Competitors, will be the President and Vice-Presidents, for the time being, of the Institute.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On Legislation affecting Life Assurance Companies, more especially with reference to the Life Assurance Companies Acts, 1870 *to* 1872, *and their Amendment.* *By* GEORGE KING, F.I.A., F.F.A., *one of the Honorary Secretaries of the Institute of Actuaries, and Actuary of the Atlas Assurance Company.*

[Read before the Institute, 30 November 1891.]

WHEN I agreed to prepare a paper for the consideration of the Institute of Actuaries, on the Life Assurance Companies Acts, 1870–72, I did not do so with a light heart. The task in any case would have been onerous, and events which have since transpired have not tended to increase my confidenee. It has become a matter of public knowledge that the Council of the Institute had under consideration the laws relating to life offices, and that it was intended to take an early opportunity to debate the question. Moreover, it has been surmised that Government had in contemplation to take action in the matter, and, possibly in next session of Parliament, to bring in a Bill for the amendment of the law. I, therefore, cannot help feeling that a great responsibility rests upon me in opening a discussion of such importance, but I am emboldened to proceed, because, even if I fail to produce anything new or important, there are many here

of far riper experience than myself, and a debate in such an assembly cannot but be instructive and useful.

The Institute of Actuaries is peculiarly fitted to deal with this intricate question. It is a body of experts, whose daily duties lead them to acquire an intimate knowledge of the law as it stands; and who, in their efforts to extend and improve the business of life assurance, become acquainted with the defects of existing regulations which require to be removed. Moreover, the Institute of Actuaries represents the independent opinion of experts. Its members are not here as delegates in their capacities as the managers or actuaries of companies, but they are here in their individual capacities, and are free, therefore, to take an unprejudiced and impartial view of the question, and to give unbiassed opinions. They can look at the matter from the point of view of the public, and entirely irrespective of the manner in which individual companies and their officials may be affected.

It is unnecessary, in so far as the members of the Institute are concerned, to explain that anything that this paper may contain is written on the individual responsibility of the author. In each number of the *Journal* issued, it is distinctly stated that the Council of the Institute, while they consider it their duty to give, from time to time, publicity to certain papers presented to the Institute, do not hold themselves responsible for the opinions put forward therein. The same remark applies to discussions held in the hall of the Institute, and neither the opener of the debate, nor any of those who take part in it, can officially or unofficially bind anyone but himself. Nevertheless, as reports of our proceedings will, no doubt, pass beyond our own circle, it may be well clearly to define the situation, more especially in view of the official position with which my professional brethren have honoured me. It must, therefore, be distinctly understood that the author alone is responsible for the opinions expressed in the paper, and that the Institute is not in the remotest degree committed to them.

The subject naturally divides itself into two parts, the first dealing with the General Principles of the Law, and the second with suggested Amendments.

Part I.—General Principles of the Law.

The general principles which have heretofore regulated the law relating to life assurance companies in the United Kingdom have been, to leave the offices absolutely unfettered in the conduct of

their business, while since 1870 requiring them to supply certain returns to the Board of Trade for the information of the public. In fact, FREEDOM and PUBLICITY have been the foundations on which the insurance superstructure in this country has been built. A similar plan has been followed in the majority of British colonies, Canada being the only striking exception. In Canada all offices transacting business are compelled to make their valuations by an official standard, and in other respects fetters have there been imposed, which in the United Kingdom are unknown. It is true that also in New Zealand there is a standard of solvency, and probably also in Victoria if the law there were strictly construed; but in these two colonies the law in this respect has never been enforced, and for practical purposes the companies are not subject to official trammels. In the United States of America, however, the case is very different. There, in theory at any rate, in almost every State of the Union each company must have a valuation made annually by a Government official appointed for that purpose; and if an office does not pass triumphantly through that test, it is compelled to close its doors. In many other directions liberty of action is very much curtailed. Investments may be made only in certain specified securities, and in the case of mortgages, the sum advanced must bear not more than a fixed percentage to the certified value of the property. The companies in some of the States are not permitted, either to decline a proposal on the life of a man of colour or to charge him an extra premium on account of race alone, and the legislators in their wisdom refuse to enter into the question of whether those of African descent have a greater or a less prospect of longevity than the white man. The law prohibits companies from favouring one class of policy-holders above another, and therefore, in theory at any rate, certain transactions in this country, which have recently caused a considerable amount of excitement, would have been impossible in America. The agents of the companies are forbidden, under heavy penalties, and even on pain of imprisonment, to share their commissions with policyholders, and these anti-rebate regulations have gone so far that one great company was shut out from an important State because it proposed to grant policies on a plan similar to that which in England would be called "half premium rates", and it was not permitted to recommence business until the form of contract, which to the official mind seemed so objectionable, was withdrawn.

In connection with insurance legislation it would, therefore,

appear that the most important point for discussion is, whether the system of Freedom with Publicity, or that of Government Control, should prevail. This question seems to me to be entirely one of expediency. I cannot admit that any abstract principle, such as liberty of the subject, should be brought in. In his memorable speech at Leeds, the present Chancellor of the Exchequer (Mr. Goschen) said that the "Government has had a "*locus standi* with regard to shipping, and has said that excessive "cargoes shall not be carried, because they are dangerous to the "safety of the public", and in the same way "the question may "arise whether the public might have the right to say that no "excessive cargo shall be carried by banks receiving deposits— "that business shall be conducted in a manner which shall be "considered safe by the community at large." If the Chancellor of the Exchequer was justified in his contention with regard to banks that the Government may have, under certain circumstances, the right to interfere in the management, it can scarcely be denied that interference would also be justified in the case of life assurance offices. The failure of a great life office is a calamity as terrible to the public as the failure of a bank, and if recklessness on the part of some banks justifies Government supervision, then so does recklessness on the part of some insurance companies. The first great question to my mind, therefore, is: In which way are the general interests of the community best served? Do the public derive most benefit from leaving the companies unfettered? or, for the national welfare, should the State exercise a paternal authority over their proceedings?

Standard of Solvency.—The advocates of Government supervision, and more particularly of a standard of solvency, rely mainly upon what many of them are pleased to consider an axiom, that, under a system of State inspection, an insurance office cannot come to grief; while, in the absence of official actuarial guidance, numerous disasters are sure to occur. They think that, such being the case, Government supervision has the effect of immeasurably strengthening public confidence in life companies; and that it is due to this fact that American offices have made progress alleged to be immensely beyond that of our British institutions. It is better to give the arguments of this school in their own words. In a recent issue of a great provincial daily newspaper there was an article which ran as follows:

"It has surprised persons who have watched the progress of assurance companies in this country and in America, to observe how

rapid and great has been the progress in England of American companies, and the extent they appear to find favour with intelligent and well-informed men. Naturally they are prejudiced against offices which have their management centred 3,000 miles away, and conducted mainly by persons who are comparatively unknown in this country. Moreover, the English Government places America at a disadvantage, by not allowing the premiums paid to American companies to be deducted from the income-tax assessment. Yet, notwithstanding the obstacles, we find the American companies making great and rapid progress. * * * * The main cause of the progress of the American companies in Great Britain is, the security given to the policyholders by the elaborate and constant official supervision of the accounts of those companies by the various American States. The Board of Trade returns in England give but a very imperfect idea of the position of the various offices, and it depends for its figures on the truthfulness or otherwise of the officials of these offices themselves. Policyholders are kept as much in the dark as possible, and the Board of Trade does not officially verify the statements of the affairs of the various companies. In America, on the other hand, the officers of all companies have to report annually, under oath, to the insurance department. On the various dates they are required to publish a list of the policies issued by them, a list of death claims paid, and a record of all the investments of the companies. These investments, moreover, are required by law to be made only in first-class securities officially named by the several States. Thus, that which is a great source of weakness in English companies is, so far as official scrutiny can make them so, an unassailable reserve fund. Many English companies are locked up in securities of very doubtful and precarious value, an enormous amount being still held on land in Ireland. Surely, it would be an advantage in the long run if the more substantial British offices were to endeavour to obtain from legislation some sort of official audit and supervision, and guarantee of publicity, which prevail in America. Until this be done, British offices will always be at a disadvantage in point of guaranteed stability. In a recent issue of White's *Insurance Register*, a list is given of 178 defunct insurance offices, and many have since been added. Not one of these offices has succumbed to excessive mortality on the part of their policyholders, but have been broken down by bad management, rash speculation, misappropriation of funds, and investments in rotten securities."

Again, in a London financial paper there was a paragraph last May as follows:

"In America the valuations are yearly, and are made by Government officials, and not by the companies themselves, and, as a consequence, they are believed in almost implicitly. One system of investigation is at work throughout, not a dozen or more systems which bring out liabilities ranging anywhere between the ratio of 60 and 100. Moreover, assurants have the consolation of feeling that assets set out actually exist, and may not be largely made up of worthless mortgages or bad debts. Under the American assurance

flag a weak office is speedily exposed and driven into liquidation, its business being gobbled up by the half-dozen or so stable *colossi.* English offices, we feel confident, would do infinitely better were they subjected to somewhat similar control, and those who oppose State supervision are no true friends of English life assurance. Given equal conditions, several of the home societies would make headway almost as fast as their American rivals. What is needed is confidence, but this feeling will never dominate the country to any wide-spread extent so long as a dozen or twenty offices rest under suspicion."

After criticizing several companies the writer goes on to say:

"People continue paying premiums, but the contemptible bonuses declared sicken both them and their friends, good offices losing business that would otherwise come to them naturally."

It would be easy to multiply quotations of a like character, but the foregoing are sufficient to show what is mainly in the mind of those in this country who cry out for Government control, and I think they include every argument of importance that has been put forward in support of the American system. The newspaper writers are often very far wrong in their facts. For instance, the writer in the provincial paper above mentioned possesses very erroneous notions as to the real nature of the American system, and the author of the other paragraph draws entirely on his imagination when he speaks of a dozen or twenty English offices being under suspicion, because at the present moment there is scarcely one, of any great importance, which experts would hold to be in a doubtful position. A scribe penning a sensational paragraph must, however, be allowed some latitude, and as his views on Government supervision are held by many who are not addicted to exaggeration of diction, they deserve respectful consideration.

In America, where Government supervision has been so complete, there are but few strong advocates of the system. The majority of the foremost insurance men distrust it, while some of them denounce it as if it were the essence of evil. Wishing to acquaint myself with the views of American experts on the subject, I ventured to write to the presidents of three or four of the principal companies; and I have to thank them for their courteous and exhaustive replies. They enable me to quote opinions which are specially valuable because they are based upon experience contrasting so completely with our own.

The Connecticut Mutual Life Insurance Company is one of the most cautiously and wisely managed institutions on the other side of the Atlantic. Its president, Colonel Greene, appears to

differ from other American managers, and to favour Government interference; and in this connection it will be interesting to quote his reasons. He writes:

"The thing which caused the failure of so many American companies, and which to-day prevents the establishment of new ones, organized upon the proper basis, was not Government supervision, nor a fixed standard of solvency: but it was, and is, the expense account. The companies which failed had to buy business at such a price that it never could have sustained itself. Those companies offered plans of no peculiar value. They had no record of results by which to win confidence, and their expenses were such that they could produce no results to their policyholders which would permanently attach them to the new corporations. The actual receipt, or the firm expectation, of dividends is a great cohesive power in the companies in this country. These companies which failed paid so much for their business at the outset, that they destroyed their power of paying any present dividends, or, if they paid them, they did so at the expense of the future, and covered it by book-keeping; and the conditions under which they started, and by which they did their business, were such as to virtually compel a great rate of expense in the future. Reform in respect of the expense account was practically out of their power: for, as will readily be inferred, they had a very slight hold upon the confidence or the attachment of their policyholders, and incurred very heavy lapses, compelling the constant replenishment of business, which was got at more than its proper cost, by new business got on the same unsatisfactory terms. Of course, under such conditions it required only a general financial and industrial stagnation and depression to carry such companies under. Their experience has been a warning which has so far proved salutary.

"So far in the history of life assurance in this country, competition among the level-premium companies has had one characteristic effect, —to increase expenses. There has been a constant tendency, almost without check from any direction. A few companies have set their faces to stem the tide, but the great majority have considered that it was perfectly legitimate to rival each other, by commissions and other expenditure for business. That tendency is in full force to-day, and was never stronger, and there is no question that, in many instances, the expense account has been so increased that the adoption of some of the Tontine methods of treating surplus has become a very substantial safeguard to the solvency of the companies. Strip them of that feature to-day, and their results to their policyholders would be very unattractive. It is from these companies and their friends that the outcry to-day very largely, though not entirely, comes against Government supervision, and a fixed legal standard of solvency. In the light of history, and of the present situation, the inference is almost irresistible that the removal of Government supervision and of a fixed legal standard, or any reduction of that standard, would be at once followed by a new spurt in competition and a further increase in the expense account up to the limit of the new standard, or, if the standard were abolished altogether, up to the limits which make a sober man tremble to think of; and there is no

doubt, in the minds of many at least, that such a condition of things would be used by those companies which are now competing for business at such enormous expense, and partially concealing and postponing the fact through their postponed dividend schemes, to prevent not only the formation and successful competition of new companies, but to crush out those which have still held to conservative standards with some success, by methods which would accomplish their purposes before the public could be educated to understand them. Government supervision and a fixed standard are the walls by which the expense account in these companies is now finding itself likely to be somewhat circumscribed. Set the wall back as far as you please, it will be reached again and again, until our public has so learnt its lessons as to compel a very different policy on the part of company managers. Indeed, it seems to me that the principal charge that can be made to-day against Government supervision is, that it actually takes the place of a real intelligent public opinion in regard to the proper management of life insurance companies. Were the thing to be done all over, it might seriously be questioned whether it would not be better for the system to develop more slowly, and for the public to learn somewhat painfully its lesson. But that is unprofitable questioning, at least for us. We began our experiment long ago. People have put faith in supervision, and ceased to watch for themselves, and that faith has, in great part, contributed to the rapid development of life assurance among us, and to the peculiar character of that development. It may have been worth while to have this development at the price of Government supervision, but however that may be we have it with us, and we have an enormous business built up on the faith of it, and any modification of supervision which shall make it to-day less restrictive would, in my humble judgment, operate, not to the introduction of new competitors, which are entirely unnecessary, but to the further exploitation of the expense account by those who have already pushed it so far."

Having now put forward the arguments of the advocates of Government supervision, it may be well to examine them in the light of experience in the United Kingdom and in America.

First, as to companies which come to grief: does Government supervision render impossible, or even improbable, the failure of life offices? It is not easy to obtain detailed information on this question, as American papers are very reticent upon it, but other unimpeachable evidence may be adduced. Mr. McCurdy, President of the Mutual Life Insurance Company of New York, an office which proudly claims to be the largest in the world, says, speaking of insurance legislation in the State of New York: "It is "an unquestionable fact that not less than half a dozen companies "in this State alone, which, but for the interference of the State, "were destined to a long career of usefulness, were destroyed "by it." This, it will be observed, does not take account of failures among really insolvent offices. Col. Greene, in the

extract from his letter given above, speaks of the failure of "many American companies", but he does not mention their number. Mr. David Parks Fackler, President of the Actuarial Society of America, a body corresponding to our Institute of Actuaries, says: "The legislation, which so needlessly wrecked *so many companies* "during the last score years, was in large degree the product of "ignorance." Mr. Richard Teece, Secretary of the Australian Mutual Provident Society, stated, no doubt after careful enquiry, that since the "introduction of Government supervision into the "United States, more than 100 life offices, and nearly 400 fire "and marine offices, have disappeared in the bottomless pit of "bankruptcy."

Many of the American offices which failed were, at the time of their collapse, small and of recent origin: but that large and apparently flourishing American offices may also come to grief was proved in the case of the *Continental* of New York 15 years ago. At the time one of the leading insurance papers of America wrote as follows:

"We begin to fear that supervision of insurance companies by the State is a failure. There is the Continental Life Assurance Company of this city, whose doors are now closed, and the bowels of whose once earnest compassion have ceased to yearn for the widows and orphans of its more than 24,000 of swindled policyholders. This company was organized in 1866. The New York Insurance Department was its sponsor, and from that time forth until now, the New York Insurance Department has annually certified to its solvency and good financial health. Meantime it has pushed its way into some twenty other States, having each its insurance department, each of which departments has also borne annual witness to the soundness of the company's claim upon public confidence."

It appears that the company was certified by the various insurance commissioners to have had on 31 December 1875 a sum of about £145,000 surplus, over and above all its liabilities. But this surplus and a great deal more had vanished before the end of 1876, through the disappearance of margins, and more, in mortgage transactions, the heavy fall in the value of real estate owned by the company, agents' bad debts, &c.

Compare this with the results of the British system. We must admit that many British companies have, in times past, failed, and that there have been a few terrible disasters; but it is not fair to charge these calamities against the *present* British system. It is only 21 years since the Act of 1870 was passed. Previous to that date the companies had liberty, but there was no

publicity; and it is only liberty combined with publicity that the advocates of the British system defend. The nature of life assurance is such that it generally takes many years for a company, even badly managed, actually to stop payment; and the 21 years which have elapsed since the passing of Cave's great Act, have not been more than sufficient to eliminate the mischief which existed previously. Although several companies have failed within the past 21 years, I believe that there was only one of them, the *Sovereign*, which, on the publication of the first valuation return, was not at once known to be practically insolvent, and the comparatively few failures which have taken place since 1870, have been those of companies in which the seeds of dissolution had been planted, and had germinated, long before. It would therefore seem that under the British plan of freedom, coupled with publicity, so far from the rate of mortality of offices being high, it is much below that which prevails in America.*

Secondly: what are the respective effects of the rival systems on companies which, though weak, are not actually insolvent? Curiously enough, the writer in the financial paper already quoted, after declaring that under Government supervision policyholders are absolutely protected, stultifies his argument by claiming it as a great advantage of the American system that weak offices are thereby "driven into liquidation." This unlucky admission is the truth, to which testimony is borne by the other authorities I have quoted.

Compare this with the effect of the British system. If companies be weak, the result of publicity, and of the free comments arising therefrom in the insurance press, and extending subsequently to the general press, is to induce them to arrange, before it is too late, for an honourable transfer of their business. Of late years this has happened in several notable instances. The transferring company has taken the step in time, and has been able to secure something more than 20*s*. in the £ for its participating policyholders, and the full face value of those policies issued non-profit. Surely this is much better than to drive such companies over the precipice. For an insolvent company to fail is a disaster, but to force a really solvent though weak one into bankruptcy

* Of the 178 companies mentioned by the provincial paper, only 27 ceased to exist after the passing of the Act of 1870. Of these, 13 were almost immediately closed as a result of the passing of the Act: 6 honourably transferred their business: 3 were reconstructed, or transferred, with reduced contracts under the authority of the Court of Chancery: and 5 simply disappeared. These 5 had an aggregate premium income of only £6,650.—G. K.

produces, perhaps, a greater disaster still. This is not perceived by the unthinking writers who call out for the immediate extinction of offices that meet with their disapproval. They do not realize the mischief that would ensue were their advice adopted; and it is well that the British public is too imperturbable to be seriously troubled by the outcry of alarmists, and allows sufficient time for prudent counsels to prevail.

Thirdly: what is the influence of the two systems on the successful companies? The history of insurance in America shows that Government supervision does not lead to a real increase of strength. It may beget confidence on the part of the public, but the question naturally arises whether, in face of the facts cited, that confidence is entirely justified. I venture humbly to suggest that Col. Greene's defence of, or perhaps it is fairer to say apology for, State supervision, is about its strongest condemnation. He shows that the tendency is for all companies to be content with barely more than the mere official standard of solvency, and to refrain from setting aside reserve funds as further security against contingencies; and he considers, judging from careful observation and experience, that if the standard were lowered the companies would be content to lower their reserves. In fact, when a Government standard of solvency is imposed, on the one hand the sense of responsibility is lost by the officers, and, on the other, public opinion adopts that standard, and asks for nothing better. The tendency, therefore, of Government interference is, to place all companies in the unsatisfactory position of having but a narrow margin to come and go upon.

The great majority of British companies at the time of the passing of the Act of 1870 were shown to be strong. What has been the effect of the Act upon them? Almost without exception, they have gradually increased the proportion of their assets to their liabilities, the grand total of the addition to the reserves, entirely irrespective of the natural increase of business, and of the higher average age of the lives assured, being enormous; and the result is that in no country of the world are the life offices so strong and so stable as in Great Britain, and nowhere do the assured enjoy greater advantages in respect of bonus and other benefits. Notwithstanding the higher rate of interest which prevails in America, the rate of bonus of the majority of British companies compares very favourably with that which obtains across the Atlantic, and it is only the Tontine system, so much in vogue in America, that conceals this fact. This most satisfactory condition

of affairs here is due in great part, I think, to the freedom and publicity which reign, because freedom and publicity result directly in the conservative policy of increasing the funds, and indirectly therefrom in enhanced profits to the assured.

It will be seen that, even as regards the stability of the companies, I give, after a long and careful consideration of the subject, my vote emphatically in favour of the British system. But that system has other great advantages over the American. The public does not lose, in the words of Col. Greene, "intelligent opinion in regard to the proper management of life assurance companies", but, on the contrary, a great educational process is in operation, and life insurance is much better understood than it was a generation ago. This educational process tells, not only in relation to the reserves of the offices, but to their general method of conducting business. In America, because of Government supervision, and the blind faith thereby fostered, little attention, beyond frequent glorification of stupendous figures, is paid to the proceedings of the offices, and it is almost unheard-of to find a comment in the press on extravagance of management. The idea seems to be that, because Government is responsible, whatever is is right; and it is only when extravagance has so advanced as to threaten real danger, that any effort seems to be made to stop it. In England, on the contrary, both in the insurance and the more general press, there are constant remarks upon undue expenditure, with the result that, notwithstanding American competition and example, a much more healthy tone prevails. There is room for very great improvement, but that improvement will come, not from Government control, which has never yet stopped undue expenditure, but from healthy publicity and a wisely-formed public opinion.

Mr. John A. McCall, at present Comptroller of the Equitable Society of the United States, and formerly Superintendent of Insurance for the State of New York, has, from his extensive and varied experience, an authority in these matters which few possess. He favoured me with a most interesting and valuable reply to my enquiries, from which, touching on the other benefits to be derived from freedom, I cannot do better than make a few extracts. He says:

"We believe that the powers vested in the Board of Trade, as a responsible body, to receive reports drawn up in accordance with the forms prescribed in the Act, and to report the same to Parliament, without endowing them with any executive power to enforce penalties,

or to otherwise control the management of companies, are wisely granted. This course is apt to encourage, by means of healthy competition, the adoption of liberal and well-considered measures beneficial to the assured; and we are assured that benefits voluntarily granted by the companies to the assured, from the field of experience, are more likely to further the interests of the latter than those enforced under unintelligent legislative enactments."

Again he says:

"Since public confidence forms a large portion of the capital upon which we trade, the business is bound to thrive in the light of a disinterested, uniform, and compulsory system of making returns to the Government. Though we possess a fixed standard of valuation in the United States, we would hesitate to recommend this plan to Great Britain, as we think the circumstances differ entirely in the two countries. This American experience standard was adopted in 1868, when the business was comparatively limited, and the bases of computing the premiums did not vary greatly. In Great Britain the conditions are quite different, and we think each company should be free to prepare its own valuation according to the table and the rate of interest it may deem best suited to its business, there and elsewhere. In fairness to all, however, we think that the elements entering into the valuation should be clearly and briefly described. If there be any lurking weakness therein on the part of any company, competition will direct attention to the same, and remedial measures must follow, or its ability to do further mischief would soon be contracted through lack of confidence."

I have reproduced the above paragraphs because they embody my own views in better words than I could select. Unfettered competition in Great Britain has resulted in many improvements being introduced, to the great benefit of the community. Some companies charge high rates, which are reduced to a low figure after a certain number of years. This plan suits those who, from their circumstances, have an income likely to grow less rapidly than their expenditure. The rates of other companies are from the beginning moderate, so that a large policy can be effected at a low present outlay. The actual return to the policyholders will be in the end about the same as in the first quoted case, but it is differently distributed, and, in a way to persons differently circumstanced, much more convenient. A man with a growing income, and desiring a large present protection, would not be suited by the first-described office, but the second exactly meets his requirements. Between these two extremes there are many gradations; and, among British offices, by making a careful selection, everyone may find insurance wares precisely suited to his needs. But companies of such various character cannot properly be valued on the same basis, and to enforce a uniform system on them

all would be most injurious. According to the nature of the business, and (although on this point some may differ from me in opinion) according to the system of division of profits, so the valuation must be made. All the circumstances of the office must be carefully considered in determining upon the basis to be adopted; and for the Government to insist on one uniform method, entirely irrespective of the nature of the insurance contracts, expressed or implied, to be valued, would be a proceeding in the last degree mischievous. From immemorial custom, and from the promises made in prospectuses, although not embodied in the policies, the assured in the various offices legitimately expect bonuses of particular kinds, and of amounts tolerably well understood; and although these benefits are not part of the written contract, yet no wise actuary would ignore them in making his valuation. He must, in one way or another, provide for their continuance, or the policyholders of his office will, in the long run, be dissatisfied, and the company will suffer in consequence. But this consideration is entirely ignored when the Government insists on a uniform system of valuation, and the official standard substitutes dull uniformity for healthy diversity. We have seen that it does not tend to secure stability, and now we may say that it does not possess any other advantages. In the words of Mr. E. J. James, an American writer in the *Cyclopædia of Political Science*, "The tendency of State supervision is to "interfere injuriously with honest and well-conducted companies, "and to afford but a feeble protection against those of a different "class; to involve the State in the odium of evils which it is "supposed to be its duty to prevent; to lessen the sense of "responsibility among those who control the offices, and the "spirit of prudence and watchfulness among the public; and to "place in the hands of public officials a power and influence "which are apt to be abused, and which are always open to "suspicion."

The plausible suggestion has been made that, while there should be no standard of solvency, each company should be required to make a test net-premium valuation by, say, the Institute Healthy Males Table at $3\frac{1}{2}$ per-cent interest, the results to be placed along-side of those by its own valuation, made on such basis as the directors might select. It is urged that by this plan, while no restraint would be placed upon freedom, the relative positions and strengths of the companies would at once become apparent, and the legitimate pressure of public opinion would be

brought to bear on the weaker companies to induce them to increase their reserves. Possibly some benefits might accrue were the plan practicable; but even so, the evils would out-weigh them. The statutory test valuation would come to be in reality, though not in name, a Government standard, with the mischievous results already discussed. But the plan is not practicable. There are several companies in Great Britain to which a net-premium valuation cannot be applied at all. How, for instance, would those companies be treated which make annual valuations, and divide the surpluses as an annual percentage reduction of premiums, varying in different series of policies? In the net-premium valuation the future reductions, which are not guaranteed, would either have to be brought in as a liability, in which case there would be a huge and fictitious deficit; or they would have to be left out of account, in which case there would be a gigantically-exaggerated surplus. In neither case would the real position of the company be even approximately shown.

If the arguments of the advocates of Government supervision be analyzed a little further, they will be found to involve much more than a mere Government standard of solvency. I must apologize for calling attention once more to the extract given above from the article in the financial print. The writer is not content to demand that the Government shall see that no company shall become actually bankrupt, but he wants the State to interfere when offices pay "contemptible bonuses." This illustrates forcibly a serious danger of State control, namely, that it has no limit. We cannot realize that evil here, but in America it is rampant. In the Legislature of each State of the Union, every session numerous insurance bills are introduced, and the companies have, at great expense, to be constantly on the watch to prevent serious mischief. It is true that but few of these bills owe their origin to the insurance departments, and only an insignificant percentage are placed on the statute book, but they are none the less a nuisance and a danger. This great legislative activity is due entirely to the idea which has gained complete hold of the American mind, that insurance companies require supervision almost as if their officers were ticket-of-leave men. Were the system of interference not already in vogue, such ideas would not enter into the minds of the legislators. It may be argued that there is no need to go to such lengths, and that a moderate interference is beneficial, but, whatever may be the case as regards the use of alcohol, there seems to be no possibility of such a thing as

moderation in restrictive insurance legislation. There must either be total abstinence or intemperance.

A very serious consideration which should weigh with Government and with Parliament is, that if the State imposes a standard of solvency, and appoints officials to see that it is properly applied, then it can scarcely draw the line at examining the offices. Persons who had insured on the strength of a Government officer's certificate, in an office that afterwards became insolvent, would have grave cause to grumble; and in justice they would have a claim for compensation from the State. Government supervision, therefore, logically implies Government guarantee. Some may think this is a far-fetched argument, but I humbly submit that it is a reasonable one, and that if the system of State supervision were once to come into vogue, State guarantee would presently advance within the range of practical politics.

Government Control and Investments.—A good deal of place has been given above to the question of a Government standard of solvency, and general arguments have been adduced telling against Government control altogether. A few words must, however, be said on the question of a Government supervision of investments. That has been advocated in some quarters, from two points of view. It has been said that Government should lay down strict rules as to the classes of investments which life offices may hold, and, further, there have been those who have demanded that Government officers should be appointed to watch that these rules are adhered to. I would submit, however, that any limitation by the State of the investment powers would be mischievous. No matter how strictly the list of permitted securities might be drawn, it would always be easy to make investments coming within its limits which would ruin any office. Some directors and managers might, not unnaturally, lose the anxious feeling of responsibility which now almost oppresses them. So long as the limits laid down by law were not overstepped, they might think themselves free from blame, and fail to exercise that minute scrutiny of each investment presented to them which is absolutely necessary for safety and profit. It may be urged with some plausibility, however, that Parliament and Chancery Judges have very strictly bound trustees, and laid down minute regulations which can be transgressed only at the risk of cruel penalties, and, if trustees are so limited, why not the executive of life offices? The cases are not parallel. Trustees receive no

remuneration for their labours, and cannot be expected to give them much time or thought. They must therefore invest in those securities which are the least subject to fluctuations, and are not likely to require careful watching. The directors and officials of life offices, on the other hand, are reputed to receive handsome salaries in consideration of the time and attention they give to their business, and in exchange they may well be called upon to take trouble and assume responsibility. By so doing they can safely secure a much higher rate of interest than can trustees, and out of the margin so gained their salaries are paid and the assured reap substantial bonuses. Again, a list of investments drawn up by Government would unavoidably exclude many securities which are most advantageous to insurance companies. A life office, so long as the capital and interest are ultimately safe, does not require any immediate income, but is all the better off if not only principal but interest also remain invested for lengthened periods. It can, moreover, average investments, the funds being large; and, although by thus exhibiting enterprise it may now and then make a small loss, yet on the average there will be a gain which will much more than counterbalance it. Lastly, it can make use of its funds to aid the life business by investing them in securities such as reversions, &c., which trustees could not touch. In these ways the life companies can safely obtain a much higher rate of interest than that ruling for the best trust securities, and, at the same time, extend their field of operations. Yet Government regulations would exclude all this, and bind the offices to a very limited class of investments. In my own humble judgment I think that in the deeds of settlement the directors should have unrestricted powers, and much more, therefore, is it my opinion that there should be no rules laid down under this head by Government.

State Supervision in Details.—If State supervision is bad as regards the great matters of valuation and investment, it would be quite intolerable as regards smaller things. To have a Government official who has, probably, been educated in red tape, interfering at every turn, would soon paralyse all energy and enterprise. He might be officious and fond of magnifying his office, or prejudiced and narrow-minded, or incompetent, or even corrupt; and it is difficult to say which class of men would exert the worst form of influence. At the best he would be narrow and ignorant as compared with the aggregate of the profession outside the Government bureau, because he, from the nature of the case, would not have had the

education and experience that are gained only by a daily intercourse with the public; and he could not appreciate the benefits of, or necessities for, the changes of plan and the new features which naturally present themselves to the active man of business. The Government official, therefore, even with the best intentions, could not help being more or less an obstructive. It would not be his duty to suggest improvements, and in many cases at any rate, he would object, at least for a time, to those brought to his notice. This is no ideal picture, but is well illustrated by the condition of insurance business in France. There there are ostensibly but few laws relating specially to life assurance, and the companies are sometimes thought to be free. But that is not the case. Before a company of any kind can be established it must get official permission; and the Government officers, being totally ignorant of the nature of life business, will not grant permission without laying down such rules that all progress is impeded. The life companies in France are groaning under this burden. In the matter of valuations, rates of premium, and methods of distribution of surplus, they are far behind the age on account of the official trammels, and they are practically unable to make progress. The recently-formed Society of Actuaries in Paris hopes, however, so to influence the official mind as to gain freedom for the companies from this irksome bondage.

PART II.—SUGGESTED AMENDMENTS OF THE LAW.

Believing, as I do, that legislation in the United Kingdom has so far proceeded on the wisest lines, and that the Act of 1870 has been productive of great good, I do not think that any fundamental changes are required, and I have no sensational proposals to make. It would, however, be affectation to assert that there are no improvements possible. The experience of more than 20 years has revealed defects, and disclosed weak points, and these may now be briefly discussed.

In dealing, on an occasion like the present, with suggested amendments of the law relating to life assurance companies, it is unnecessary to enter into minute details, and to discuss mere verbal alterations of the clauses of the Act, and of the schedules. It is better to lay down principles, and, when a general consensus of opinion has been arrived at as to what changes are really desirable, it will be time enough to consider what exact wording should be adopted.

Prompt Publicity.—Publicity being the main desideratum, it almost goes without saying that promptitude and completeness should be aimed at. As regards promptitude, there is room for great improvement in the United Kingdom. To take an example: The blue book containing the statements deposited with the Board of Trade during the year 1890 was not available to the public until 4 August 1891, and the accounts included in it were, in the great majority of cases, for the year ending 31 December 1889, so that almost all the figures in the volume were more than eighteen months old. The returns of several companies were only up to some date in November 1889, while the accounts of two offices only came down to 31 July 1889, and of one only to 31 May 1889; yet all these accounts were deposited within the statutory period. As a matter of fact, it is quite possible, under present arrangements, that the accounts of offices may be available only after a period exceeding two years and three months from the date to which they refer; and, at such a distance of time, publicity ceases to be of much use. To search even the latest blue book is like exploring some ancient geological formation for fossils.

It may be pleaded by the Board of Trade that the delay to 4 August was caused by the fact that in several instances correspondence had to be entered into with the offices. Looking, however, at the dates of the letters which appear in the blue book, it would seem that the public officials conducted that correspondence with great deliberation, and, moreover, I would, though not without hesitation, suggest that at least part of that correspondence arose from excess of zeal, and a mistaken view on the part of the Board of Trade of their functions. Have they not in several instances criticised, not the mere form in which the accounts are rendered, which of course it is their duty to do, but also the propriety of some of the methods in which business has been conducted? This is not an occasion on which to hazard an opinion as to whether the views of the Board of Trade or those of the directors of the companies concerned were the more sound and reasonable; but surely, under present law, if Government officials do more than merely insist that the accounts shall be in statutory form, they exceed their duties.

Contrast the British with the American methods. Before the end of April 1891 there were in London, from two States of the American Union, complete records of the transactions of all American companies for the year ending 31 December 1890, these

records including not only the revenue accounts and balance sheets, but the results of a complete actuarial valuation of each company. It would thus appear that in America it does not take more than three months for the companies to file their statements, including valuation reports, and for the Government to publish them. In order that we might enjoy a similar advantage in Great Britain, it would be necessary that all companies should make up their accounts to a uniform date, say 31 December in each year, and that valuations, when made, should also be to 31 December. It would give the officials of the companies sufficient time to complete their calculations and file them with the Board of Trade, were they obliged to have them ready by 31 March in each year; and if three months more were given to the Board of Trade to make the necessary scrutiny, and to conduct such correspondence with the companies as might be found unavoidable, and to put the accounts in print, that would, from American experience, be an ample allowance of time. On the first occasion of changing the dates of companies making up their accounts, there might be a little trouble and inconvenience, but that would occur only once, and thereafter everything would go smoothly. It is true that some companies hold their meetings at a considerable interval from the date to which the accounts are made up, but that need not interfere with the statutory statements, and, after all, no great harm would ensue, if the custom became universal, of holding the annual meetings promptly on the closing of the books. If these things can be done in America, why should they be impossible here? The gain to the public would be so great that I venture to hope the Legislature may adopt some such regulations as those above suggested. The only serious objection that presents itself is, that some companies transact foreign business, and that it would not be easy to obtain the necessary returns from distant agencies in time to embody them in the schedules previous to 31 March. That, however, could be overcome by permitting the books for foreign business to be closed three months earlier. This plan is already in force in several companies.

Supplying copies of Returns.—The Act, as it at present stands, seems intended to protect shareholders and existing policyholders; but, except for the deposit of the returns with the Board of Trade, the general public has no statutory access to the accounts. I think that, in the case of at any rate companies seeking new business, sections 11 and 13 of the Act should be modified, so that

anyone, and not only shareholders and policyholders, should be entitled to copies of the deposited statements and of the deeds of settlement of a company, although it would perhaps not be unreasonable to permit the companies, in the case of strangers, to make a small charge for their accounts and statements, as well as for their deeds of settlement. When a man is invited to become a policyholder in an office, either by advertisement or by direct solicitation, he is surely entitled to official information regarding the standing of the company; and it is like locking the stable door when the steed has been stolen, to insist that the man must be a policyholder before he can claim a balance sheet.

List of Governing Body.—A difficulty has arisen in the case of mutual offices, and injustice has sometimes been done, in that policyholders have no access to a list of the governing body of their company, so that a great, and, it will probably be admitted, undue advantage is given to the directors. A dissatisfied policyholder, however good the grounds may be of his dissatisfaction, has to trust entirely to the generosity of the board, with which perhaps he is at war, in order to communicate with his fellow members; and he is thus, even if the board be following mischievous courses, at a very great disadvantage in opposing them. It would be unreasonable, perhaps, to suggest that a list of the policyholders entitled to vote at meetings of the company should be open to inspection, because in these days of keen competition unfair advantage might be taken by representatives of rival offices; but it would be only right, I think, that the companies should be under obligation, at the expense of course of the applicant, to post to every member of the governing body any communication which might be desired by one of that body. In this way a member would be able to communicate with his fellows; and while the board would of course retain that possession which is nine points of the law, it would not possess the unfair advantage of being practically able to silence malcontents.

Separation of Funds.—Section 4 of the Act, under which the life funds of a company must be kept in an entirely separate account, is a very important one, and the principles it involves might with great advantage be extended. In every British company there is a participating section of policyholders. In some, participating policyholders share in the profits that may be derived from non-profit business, or from annuities; while in others, participating policyholders share in the surpluses of their own section only, the profits on the non-participating business

and on the annuities belonging entirely to the shareholders. Now, for many reasons, some of them cogent, it seems to me desirable that separate statements should be made of the business in which policyholders participate, and of that in which shareholders alone are concerned; and not only do I think that separate revenue accounts should be given, but I would go the length of saying that it is very important that there should be separate balance sheets, and separate investments. This need not in the slightest degree interfere with the arrangements that the directors may make for investing the money, because the policyholders and the shareholders respectively might be, as it were, contributory mortgagees, or tenants in common, of the securities; and it would be necessary only to earmark certain securities to each section, or, where one security belongs to both, to say how much of it is the property of the one, and how much of the other. Were this course adopted, much light would be thrown on transactions which are now often obscure. In some companies, from the shareholders' and policyholders' affairs being inextricably mixed in the accounts, it is impossible to say how much the shareholders really draw from the policyholders. No restriction should be laid down as to what proportion of the profits shareholders should take, but it should be clearly shown in the accounts how much they actually do take. For instance, at present with some offices the shareholders receive out of the funds, interest on their capital in excess of that which the capital actually earns, but there is nothing to show how much the excess is, and how much, therefore, the policyholders contribute. With separate revenue accounts and balance sheets, however, an entry would have to appear in the policyholders' statement, showing how much of their interest was transferred to make up the interest on the capital of the shareholders, and in this way everyone would see what was being done. Then, again, it has not been unknown, in the history of amalgamations and transfers of business, for very large transactions to be carried out practically in total darkness. Hints have been thrown out in the press that sometimes the policyholders of the purchasing company have thereby seriously suffered, but that is not the matter now in question. I do not ask that restrictions should be placed upon amalgamations, or that improvident purchases should be prohibited, but only that the true terms should appear in the accounts: and that end cannot be attained under the present system of one balance sheet and one set of investments. Again, investments suitable, for

instance, for fire and life business respectively, differ very materially in their character. In the case of fire business, where there is a liability to sudden heavy demands, easy convertibility of a great part of the funds is essential. In a life company, on the contrary, it is impossible that any unforeseen call of large amount can suddenly arise, and therefore the company can with safety lock up its funds to a much larger extent than in almost any other class of business, and can thereby secure a comparatively high rate of interest. Such being the case, when there is only one balance sheet and one set of investments for different classes of business, it becomes a difficult question to apportion the interest earned, and I do not know of any case in the whole blue book where it is explained on what principles the apportionment is made. The obscurity thus arising would be obviated by the separation of investments.

Under the present Act it is optional whether companies shall keep a separate annuity fund or not. The nature of annuity business differs so much from that of assurance business, that a separate annuity fund might well be made obligatory, although it does not appear to be necessary to have a separate balance sheet or even a separate revenue account. Careful investigation, by means of the published returns, of the annuity business of many companies shows that it has often been carried on at a loss. This, however, is not superficially apparent in the annual accounts, or even at the periodical actuarial investigations; and it is quite possible that the directors and officials themselves of some of the companies are scarcely aware of the fact. Companies should not be prohibited from carrying on annuity business, even at a loss, but it is certainly desirable that the actual results should be known.

In some companies different sections of policyholders are, in so far as divisions of profits are concerned, treated as if constituting separate companies. For instance, there are offices which have temperance and general sections, and there are others which keep entirely separate such classes as those assured for the whole of life and those who hold endowment assurances. It would certainly be convenient if in all cases the funds of these separate sections were treated separately, and surely the policyholders, who are partners in the business, are entitled to the information which would thereby be afforded. Of course there need not be separate balance sheets and separate securities, but merely revenue accounts showing the operation of the different funds. No doubt

it would be voluminous for the company to print so many separate accounts, but the difficulty could be obviated by a system of parallel columns. This is already done with great convenience by some offices, and might be done by all which are in the same category.

Lastly, under this head, it seems not unreasonable that industrial business, which in its nature differs so much from ordinary business, should be kept quite distinct. The more important of the industrial companies have already carried out this arrangement on their own initiative, and the rule should be extended to all.

Revenue Account.—Touching minor details of the Revenue Account, it would be very desirable that the single premiums and commutations of premium should be shown separately from the annual premium income; that the amount of income tax should be shown, either as an item on the expenditure side of the account, or in an inner column on the income side by deduction from the interest; that the claims under life policies arising from endowments matured should be shown separately from the death claims; that the reductions in premium by bonus should appear on the expenditure side, the full contract premiums being entered on the receipt side; and that the surrender of policies should be given separately from the surrender of bonuses. These, and various other points of more or less importance, were very fully discussed by Mr. Sprague in his valuable work on Life Assurance Accounts, published in 1874, and I need not refer to them further here. In order to put my suggestions into concrete shape, I here append a draft of the three revenue accounts which I would suggest to take the place of the first schedule or the third schedule respectively, as given in the present Act. In the case of a mutual life company only the first of these revenue accounts would be necessary; but in the case of a proprietary office, where the participating policyholders shared in the profits of the whole of the life branch, accounts 1 and 3 would be necessary; while, in the case of a company in which the shareholders took the entire profits of any section of the life business, all the three revenue accounts would be called for. In addition to the revenue accounts now suggested, there could be no objection to the option, which the present Act gives, of the company also preparing a profit and loss account.

No. 1.—*Life Revenue Account (Policyholders' Branch) of the for the Year ending* 31 *December* .

	£ s. d.		£ s. d.
Amount of Funds at the beginning of the Year, namely:		Death Claims	
Life Assurance Fund . . £		Endowments Matured	
Annuity Fund . . .		Annuities	
Other Funds (to be specified) .		Surrenders of Life Policies	
Total		Surrenders of Bonuses (including Bonuses paid in Cash)	
Single Premiums and Commutations of Premiums under Life Policies		Surrenders of Annuities	
Periodical Premiums under Life Policies . . .		Bonuses in reduction of Premiums	
Single Premiums and Commutations of Premiums for Annuities		Commission in respect of Life Policies . . .	
Periodical Premiums for Annuities		Commission in respect of Annuities	
Interest, Dividends, and Rents arising from the Funds in this Account . £		Expenses in respect of Life Policies	
Less Income Tax . .		Expenses in respect of Annuities	
Other Receipts (to be specified)		Dividends and Bonuses to Shareholders . . .	
		Other Payments (to be specified)	
		Amount of Funds at the end of the Year, namely:	
		Life Assurance Fund . . £	
		Annuity Fund . . .	
		Other Funds (to be specified) .	
		Total	
£		£	

Note 1.—This account is to include all life assurance and annuity transactions in respect of which any of the policyholders share in the profits. But see Note 2.

Note 2.—The annuities may be returned in a separate account.

Note 3.—The various items in this account are to be entered net of re-assurances.

No. 2.—*Life Revenue Account (Shareholders' Branch) of the for the Year ending* 31 *December* .

	£ s. d.		£ s. d.
Amount of Funds at the beginning of the Year, namely:		Death Claims	
Life Assurance Fund . . £		Endowments Matured	
Annuity Fund . . .		Annuities	
Other Funds (to be specified) .		Surrenders of Life Policies	
Total		Surrenders of Annuities	
Single Premiums and Commutations of Premiums under Life Policies		Commission in respect of Life Policies . . .	
Periodical Premiums under Life Policies		Commission in respect of Annuities	
Single Premiums and Commutations of Premiums for Annuities		Expenses in respect of Life Policies	
Periodical Premiums for Annuities		Expenses in respect of Annuities	
Interest, Dividends, and Rents arising from Funds in this Account . . £		Dividends and Bonuses to Shareholders . . .	
Less Income Tax . .		Other Payments (to be specified)	
Other Receipts (to be specified)		Amount of Funds at the end of the Year, namely:	
		Life Assurance Fund . . £	
		Annuity Fund . . .	
		Other Funds (to be specified) .	
		Total	
£		£	

Note 1.—This account to include only those life assurance and annuity transactions in respect of which the shareholders take all the profits. But see Note 2. This account is not required of those companies in which policyholders share in the profits of all life assurance and annuity transactions.

Note 2.—The annuities may be returned in a separate account.

Note 3.—The various items in this account are to be entered net of re-assurances.

No. 3.—*General Revenue Account of the* *for the Year ending 31 December* .

	£	s.	d.
Amount of Funds at the beginning of the Year, namely:			
Shareholders' Capital paid up . £			
General Reserve Fund . .			
Fire Fund			
Marine Fund			
Profit and Loss (if any) . .			
Other Funds (if any), to be specified			
Total			
Fire Premiums			
Marine Premiums			
Other Premiums (descriptions to be specified) . .			
Interest, Dividends, and Rents arising from Funds in this Account . . £			
Less Income Tax . .			
Other Receipts (to be specified).			
£			

	£	s.	d.
Fire Claims			
Marine Claims			
Other Claims (to be specified)			
Expenses relating to Fire Business			
Expenses relating to Marine Business . . .			
Expenses relating to other Insurance Business (to be specified)			
Other Expenses			
Commission relating to Fire Business . . .			
Commission relating to Marine Business . . .			
Commission relating to other Insurance Business (to be specified)			
Dividends and Bonuses to Shareholders . . .			
Other Payments (to be specified)			
Amount of Funds at the end of the Year, namely:			
Shareholders' Capital paid up . £			
General Reserve Fund . .			
Fire Fund			
Marine Fund			
Profit and Loss (if any) . .			
Other Funds (if any), to be specified			
Total			
£			

Note 1.—This account is not required of mutual companies transacting only life and annuity business.

Note 2.—The various items in this account are to be entered net of re-assurances.

Balance Sheet.—Passing to the annual balance sheet, it has already been shown above in which case it is desirable to have separate balance sheets for the various funds. A company preparing only the first of the three revenue accounts would require only one balance sheet; whereas a company making up either two of the revenue accounts, or all three, should have two balance sheets, one for the policyholders' branch, and the other for the branch belonging entirely to the shareholders. Although in this latter case the whole of the business carried on for the shareholders need have only one set of investments, yet it would be necessary to retain the provision that the life assurance and annuity funds included in the shareholders' balance sheet should, as at present, be liable only for the life assurance and annuity contracts.

It is unnecessary here to give forms of balance sheet, as they need not differ much from those already supplied by the Act of 1870. A few alterations might be suggested. On the debit side of the account, claims announced but not yet admitted should be included among the liabilities. True, under certain very rare circumstances, such claims will never have to be paid. The announcement of death may have been premature; or the company, from fraud or breach of contract on the part of the assured, may be relieved from liability. Nevertheless, in such very rare instances, the matter could be put right in the next account, and it is in conformity with the best principles of finance that every known and even every surmised liability should be provided for.

On the credit side of the account the assets might certainly be better classified than at present. It is sometimes very difficult to say in which category a particular security should be placed. The item of outstanding interest should be subdivided, and it should be made clear, by placing it in a separate line, that interest accrued but not yet due is to be brought in as an asset. This item has been very differently treated by different companies, but recently the Board of Trade has been insisting that accrued interest shall be included. Moreover, it is of great importance to the public to know how much interest actually due and outstanding is really in arrear. It is customary to give a certain period of grace for the payment of interest, just as for the payment of premiums; and interest which has only just become payable can scarcely be said to be overdue. Thus, interest due at the Christmas quarter, while it may be still *outstanding* on the

31st December, is not *in arrear,* but there is nothing in the present form of accounts to distinguish this from interest long overdue. If, however, the item of outstanding interest were subdivided, so that interest which has been due for, say, three months, and is still unpaid, should appear separately, much useful light would be thrown on the quality of the investments of the companies; and, without any Government interference with their methods of conducting business, a legitimate pressure would be brought to bear in the direction of securing that good investments only should be made, and that these should be well looked after.

A very great defect of the present law is, that while fairly full particulars of the liabilities of the company are demanded, the assets are left in obscurity. In order that full benefit may be derived from publicity, equally complete information should be available regarding the two sides of the balance sheet. Under the American law, companies must file detailed lists of all their securities, and this regulation is worthy of imitation. The British offices should be required to give complete information regarding each of the investments. There should be shown the nominal amount of each stock held by the company, the value at which it stands in the books, and the market value on the day of closing the accounts; and a column might be added showing the annual interest receivable. So also, but within limits, information should be required regarding loans and mortgages. If this information were not asked for each year, it should certainly be required of the offices on the occasions of the actuarial valuations and divisions of profits.

One or two offices have an item in their balance sheets of doubtful character, the expenses which have been incurred in extending the business, or in purchasing connections, being treated as assets. It is, however, a matter of business policy, with which Government should not interfere, whether an asset of this description should be retained; and the only precaution which the Legislature should take is, to see that exactly the true facts of the case are known. To secure this end it is necessary that the rule be rigidly laid down, that all expenditure of every kind, whether incurred for acquiring or extending business, or for any other object, shall appear in the revenue account; although, subsequently, sums may thence be carried into the balance sheet. At present, cases could be mentioned where large sums spent are carried directly as assets into the balance sheet, without appearing

in the revenue account at all: and very minute scrutiny of the accounts for more than one year is required in order to arrive at the truth.

No provision is made by the Act for the audit of the revenue accounts and balance sheets, and this omission should be supplied. I would suggest that it be enacted that chartered accountants be employed to conduct the audit, and a suitable form of certificate might be prepared to which the auditors should conform.

Returns of New Business.—A further very important piece of information, which the present schedules of the Life Assurance Companies' Act do not demand, is the amount of new business transacted by the companies. It has been suggested by some, that information under this head should be secured by means of suitable items in the revenue account, and in some of the Colonies this plan is followed. But difficulties here present themselves. Not only the amount of new business, but, in broad groups, the nature of the new business should be shown; and it is not easy to prepare a revenue account to this end. Moreover, the whole of the first year's premiums on new business transacted in a year is not necessarily payable in that year; and, therefore, either the new sums assured shown would not correspond to the new premiums, some of the new premiums being instalments in respect of business of the previous year, or else the new premium income would be understated. It would be better to have, as in America, a separate annual statement of new business, showing in three classes, namely, ordinary whole life, endowment assurances, and other descriptions, the number of policies issued, the amounts less re-assurances, thereby assured, and the annual premiums, less re-assurances, payable thereunder. Similar information might be asked for in respect of policies discontinued from every cause during the year; and, lastly, there should be shown the amount of business remaining on the books at the close of the year. Information under these heads, either in the form now sketched out, or in some equivalent form, is essential for a just judgment to be passed upon a company. An office having a proportionately large new business, of necessity incurs a comparatively high rate of expenditure; and although the views of actuaries differ very widely as to the proper method of treating the expenditure for new business, it cannot be denied that to institute a fair comparison between offices, the amount of new business, as well as the total premium income, must be known.

Although not of importance as regards the financial standing

of companies, yet, for statistical purposes, it would be interesting to know the amount of business transacted within the United Kingdom, and the amount outside its borders. The American States for the most part make this requisition, and I do not see why, in the United Kingdom, we should not do likewise. It will be seen lower down that, as regards foreign and colonial companies, there are strong reasons for requiring that the business transacted in the United Kingdom shall be returned separately.

Actuarial Returns—Fifth Schedule.—By section 7 of the Act, an investigation must be made at periodical intervals into the financial condition of every life assurance company "by an actuary", and an abstract of the actuary's report is to be made, in the form prescribed in the Fifth Schedule to the Act. Carrying out that clause, the descriptive heading to the Fifth Schedule runs: "Statement respecting the valuation of the liabilities under "life policies and annuities of the , to be "made by the Actuary." Notwithstanding that both in the clause itself, and in the heading of the schedule, the word "actuary" appears, there is nothing in the Act defining an actuary, or stating what his qualifications are to be. The actuary, unless he be the principal officer managing the life assurance business, is not required to certify to the valuation, and, in fact, his name need not appear at all. In any amendment to the Act, the term actuary should certainly be defined, and so defined as to secure professional competency; and I would suggest that the definition be, "a Fellow of the Institute of Actuaries, or a Fellow of the Faculty of Actuaries." Now that both these bodies have charters, and are the only chartered actuarial bodies, there could be no valid objection to this definition. Provision having been made to secure that a competent man shall make the valuation, his name should appear, and he should be personally responsible for his work, and it would be a useful provision, and so prevent the real opinion of a competent man being concealed, or an incompetent man being employed merely to carry out instructions from possibly ignorant persons, if the actuary, in addition to signing the schedules, were required, as in the case of friendly societies, to make a separate report, in form to be decided by himself, such report to be filed with the Board of Trade, along with the other statements. It would be an innovation, but a salutary one, if he were required also to certify that not only were the calculations correct, and made on the principles explained in the schedules, but that he approved of these principles. In the

valuation, however, there are two sets of facts to be certified to, namely, the particulars supplied for the valuation, and the results of the valuation itself. If the actuary is not himself responsible for the particulars extracted from the books, then the official who is responsible for that part of the work should certify as to its accuracy. It would, of course, not be fair to impose upon a consulting actuary responsibility for the correctness of particulars supplied to him by other persons.

In respect to heading No. 2 of the schedule, which asks for an explanation of "the principles upon which the valuation and distribution of profits among the policyholders are made", many companies have been in the habit of giving information of a very vague and unsatisfactory character. I do not know if an amendment to the Act to ensure greater explicitness is required, the present wording being probably sufficient, but if so, then the Board of Trade should see that the answers given are full and clear. It very frequently happens, more particularly with regard to the distribution of profits, that really no information is conveyed as to how the surplus has been distributed, and a vague idea can be formed only by referring to the specimen bonuses required by heading No. 9.

In order that the nature of the valuation may be thoroughly understood, more particulars must be supplied than are demanded in the Fifth Schedule. A table of the annuities employed in the valuation, and of the net premiums adopted, should, I think, in all cases be supplied; and it should be explained how both the ages at entry and the ages at valuation are arrived at. Companies differ very much in their methods of calculating these ages, and the differences have very considerable effect upon the amounts reserved. Then, as in the case of the Friendly Societies' Act of 1875, specimen reserves for policies of different kinds and of various durations should be given, especially in respect of recent business, and a full table of policies carrying negative values should be appended, showing under each age attained the number and the amount of such policies, and the premiums payable thereunder, and the amount of negative values included in the valuation. The method of valuing re-assurances should also be explained. I could name one case, of a now defunct company, where many policies must have had large negative values, so that they were treated as assets in the valuation, instead of liabilities; and where the re-assurances corresponding to these very policies were taken again as an asset at the surrender-values of the

re-assuring offices, so that, in these particular cases, and they were of serious amount, the policies were treated as assets twice over. I do not here say that such a course should be forbidden, but I merely assert that if the course be followed, it should be fully explained to the public.

The distribution of the premium income is of very great importance when policies are valued in groups, or by annuities with average adjustments, and I think a statement should in all cases be asked for, showing the premium income of the company for each month of the year. An enquiry should also be included as to the adjustments of the annuities employed in valuing the premiums. An examination of the statements and accounts filed with the Board of Trade speedily convinces one that the most diverse practices obtain in this respect, and that the differences in the reserves of the companies may be very great through a slight difference of procedure. Yet, under the present regulations, it is very difficult to ascertain exactly what the companies have done.

The consolidated revenue account should, of course, be made to correspond to the annual revenue accounts, so that were a change made in these, the consolidated revenue account should be altered too.

The heading No. 7 of the Fifth Schedule asks for a summary and valuation, in a particular form, of the policies of the company. That form should be, to a certain extent, re-cast. It would be well to separate the sums assured from the bonuses. It should be made clear that the office annual premiums are the original contract premiums, and a column should be added, both in the particulars for valuation and in the results of the valuation, giving the permanent reductions of premium by bonus. The whole-term policies should be distinguished into those by uniform annual premiums, and other classes. At present, companies generally place policies by limited payments in a group by themselves, but the Act does not require that to be done, and some of the companies include all the whole-life policies together in one item. The term "extra premium" should be better defined, the footnote to the form not being sufficiently explicit. It is not at present clear whether it includes the permanent addition made in respect of deteriorated health at the date of assuring, or whether that is an extra which is not intended to come within the term. Separate forms should be supplied for that business in which any policyholders share in the profits, and for that in which the shareholders alone are interested. In fact, the form referred to

under heading No. 7 of the Fifth Schedule should, in this respect, correspond to the annual revenue accounts. Moreover, in the note to this form, where it is stated that "if policies are issued in or "for any country at rates of premium deduced from tables other "than the European mortality tables adopted by the company, "separate schedules must be furnished", an alteration of the language is required. The important point is, not the original mortality table on which rates have been calculated, but whether rates of premium other than those used in Europe are employed. For instance, some companies transact colonial business at lower rates than they charge for their business in the United Kingdom, but both sets of rates may be based upon the Institute of Actuaries' Tables, only the loading being different, and, therefore, according to the letter of the Act, although not, I think, according to its spirit, only one summary and valuation is required, and, as a matter of fact, only one is, in such circumstances, usually given.

After heading No. 7, another should be added to include particulars of the investments. This point has already been touched upon in dealing with the annual balance sheet, and it is only necessary to add that full information should be given as to how reversions and other securities of a like contingent character are valued.

Heading No. 9 of the Fifth Schedule requires amendment. The profit made by the company in the quinquennium is not necessarily the amount of surplus shown by the valuation. The basis of valuation may have been altered, and the calculated reserves may have been thereby either increased or diminished, the surplus being correspondingly diminished or increased. I think, therefore, that the company should be asked to show, not only the net liability under the system of valuation now adopted, but also what would have been the net liability had the system of valuation in use on the last preceding occasion been adhered to. In this way very useful information would be furnished as to the position of the company, and as to whether it was strengthening that position or the reverse. In other respects the question, as it stands, is obscure. It asks how much profit has been divided among the policyholders, but it should also require similar information as to the shareholders. This matter may seem very simple, but in reality it is very difficult to find out from the Board of Trade returns exactly how much surplus is divided, and how much the shareholders take.

The object, I should say, of asking for specimen bonuses is

that the insuring public may judge what is likely to be the bonus-giving power of the company in the future, and, if this view be correct, it should be made clear that the specimen bonuses are to be hypothetical, and to represent those resulting from the present valuation only. They should be calculated as if the rate of bonus had been the same previously as on the present occasion, and as if it would always remain the same in future. A company, for instance, operating by a recently-opened new series of policies, should nevertheless give specimen bonuses for policies of long duration, it being explained that these are the bonuses which would have resulted had the fund been in operation for a sufficiently long time, and had the bonus always remained the same. In fact, I conceive that the specimen bonuses required are to represent those which, on the showing of the particular valuation, are likely to be given in the future, rather than that they are to be a mere record of the past. Enquiry should be made as to how many premiums have to be paid in order to secure the bonus. With some companies, for instance, six premiums must be paid, and with others only five, in order to secure the first quinquennial bonus, but this difference is not apparent in the answers to the question as it at present stands, and comparisons between companies are thereby vitiated. A question should also be put as to how the company acts in the matter of interim bonuses. At the present day endowment assurances are becoming a very important class of business, and specimen bonuses for that class should, I think, also be asked for.

Sixth Schedule.—The Sixth Schedule is meant to supply particulars which will enable an outside actuary to check, with some degree of accuracy, the valuations made by the officials of the companies. It is of far more importance that this schedule should be made complete than that a Government actuary, as in America, should be called in to make an official valuation. As a matter of fact, in England the particulars supplied under the Act of 1870 are much more full than those which American offices furnish to the public through their governments: and there is thus a better guarantee for the good management and the solvency of British companies than of those on the other side of the Atlantic. Under the American plan the public must merely take on faith the assertion of the Government that the liability by a particular method of valuation amounts to so much. In England, on the other hand, an independent actuary can, with some confidence, test the figures for himself. In order, however, to carry

out the British system to perfection, the Sixth Schedule requires to be modified, but that would not really entail increased labour on the companies' officials. Headings 2 and 3 might be combined, the answers having been generally given in one table in the blue books. It should be made clear that the "amount assured on lives for the whole term of life" does not include policies by limited payments; and a separate return, to be given in a form as explained below, should be asked for in respect of those policies under which payments have been limited, or the term shortened, by bonus. At present, such policies may all be included in the one statement, and when that is done it is impossible to test the valuation. The premiums receivable annually should be the original contract premiums, and columns should be added giving the bonus reductions and the net premiums. Lastly, separate statements in respect of headings Nos. 2 and 3 should be given for each of the classes of business valued by separate mortality tables, so that, for instance, where the combined H^{M} and $H^{M(5)}$ tables are used, separate statements should be given for that portion of the business valued by the H^{M} annuities, and for that by the $H^{M(5)}$ annuities, respectively.

Headings 4 and 5 might similarly be combined, and should include everything except ordinary whole-life business which from the commencement has been, and which still is, subject to uniform annual premiums. Limited payment policies, for instance, whether originally limited or having become limited by bonus, should come under headings 4 and 5. Heading No. 6, which asks for the total amount of premiums which has been received from the commencement upon all policies under each special class, is perhaps that which produces the most irritation among the companies' officials. The information is very troublesome to extract, and is of little use. I would suggest that that information be no longer required, but that instead, such particulars of the endowment-assurance policies and of policies by limited payments, whether they were originally of these descriptions or have become such by bonus, should be given as will enable an approximate calculation to be made of the respective reserves.

As regards endowment assurances, a return might be supplied, showing in columns the sums assured and the reversionary bonuses, respectively, maturing in each calendar year; and the office premiums, the net premiums, and the bonus reductions of premium corresponding thereto, and in another column the

average present age of the lives assured in each yearly group. The following is the form which might be adopted:

Year of Maturity	Average Present Age of Lives Assured	Sums Assured	Reversionary Bonus	Office Annual Premiums	Bonus Reductions of Premium	Net Annual Premiums
1892	...	..	...	...	...	...
1893	...	..	...	...	...	...
1894	...	...	...	...	...	...
:	:	:	:	:	:	:

The average present age will be arrived at by adding together the ages of all the lives assured under policies maturing in, say, 1892, and dividing by the number of the policies.

By experience I can say that a table prepared on this plan enables a valuation to be checked with very great accuracy, and with very little trouble.

Similarly for limited-payment policies. The sums assured and bonuses should, as in ordinary cases for the whole term of life, be arranged under present ages, but, as regards the premiums, the policies should be grouped as shown above for endowment assurances. In this way the valuation of policies by limited payments could easily be checked.

Other groups of special policies are too unimportant to require attention, and they might be altogether dropped out of heading No. 6.

Heading No. 9, asking for the average rate of interest, has been a source of great perplexity. A rule should be laid down to say exactly how the average rate is to be arrived at, but if the particulars of investments already suggested above were given, this heading would be no longer required.

Two matters of great importance have still to be considered, namely, the admission of companies and the retirement of companies.

Companies included in the Act.—In the interpretation clause of the Act of 1870, the term "company" is so restricted that the Act applies only to those which issue or are liable under policies of assurance upon human life within the United Kingdom, or which grant annuities upon human life within the United Kingdom, and are not registered under the Acts relating to friendly societies. Of late years several companies, which from the magnitude of their operations have become important, have

been organized to provide fixed benefits at the end of a term of years certain, in consideration of periodical payments made in the interim. For the protection of the public, all such business should be brought within the Act, so that offices like the Provident Association of London and the Rock Freehold Land Society should have to make returns. To a large and growing class of the community institutions of this class are quite as important as life assurance offices, and their mismanagement produces quite as much mischief. From this rule genuine registered building societies and savings banks should, of course, be excepted. It is rather anomalous that industrial insurance companies come under the Act, while the large collecting friendly societies which transact life assurance business of precisely the same character are exempt. The Act should be extended to include companies of every kind transacting business of this nature.

Deposit.—By clause 3, every company "established after the passing of this Act within the United Kingdom" must make a deposit. This enactment has been evaded in certain instances by the resuscitation of companies registered before the passing of the Act, and which, for all practical purposes, had become defunct. It has been found that by reviving these, the necessity for a deposit does not exist. I do not know whether it is still possible at this distance of time to escape from the provisions of the Act in this manner, but, if so, the Act should certainly be amended, so that every company commencing to carry on the business of life assurance within the United Kingdom, no matter when it may have been established, shall make the deposit. The deposit is returnable to the company so soon as its life assurance fund, accumulated out of the premiums, shall have amounted to £40,000. It is not clear from this whether the annuity fund is to be included in the life assurance fund; and, seeing that elsewhere in the Act "premiums" are kept distinct from "consideration for annuities granted", it might have been thought that only premiums under life policies are in this connection to be taken into account. In practice, however, the whole accumulated funds of the company, whether arising from life policies or from annuities, are allowed to count towards the £40,000. One curious anomaly has occurred in the case of the transfer of the business of the Scottish Economic office to the Scottish Metropolitan. That transfer was effected before the Scottish Economic had accumulated £40,000. After the transfer was

completed, when the Scottish Metropolitan applied for the deposit to be returned, it was ruled by the Court that that cannot be done until the funds of the joint company shall have increased by £40,000 after the date of the transfer. The funds of the joint company were already far in excess of the stipulated figure, but that was held to be not sufficient. It might thus happen that if a new company, having not yet withdrawn its deposit, were transferred to an old office, with millions of assets, but which, from the age of its business, was not making further accumulations, the deposit might never be returnable at all. Surely this is an anomaly that should be removed.

The deposit of £20,000 must, under the Act, be made by foreign and colonial offices commencing to transact business in the United Kingdom, but if the accumulated funds of these offices already amount to £40,000, the deposit is returned at once, and the proceeding becomes a mere farce. It would be much better to insist that the deposit shall remain in the hands of the Court in the case of all companies, whether home, colonial, or foreign, at any rate until the accumulated funds arising from British business, and retained within the United Kingdom, shall have amounted to £40,000. But I would go even further than this. I think that all companies, whether home, colonial, or foreign, should be required to retain in the names of trustees—British subjects, and domiciled in the United Kingdom—at least an amount equal to the calculated liabilities existing under the business transacted through offices or agencies within the United Kingdom, such assets not to be withdrawn from the United Kingdom as long as any of such business remains in force. There need not be any restrictions as to the investment of these funds, except that the investments must be such as can legally be held by British subjects. Thus, for instance, it would not be permissible for the British trustees to hold land, or mortgages of land, situated in countries where aliens are not allowed to possess real property; but otherwise the powers of investment should be unrestricted. At present there is nothing to prevent a foreign life company closing its British office, and leaving its British policyholders to follow it to its own domicile. By the time these unfortunates had found it, their policies would probably have lapsed, and they would have no remedy, and the whole of their past premiums would be forfeited. So far as I know, this has never happened in the case of a life office, but it has in the case of some foreign fire offices, and the public should be protected.

There would be involved complete returns of business transacted through offices or agencies in the United Kingdom, both as regards the annual statements and the valuations, but so far from this being objectionable, it would be an advantage. In the case of companies not domiciled in the United Kingdom, the trustees in whose names the British investments should stand should be attorneys, to sue and to be sued on behalf of the company. These suggestions may appear to be radical, but they do not really affect the methods of transacting business, or limit the powers of investment. They merely provide protection for British policyholders where such protection is required.

Amalgamations and Transfers.—Probably the sections of the Act bearing upon amalgamations and transfers, and upon the liquidation and re-construction of insolvent companies, are those which have proved most defective, and which stand most in need of amendment. Clause 14 is supposed to regulate amalgamations and transfers, but it is nowhere explained what an amalgamation is, and I believe lawyers are unable to give, with confidence, a definition of the term. Where two companies are to be amalgamated, application has to be made to the Court by petition to sanction the proposed arrangement, and certain documents giving particulars of the transaction have to be sent to each policyholder of both companies. When, however, the business of one company is to be transferred to another, while in other respects the procedure is the same, only the policyholders to be transferred are to be favoured with the information. Seeing that the great bulk of policyholders are practically partners in the business, I think that, whether there is to be an amalgamation or a transfer, all the policyholders of both companies should receive the papers. Some transfers, while they may lead to an increase of the total surpluses to be divided, might very likely result in a reduction in the rate of bonus to policyholders in the purchasing company, a great many new partners being admitted by the transfers who do not bring with them a corresponding amount of funds. This is not fair to the policyholders of the purchasing company, although it may be profitable to the shareholders; and the policyholders should have a voice in the matter, and a right to object. Under the present Act they receive no notice of the transaction, and presumably have no *locus standi* before the Court.

The clause enacts that the Court, after hearing the directors and other persons whom it considers entitled to be heard upon the

petition, may confirm the same; but the Court does not take any initiative, and simply listens to the statements made to it. Since the passing of the Act we have seen the transfer of the business of one insolvent company to another which was not in a much better plight, and which itself has since been obliged to close its doors, and transfer its business. The statements presented to the Court gave glowing accounts of the position of the purchasing company; but if any independent expert had been called in, a very different tale would have been told, and the transfer would no doubt have been vetoed. It seems to me, therefore, essential that the Court should have advice entirely independent of the officials of the companies directly concerned, and I think it might well be enacted that, before sanctioning the transfer, the Court shall take the opinion of two members or ex-members of the council of either the Institute or the Faculty of Actuaries. In this way, no reasonable amalgamation or transfer would be interfered with, whilst any such as that mentioned above would be rendered impossible.

Clause 14 appears on the face of it to be very explicit, and to make it impossible for amalgamations and transfers to take place without the concurrence of the Court of Chancery, especially in view of the sentence which runs: "No company shall amalgamate "with another or transfer its business to another unless such "amalgamation or transfer is confirmed by the Court." Also, by this section, the Court is limited in its powers, because it is enacted that: "The Court shall not sanction any amalgamation or "transfer in any case in which it appears to the Court that "policyholders representing one-tenth or more of the total "amount assured in any company which it is proposed to "amalgamate, or in any company the business of which it is "proposed to transfer, dissent from such amalgamation or "transfer." Clause 15 provides that certain very important statements shall be filed in case of amalgamation or transfer—statements which are intended to prevent the gross abuses which in former days prevailed. There is no prohibition of exorbitant commissions and fees, but seeing that all such payments have now to be disclosed, the practical effect is to bring the cost of the transactions within reasonable bounds.

Notwithstanding the apparent explicitness of these clauses of the Act, and the probable intention of the Legislature to include all transfers, lawyers have found other means of carrying through such transactions. Various transfers of business have of late years taken place without the sanction of the Court. For example,

recently the British Empire Mutual office took over the Western Counties; but a way was found of doing so without application being made to the Court of Chancery or to any other Court, and although the statements required by Clause 15 were filed, there was apparently no legal obligation to prepare them. It was found that by placing the transferring company in liquidation, by appointing a liquidator, and by previously passing certain resolutions instructing him to carry out a certain agreement, the business of the transferring company could be taken over without any petition to the Court. Apparently, several other similar transactions have taken place, and the Board of Trade has had no power to intervene. I do not discuss here the propriety of these transactions. Nevertheless, I think secrecy is to be deplored, and should be impossible. The meshes of the Act should be so narrowed that no description of transfer should slip through, and the procedure intended by the Legislature should in all cases be carried out.

The restriction on the power of the Court, whereby a transfer is made impossible if one-tenth of the policyholders of one of the companies object, sometimes acts prejudicially. This was found to be the case when the first effort was made to transfer the business of the insolvent Sovereign office to the Sun. The Court of Chancery found itself compelled, instead of authorizing reconstruction and a transfer to the Sun office, to make a winding-up order, and to reject the policyholders' scheme for reduction of contracts and transfer. The grounds stated were (1) that the Sovereign had no power to transfer, and (2) that under section 14 of the Life Assurance Companies Act the Court has no power to sanction a scheme if one-tenth in value of the policyholders are against it. In this particular case the policyholders opposing were the annuitants of a company which had been purchased years ago by the Sovereign, and it seemed very hard that the other policyholders should thus be debarred from carrying out an arrangement eminently to their interest, and which need not have affected the position of the annuitants at all. By means of "The Joint-Stock Companies Arrangement Act, 1870", the difficulty was, however, overcome. The scheme was again brought forward under that Act, with the approval of the statutory numbers and amounts of the creditors—namely, a majority in number and three-fourths in value,—and it was finally sanctioned by the Court. The transfer thereupon took place, the reduced contracts being fixed in the first instance at the amounts which would be supported at the

advanced ages by the future premiums payable. The liquidation was still to proceed for the purpose of defining the rights of the various creditors, policyholders to rank for dividends according to the values of their policies, these dividends to be handed over to the Sun life office in order to provide additions to the reduced contracts.

In this way, while the Act of 1870 proved unworkable, a most useful means of escape was found, and an arrangement beneficial to all parties was concluded.

I have given this case in a little detail, from information kindly supplied to me by the actuary of the Sun, as it seems to me to point out the way to a very useful amendment of the Act. In so far as life offices are concerned, all Acts bearing upon the questions of amalgamation, transfer, liquidation, and reconstruction, should be consolidated, and a free hand should be given to the Court, acting under competent advice, to do whatever may seem best for those concerned.

Insolvent Companies.—The Act of 1870 has been found to work most unjustly in the reconstruction of insolvent companies.

By Clause 21, if a company is proved insolvent the Court shall suspend further proceedings on the petition for a reasonable time, to enable the uncalled capital, or a sufficient part thereof, to be called up; and it is only if, at the end of the original or any extended time for which proceedings shall have been suspended, such an amount shall not have been realized by means of calls as, with the already invested assets, to be equal to the liabilities, that an order is to be made on the petition as if the company had been proved insolvent. It is not here said that this capital called up for the purpose of making good deficiencies in the assurance fund shall be written off, and shall become the property of the policyholders. Indeed, it still remains the property of the shareholders, and does not form an integral part of the assurance fund at all. Also, the reduction of contracts authorized by the Act is to be "in place of making a winding-up order." The result of this is that when a company has been reconstructed with reduced contracts, and so has been made solvent at the expense of the policyholders, the shareholders again step into possession, and become entitled to whatever rights they had before under the deed of settlement of the company. Thus they take an interest in future surpluses, and take possession of any assets which may be recovered, and which had not been specifically included among those on which the reconstruction was based. It thus may very

well happen that a company which has paid only a few shillings in the £ to its unfortunate policyholders, may remain a very good thing for its shareholders. This is not as it should be, and the Act should be so amended that the reduction of contracts shall not be in place of winding-up, but as a means towards winding-up; and no shareholder should take a penny until every policyholder has been paid in full. An insolvent insurance company which has been reconstructed is in a very different position to a discharged bankrupt. The creditors of a discharged bankrupt are not compelled thereafter to trade with him, and they are not compelled to leave the dividends from his estate in his hands in view of future trade. It is not unreasonable that a bankrupt after his discharge should retain such profits as in the future he may make. In the case of a life office, however, policyholders, when the company has been reconstructed, are compelled to go on paying premiums, or else to forfeit all their previous payments; and they are compelled to leave their dividends in the hands of the company, or rather to take them in the shape of insurance and not as cash. Therefore, it is only right that until the policyholders are paid in full the shareholders should take nothing.

Another great hardship which affects some of the policyholders of insolvent companies is, that although the company has broken its contract with them, they are held fast to their contracts with the company. If they have received loans against the surrender-values of their policies, they are compelled to repay the loans, and are not allowed to set-off against them even the reduced values of the policies. I think that this should be altered, and that where a company has become insolvent, policyholders who have had loans within the surrender-values of their policies should be relieved of their debt if they discontinue the assurances.

The present procedure in the case of an insolvent company is so slow that great damage is done. The proceedings drag wearily along, and perhaps for two or more years policyholders have to pay their premiums into a suspense account, and are kept in ignorance as to their future. I think the Act should be so amended that much greater despatch may be secured, and especially I would suggest that the precedent set in the case of the Sun and the Sovereign should be hereafter always followed, and that reconstruction should, in the first instance, take effect promptly on the future premiums alone, dividends from the bankrupt estate being applied afterwards, as they come in when the securities have been realized, to increase the reduced contracts.

Although the reduction of contracts to commence with would, under this arrangement, be great, the promptitude with which the reconstruction could be carried out would make ample amends.

While I would earnestly deprecate any right of supervision or interference in management of the companies being granted to the Board of Trade or any other Government department, I think the powers of the Board of Trade should be extended in the direction of enabling it by summary process to insist on the proper accounts being filed within the proper time, and generally on the Act being carried out to the letter. At present the Board of Trade is powerless, and may be defied with impunity. This great defect of the Act is very apparent in the 18th section. There it is enacted that if a company makes default in complying with the requirements of the Act, it shall be liable to certain penalties; and if default continue for a period of three months after notice of default by the Board of Trade, and after publication of that notice, the Court may order the winding-up of the company *upon the application of one or more policyholders or shareholders.* Thus, both the Board of Trade and the Court are powerless, unless some policyholders or shareholders can be found to act as cat's-paws. It would be much better to give the Board of Trade itself the right of initiative. The principal scandal of our existing law is, that companies which are known by every expert to be insolvent, and the insolvency of which is freely canvassed in the insurance and general press, are yet allowed to go on for years collecting premiums, and wasting their resources, it not being the duty of anyone in particular to stop them. The directors have, no doubt, a petition all ready in their pigeon-holes, to be presented should any hostile action from outside be threatened; but, so long as no such action is rumoured, they go on, and allow the company to get deeper and deeper in the mire. It is only when the case becomes extreme, and speculative lawyers and accountants think it worth while to take it up, that the downward career is brought to a close. It should not be left to individual policyholders or shareholders to take the necessary steps. None of them have sufficient interest in the concern to make it worth their while, for that alone, to risk the costs, or to incur the loss of time. I would not give to the Board of Trade, or any other Government department, the power on its own motion to stop companies which are known to be insolvent, or the power to make inquisition, practically with closed doors, as to the standing of doubtful companies: but I think that the Board of Trade might

have the duty laid upon it of itself petitioning the Court of Chancery for the winding-up of a company which there was every reason to think was insolvent. Of course, the Board of Trade would have to prove insolvency to the satisfaction of the Court, but, in those cases where intervention was considered necessary, that would not be difficult, while, on the other hand, long-continued scandals would cease to be possible. Knowing the power vested in the Board of Trade, the Directors themselves would take action in time.

This paper has extended to a much greater length than I intended, and I must apologize for its prolixity. The subject is, however, a very large one, and there are still many points of importance on which I have not been able to touch. As stated in the opening remarks, I have felt great diffidence in the performance of my task, but the kind forbearance of the Institute is proverbial. I have presented the subject as one for consideration, in the firm belief that in the multitude of counsellors there is wisdom; and if I have sometimes expressed myself in a way that may be thought to be emphatic, I hope to be forgiven. It is not with any intention of forcing my own views upon others, but rather that the views of others may be elicited. By free discussion only can a sound opinion be arrived at, and it is very likely that when the experience of many is brought to bear, each of us will see cause to modify in many respects his previous conceptions.

Discussion.

The President (Mr. B. Newbatt) having called upon the referees of the paper, Mr. Bailey and Mr. Higham, to open the discussion,

Mr. A. H. Bailey said that some light might be thrown upon the subject of the paper by referring to the state of the insurance world before the Act of 1870 was passed, and by a consideration of the evils which it was the object of that Act to remedy. Forty or fifty years ago, new life assurance companies sprang up like mushrooms. They commended themselves to company promoters for the reason that they did not require any large outlay of capital. In the first instance it was all receiving money, the payments being deferred for a convenient time, so that small sums of money only were required to start life assurance companies. Of course, the number of the companies so started was far beyond what the public required. They struggled on, but were simply eaten up by expenses. There being too many, the next step was for amalgamations. Some of these amalgamations were carried out honourably and judiciously, but with

others the professional amalgamator appeared upon the scene, and succeeded in uniting one insolvent company with another, considerably diminishing the assets for his own remuneration. This went on until 57 companies were reduced to two, the "Albert" and the "European", the premium income of each exceeding £300,000 a year. One morning the "Albert" found itself unable to meet its claims, and the "European" shortly afterwards followed. This being the failure of 57 companies, it created a great sensation and forced itself upon the attention of the Government, and the Board of Trade found it necessary that something should be done. Mr. Cave was at that time vice-president of the Board of Trade, and the Bill which afterwards became the Act of 1870 was drawn. The real author of that Bill was a well-known member of the Institute, the late Mr. Pattison, who was Mr. Cave's adviser throughout. The fifth and sixth schedules were entirely drawn by Mr. Pattison, and were not, he believed, altered in their passage through Parliament. The first object of the Act was to insist on the publication of accounts and statements, in forms which would show the real position of the company. At that time, companies established since 1844 were required to publish accounts, but in any form they pleased, while those established prior to 1844 were not required to publish anything. Next, it was provided that the establishment of new companies should be checked by the requirement of a deposit of £20,000 with the Court of Chancery, and a very effectual spoke in the wheel that has proved to be. Thirdly, it was considered that the scandals which had attended amalgamations should be prevented by requiring the concurrence of the Court of Chancery, and by providing that statements should be sent to the persons interested, showing the terms on which the amalgamations were to be effected, and in particular the sums paid to any person promoting those amalgamations. Fourthly, it was enacted that companies, avowedly insolvent, should be allowed to continue by reducing the sums assured which they had contracted to pay. In the main the Act had worked well, but it was not at all surprising that after 21 years' experience, improvements could be effected. He would take each of the four objects in their reverse order, commencing with the reduction of contracts. That was a matter on which the Act urgently needed amendment. At the breakdown of the "Albert" it was necessary to have a special Act of Parliament to wind up its affairs, and the late Lord Cairns was appointed for that purpose, and he (Mr. Bailey) had the honour to be Lord Cairns' actuarial adviser. When Lord Cairns was asked to reduce the sums assured, he replied that that could not be done, that the clause in the Act had been wrongly drawn, and that instead of the words "in place of a winding-up order" there should have been inserted "as a mode of winding-up." There was only one case, namely, that of the "Great Britain", a mutual society, where the reduction of contracts had been successfully carried out, there being no conflicting interests of shareholders and policyholders. As to other cases of insolvent companies, he had never been able to understand the process of liquidation. An official liquidator was appointed. A company being avowedly insolvent, it would be thought that almost the first act of an official liquidator would be to make a call upon the shareholders. That was not done,

and, contrary to all their ideas regarding insolvent companies and individuals, the working remained in the hands of the proprietors. Then, as to amalgamations, they required a little assistance from their legal friends. Mr. King had said it was difficult to understand the distinction between "amalgamation" and "transfer." An "amalgamation" meant a partnership, and a "transfer" meant the giving up of one company to another. Taking the analogy of banks, quite recently Messrs. Prescott and Messrs. Dimsdale had amalgamated—entered into partnership—and continued as Messrs. Prescott and Dimsdale. Several years ago the business of Jones, Loyd and Co. was transferred to the London and Westminster Bank, and Jones, Loyd ceased to exist. To the mind of a layman there was a clear distinction between "amalgamation" and "transfer", but the clauses might not have been properly drawn. What was really wanted was some outside authority, the Court of Chancery or other, to see that in amalgamations or transfers the interests of the assured were properly attended to. As to the deposit of £20,000, he thought the time had come when that might be discontinued. There was now no fear of any great number of new companies, and he, for one, should be glad to see the number increase, but in the present state of the law it was practically impossible for a new mutual life assurance company to be started. The most important matter of all was as to the schedules, in regard to which he differed from Mr. King. His own idea was that the requirements should be materially reduced, only that being given which was required to show the position of the company, details being omitted. The revenue accounts required little alteration. No doubt, as Mr. King had noted, the premiums should be the original contract premiums, and the income tax should be deducted from the interest. He thought there was no need to distinguish claims by death from claims by maturing endowments. As regards the forms of account, he would like to ask anyone connected with marine insurance, if they would submit to marine premiums and fire premiums, and marine claims and fire claims, being combined. There ought to be a separate account for each department of the business—fire, life, marine, accident, guarantee, or other. It would be impossible for anyone to judge of the value of the assets from a printed statement. The accounts of the Bank of England were published weekly, the assets being comprised in four items only—Government securities, other securities, notes, coin—amounting to a total of over £51,000,000. Some such form, perhaps a little extended, was wanted for assurance companies. He could not agree with Mr. King, that claims announced but not admitted ought to be included in the revenue account of the year; but that was a small matter, not worth discussing. Regarding the delay in the publication of the accounts, it would be easy to require that the accounts of all insurance companies should be made up to the 31st December of each year, and some companies had lately been altering their arrangements for that purpose. Then the accounts should be delivered to the Board of Trade before the 31st March. As to the valuation schedules, he was a radical reformer. He wanted to get rid of the sixth schedule altogether. The object of that schedule was supposed to be to check the statements of the fifth schedule, but he

did not believe that was practicable. All that was wanted was, that the principle on which the valuation was made should be clearly set out. In the fifth schedule the present value of the gross premiums payable should be required. He was astonished that an estimate of the liability under a policy should sometimes be made, taking no account whatever of the actual premiums payable. He differed from Mr. King as to a statement regarding the new business. That was a matter with which the public had no concern whatever. The craze of the present day, no doubt, was for a large premium income, and to that the interests of the assured were often considerably sacrificed. But this should not be encouraged by requiring any official publication of new business statistics.

Mr. C. D. Higham agreed with Mr. King in his distrust of any Government valuation or official standard, not only because of the great recuperative power often seen in weak offices and friendly societies, but also because of the difficulty there would be in changing any standard that might be fixed. For instance the only approach under the Act of 1872 to a standard of any sort—Seventeen Offices' Mortality Table, 4 per cent—would not now either in mortality or interest be considered the right one. While undoubtedly those who were in an office had every right to make enquiries, he did not admit any such right on the part of the outside public. Many difficulties would be met if the Blue Book were published more quickly than it had been in the past. All needs of possible proposers could safely be left to the competition of the different companies, and to the energy of agents in finding out weak spots. Mr. King's idea as to posting circulars to members was, he thought, impracticable. If anything of the kind was needed, it should be in the form of the clause in the Act relating to shareholders providing that names and addresses might be obtained at a cost of 6*d*. for every 100 words. He wished companies would be bold enough to publish the names and addresses of their members, as was done by some already. Referring to the accounts, there should certainly be a revenue account for every fund mentioned in the balance sheet, but he could not agree that there need be any separation of the assets. The great point was to let the public know how much the shareholders took and how much the policyholders, and then it was not necessary to state whether investments belonged to one fund or to the other. He did not agree with Mr. Bailey as to "claims announced." If they included interest accrued but not then payable, so they should make provision for expenses incurred and not then payable, for dividends which might have to be paid out of the year's profits, and for claims announced but not admitted, though under the last head only the difference between sums assured and reserve value need be entered, as the policies would not have been written off. A list of investments such as Mr. King spoke of would, he believed, be useless. The most important of all were the mortgages, and as a list of the mortgages could not be given, they might just as well leave the others alone. He was not an advocate of too low a valuation of assets, as it was unfair, but if they were to publish a list of everything, with the present prices, they were likely to have requests for more dividends and bonus, even though the margin of value was only what most of those present would consider desirable. He

thought there should be more subdivisions and more guidance as to the category in which certain investments should be put: for example, Indian railway stocks guaranteed by Government, mortgages on rates, &c. If Mr. Bailey would allow the sixth schedule to remain, he thought the order of it should be changed with the fifth, as obviously the particulars for valuation should come before the valuation itself. Under the fifth schedule, how claim acceleration reserve (if any) was provided for should be shown, and instead of valuing any net premium, the loading should be shown as a liability. Not only would the margin for expenses and profit then be evident, but all whole continuance policies could be thrown under one heading, and a proper reserve would then be made for single and limited premium policies. He could not agree that to call for two valuations for the mere purpose of showing the effect of any change of table was justifiable, nor the prolonged bonus table Mr. King had suggested; indeed, he questioned whether the Act required any information as to any bonus save the one under division, though he preferred two columns, showing the old and the new bonus at each age and duration. If the sixth schedule had required a copy of a whole continuance and endowment assurance policy, probably some of the unfortunate Albion policyholders might have been saved. He could not think a policyholder with a loan from an insolvent company should be spared its repayment, for, if he might write it off from the office value on which a dividend was payable, he would escape a loss at the expense of his fellow members. He would remind Mr. King that Solomon did not say that in the multitude of counsellors there was *wisdom*. He said *safety*, and they all knew cases where numbers had procured safety when a bolder course would have been the truer wisdom.

Mr. Sheppard Homans (of New York), after expressing the great pleasure with which he attended the meeting, and referring to occasion when he first had the pleasure of appearing before the Institute, 30 years ago, said he believed that the hearty welcome which had been given to him that evening was due, in large part, to the friendly feeling that was entertained to their younger sister society, the Actuarial Society of America, which was striving to emulate the example of its older sister, and to do its part in the advancement of the science to which their lives were devoted. With regard to the paper to which they had listened, he looked at the subject from a somewhat different point of view. He was unable, from want of familiarity, to deal with the schedules to the Life Assurance Companies Act, but, he asked, what was the object of any interference by Government? There must be some interference. A life insurance company was the creation of the Government. It would surely be unsafe to allow any man or set of men to get up a company at will. It must have a charter, and it was only right and proper that some provision and control should be exercised by the Government. He took it as an axiom that the best Government in the world was that in which there was least interference with the individual. In America they had one great misfortune—they had 42 masters, to each of whom they must pay their obedience in the form of an annual return. The main objects of any interference by

Government were two. One was to point out in time the danger of the insolvency of a life assurance company. How could that be done with the least possible interference on the part of the Government? The second object was, if insolvency or impairment had occurred, to show how the misfortune could be lessened or alleviated. He was far from being an advocate of the net-premium system of valuation. As a test of solvency he believed that every gentleman would agree that it was a failure. But the net-premium system had its value, which was to test, first, whether that portion of the premium which was intended to be laid aside and accumulated was intact and added to the reserve; secondly, whether the amount which should go to the payment of death claims had been sufficient; and, thirdly, whether the expenses had been within the margin provided for that purpose. They might graft on to the net system of valuation the present value of the loading, and offset that by the present value of the future expenses and contingencies, but that he considered of minor importance. While there were many defects of omission and commission in the State insurance departments of the country (and he was sorry to say that he thought the balance was against them), yet it would be unwise to condemn them entirely. It was true, as Mr. King had said, that Government interference involved, logically, Government guarantee. The members of a life assurance company, and especially of a mutual life assurance company, were entitled to certain information. That information should not be in detail, and it was not necessary that the company should give to individuals who came to the office detailed information with regard to anything, but they were entitled to know whether the company was on a sound basis, whether the funds were well invested, and whether the management was honest and skilful. This could never be done by Government interference. The policyholders should have some recognized representatives; they should select two or three men of the highest character in the community, who would be charged with the duty of receiving proxies as a sacred trust, for which they should be properly compensated. They should agree not to be benefited in any shape or way except by the compensation given to them. Of course, the policyholders could revoke those proxies. As it was now, the policyholders were not represented, and the danger was that either the officers would be automata, or that some active agents would gather proxies together and overturn the manager, as had been done frequently in his own country. In Great Britain, if a company were insolvent or in trouble, the Court of Chancery might decide what should be done, and there was no appeal from that decision. In the United States they had, as a clause in the constitution, provision that no State, no legislature, no court, and no power should interfere with the obligation of contracts, and no matter what the advantage might be, no power in the United States could compel a man to break his contract. There was the case of a company with $13,000,000 assets and a splendid corps of agents, and with very light mortality, and, as an insurance institution, everything that could be desired. The president of the company unfortunately invested $5,000,000 injudiciously, and something had to be done. The Commissioner of the State called in experts instead of putting the company in the hands of a receiver—the usual course,

unfortunately, in America. The plan adopted was that which, he believed, was in vogue in Great Britain, namely, to induce the policyholders to scale down their contracts, so that the reduced insurance would be met by the tangible securities. He was consulted by the officers of that company, and he told them that, in his judgment, that was the wrong course to pursue, for the following reasons. In the first place, the company was lessening its income at a time when it wanted every dollar. It was diminishing public confidence by a partial confession of failure, when, by every means in its power, it ought to restore and maintain public confidence. It was asking its agents to cut down their commissions, for which, of course, they would demand compensation. Above all, it was asking men whose lives had become impaired to give up that which could not be replaced, whilst they ask men in good health to give up that which cost them nothing. On being asked for a remedy, he suggested that as the company was as an insurance institution all that could be desired, but as a banking institution which had received certain moneys for accumulation was a failure, the true remedy, theoretically, would be for each policyholder to give a cheque for his share of the deficiency. That would make the reserve intact, and all would go on smoothly, but, of course, it was out of the question. But the next best thing could be done—namely, that each policyholder, to protect his own interest, should give a certain obligation against his share of the reserve. Had that course been adopted, there was no doubt in his mind that that company would not only have been solvent to-day, but would have been doing a successful business.

Mr. R. P. Hardy said that Mr. Bailey and Mr. Higham had dealt chiefly with certain proposed alterations in the forms. They would all be prepared to leave that matter to be settled by a small committee of the Council. But he would like to say that if the salvation of the policyholder, or, indeed, of anybody else, was supposed to depend upon a mere form they would be leaning upon a broken reed. No inspection was of any avail unless it descended into the strong rooms of the companies and examined the securities in detail. Even then, when they had exercised their opportunities of inspection to the greatest possible extent, there was nothing that would supply the place of those elementary conditions of honesty and discretion in management. But, apart from these details, there was a preliminary question to be considered. It was idle to conceal that there was a strong opinion amongst many gentlemen present that the Council would not have allowed this important subject to come forward, nor would Mr. King have devoted the great pains he had done to the preparation of his paper, unless it were believed that the Government had some legislation in view of a coercive nature. If that was the case, one would like to know what was the catastrophe that had recently happened or what was it that was pending in the future, and why was it suggested that they should mobilize their forces for what he believed to be a contemplated attack upon their liberties of administration? Was it a renewal of the attack made not long ago, but which failed so ignominiously, upon the industrial assurance companies, those bright examples of the administration by

the people of their own affairs as contrasted with the ghastly failure of the Government's own scheme? Were the liberties of the working class assurance associations to be threatened—was that at the back of this contemplated legislation; or was it that some of the companies. with a view to afford full facilities to the public and to put themselves a little more in the front, were going a little too fast for the average official mind? To an official they knew that there was no *bête noir* anywhere in the world like the insurance manager with any ideas. The official forms were far too small to take notice of fresh ideas. The ready-made letters would not fit the case of a man of that particular description, and one who was always giving trouble and requiring you to write letters was a man to be put down by legislation if possible. He would ask, and on that question he should find himself at issue with Mr. Homans, upon what principle did the State propose to interfere in private partnership? Mr. Homans said that insurance associations were the creations of law: so was every right in the world the mere creation and exhibition of law. Government had given insurance companies no privileges; they were taxed without mercy; and now it appeared that worse things were to happen. What did Government do for them which they did not do for the London and Westminster Bank, or for the rottenest mining company brought out in the City of London. If the great insurance companies of this kingdom, the pride and model of the world, were to be regulated, put under rules as he believed only two other trades were, namely, pawnbrokers and licensed victuallers, if they were to be subjected to police regulations and have the quality of their goods sampled and gauged by some official standard, why should they not go into the London and Westminster Bank and inspect the contents of the bill case? Why allow mushroom companies to carry on their plundering? Whatever was sauce for the goose was sauce for the gander, and if they were to be put under inspection and regulation, let all the other companies be treated in the same way. He agreed with Mr. King that a very serious amendment was required in the treatment of insolvent companies. He would not, for obvious reasons, say much upon that point, but he would say that when the real state of the law was brought to his knowledge, and to the knowledge of some gentlemen with whom he acted, it was a profound disappointment. But if the Government, occupied with trifles like Home Rule, had any surplus energy to work off, he would suggest some very useful directions for its employment that would not trench upon their liberties. Why did they leave the law respecting the assignment of policies in its present disgraceful muddle? Why did they not strengthen the common law, and, if necessary, introduce a new law of conspiracy to bring to book directors and auditors and officers who signed balance sheets which, if not false to their knowledge, were within their own means of verification? Why should these persons be allowed to snap their fingers at the law? Let the Government take up that question, and he was sure they would find plenty of support in that room. Why did they not take up and require some exposure of those assessment companies which were interfering largely with the legitimate assurance business of the country and were paying no taxes at all? There were some other points upon which

they might employ themselves, but he would not allude to them as they might come a little home to some of their friends. He would only say this, that he claimed on behalf of a most important trade —it might almost be called a trust—as much liberty as the late Mr. John Stuart Mill laid down in his treatise: the most unrestricted liberty to conduct their affairs so far as they did not interfere with the liberty of others—in a word, a liberty not merely to manage but, if they chose, to mismanage their affairs. Above all, he asked the support of the Institute for the principle of commercial liberty by which this country had largely achieved its present distinguished position, and which principle would carry it through many troubles, to which less happy and more bureaucratic countries were constantly exposed.

Mr. T. Y. STRACHAN thought it was a little late in the day to object to legislative interference, and so he could hardly follow Mr. Hardy. He was opposed to legislative interference, but they must recognise that it had existed for twenty years with regard to life assurance companies. That was a fact they could not get rid of, and they must therefore direct their attention to the question. Reading the early part of the paper, in which the author objected to Government supervision, he thought it was one to which he could give an entire adhesion; but he was utterly disappointed to find at the end, that it was proposed so to legislate in the future as to enable each opposing company to know, if it chose, as much about the business of its competitors as the managers of the companies themselves. The public would not understand information proposed to be given, for they could not shut their eyes to the fact that life assurance accounts were beyond the ken of the general public. What was asked was that they were to put every opposing company in charge of the management of another company. Was that a correct principle for working either insurance or any other business? They were continually told by joint-stock companies that it was not wise to give full revenue accounts, because it disclosed the state of affairs to competitors, and yet the only principle at the bottom of this recommendation was that they were to give details which would enable all sorts of people to raise objections. Everyone connected with the management of a company knew that if they were to give a list of their securities they would be subject to complaints from the shareholders, that this had been gone into and something else left out. With reference to Mr. Bailey's remarks, he thought there were advantages to the public in showing the gross premiums, and in showing how much of the gross premiums had not been valued compared with the cost of conducting the business. That was a question for the public as tending to indicate the bonus-paying power of the office. If the returns could be made more uniform it would be an advantage. He hoped the voice of the Institute would not be in favour of publishing all the details suggested in the paper, but only what was useful and not what would merely enable an opponent to understand and attack a company.

Mr. G. S. CRISFORD wished to draw attention to one sentence in the paper, which was perhaps one of the best comments they could have upon the question whether legislation, or an alteration of the

Life Assurance Companies Act, was much needed. Mr. King said: "The result is that in no country of the world are the life offices so strong and so stable as in Great Britain, and nowhere do the assured enjoy greater advantages in respect of bonus and other benefits." That had been brought about a good deal by the passing of the Act of 1870, and it was fair comment to say that the Act was so well drawn, and contained so much that the public needed, that very little alteration was now required. There might be some small matters, such as Mr. Bailey had referred to, with regard to the winding-up of companies, and so on, which the Council of that Institute could, perhaps better than any other body, advise the Legislature upon. But what was wanted was simplicity, not complexity. The more they simplified instead of increased the schedules and returns, the better would the public be able to understand the figures that were laid before them. The object of the Act was to give information to the public. It was not intended for experts, who could use their own means of investigating the figures of a company, but it was to give the information in such a simple form that the public, as far as possible, might be able to understand the position of a company by looking at the returns. He believed an immense amount of good had been done by the Act, and it would be a great misfortune if they were to let it go forth from that room that many alterations or amendments were required. The more the schedules and returns were complicated, the greater would be the difficulty of the public realizing the benefit the Act was to them at the present time.

Mr. A. G. Mackenzie said that he read Mr. King's remarks as to the schedules and their amendment with something like fear and trembling lest the members of the Institute should endorse them. The balance sheet and revenue account might certainly be simplified, but he should not like to see them amplified to any considerable extent. With regard to what had been said about prompt publicity, as the representative of a company with an agency in India he might be permitted to say that the period of three months was a little too short. Companies should make up their accounts to the 31st of December, but in the case of a company having an agency in India or the Colonies, particularly in a valuation year, three months was a very short period, and he scarcely thought that the suggestion that Indian and Colonial accounts might be made up to September was a good one. What the public wished to see was the position of a company at a particular time. Mr. Bailey desired to abolish the deposit that was required at present from life assurance companies. He believed that even now that security to the policyholders could be skilfully circumvented. In a recent case a friend of his was asked to subscribe to a new life assurance company, and was offered a debenture upon all the property of the company, including the deposit. He was a very shrewd solicitor, and was of opinion that he would have a valid charge. He did not think any company ought, as a matter of principle, to assume in its valuation that it would in the future manage its business at a lower rate of expenditure than it was doing at the present moment. If some alteration were made, and it was insisted upon that the company should not value in that way or

assume a lower rate of expenditure than they had experienced on the average for the last five years, it would have a tendency to reduce expenditure, which would very much lessen the difficulties with which young companies had to contend. Mr. Bailey had expressed the feeling, in which many would sympathize, that there were too few young companies, and he hoped their interests would be considered in any future legislation. Amalgamation seemed to be the order of the day, and the offices were 15 per-cent less than they were 20 years ago. If that continued they would have, before long, a few large offices doing the whole insurance business of the country, and he did not think that would be very satisfactory to the insuring public.

Mr. S. G. Warner said that existing legislation had, in the first place, been framed with a view of giving the assured reasonable ground for judging of the solvency of the various companies, and, so far, most would agree that such legislation was desirable. But there appeared to be another element embodied in the schedules of the Act of 1870, with the object of assisting them in their competition with one another. With regard to such part of the schedules, for instance, as the table of bonuses, it was difficult to understand how that had been introduced, except with some idea of giving the public the means of judging as to the comparative advantages of various companies. That seemed to be a much more questionable thing for legislation to deal with, and surely it might fairly be left to the enterprise of individual offices, in their competition, to bring quite actively enough before the public the respective advantages they had to offer. With regard to the table of bonuses, it had been suggested in the paper that, for the future, it should be entirely on a hypothetical basis. It was a little difficult to see what advantage such a table, on such a basis, could possess. Suppose an office which strengthened its valuation basis declared, in consequence, for one quinquennium, a bonus rather smaller than previous or subsequent ones. That would be quite a temporary matter, and yet, upon so slender a basis, an imaginary table would be built up, extending for 30 or 40 years. Surely this would be a waste of labour, and, if the table be retained at all, it should not be hypothetical, but should be, as far as possible, based on actual fact. The questions of the incidence of premium income during the year, the precise annuity employed in the valuation, and so forth, seemed to be things that the Government should hardly interfere with. It was difficult to say that they could ever affect the great question of the solvency or safety of a company, and it was a rather questionable expedient to introduce into the schedules refinements of that kind. One very suggestive thing in the paper under discussion had been the distinction drawn between the two ways in which the Government might go far to secure the safety of the public. One way was the publicity of accounts—but another very powerful way was to give the policyholder increased rights against his office, and that seemed to be desirable for many reasons. Let the policyholder by all means have the most stringent rights practicable against his company, whether in the way of demanding, on certain conditions, the addresses of his fellow policyholders, or to raise any

questions as to the position or policy of the company. All these things could be done without any of the difficulties attending minute publicity as to accounts or investments, and with this immense advantage, that the differences of a company could thus be settled, so to speak, domestically. After all, the safety of each company concerned only a very small section of the public—those insured in it, and those only. If they could be protected by giving them greater rights and privileges as against the office, it would be more desirable to proceed in that way than by the more roundabout method of attempting to give publicity to an infinite amount of detail with regard to the accounts.

Mr. T. E. Young said that he agreed with Mr. King in the two essential principles of publicity and freedom, but, unfortunately, when coming to apply his principles to actual instances, he found himself differing very materially from him. He would only refer to two points, and, in considering them, he was guided by the principle that no requirements should be exacted from a company unless a definite and substantial benefit to the policyholder would be thereby secured. Applying that principle to one point for example, a detailed statement of the new business, and the expression in a broad degree of the sources from which that business was derived, they would be furnishing information which would afford no help or assistance whatever to the policyholders, and would, in his opinion, be doing serious damage by emphasizing that impression which competition had unhappily confirmed in the public mind, that the new business of a company was the test of its prosperity. With respect to another very essential point, he took the strongest possible exception to any minute statement of the investments. Such a statement must necessarily be of a most partial and incomplete character, and thus would be entirely opposed to the principle of conferring any solid benefit upon the policyholder. He found that mortgages and rates, investments in ground rents, and so forth, comprised about 63 per-cent of the total investments of life assurance companies, while those in Stock Exchange securities amounted to only 28 per-cent. It would be only possible, therefore, to furnish information respecting the market value of securities in respect to 28 per-cent of the investments. But how were they to give the public the faintest possible notion of the solidity and value of the mortgages, and the investments in life estates and reversions, on which essentially depended the solvency of the company. It would be utterly impossible to effect this object, and he was the more confirmed in his opposition to such a proposition as not simply being fruitless and useless in itself, but on this ground, that they would find competition, which at the present date was uncompromising—perhaps he ought rather to designate it by the term compromising—being increased in its virulence and unfairness by a competitor pointing out some one inferior investment as a specimen of the investments of a company generally, while those investments whose present value could not for the reason stated be fully represented, might be of the soundest character.

The President said that more than one-third of the paper dealt with the question of State interference, after the American fashion. Although that disquisition was of interest in itself, and put before

them clearly the actual condition of affairs in America, and also afforded some excellent reasons why they in England should not follow the example of their American cousins, for the practical purpose before them he did not think it was needed. He had every reason to hope and some reason to believe that there was no intention whatever on the part of the Government to introduce any of those coercive measures which would unite every man in that room in the strongest possible opposition to them. But the want of intention on the part of the Government to introduce any drastic measure did not disentitle them to come and say, "We have found this Act in some respects defective, and we ask from you some guidance as to the direction in which you, with your knowledge of the subject, would say that an amendment should take place", nor, as he understood, did anyone attempt to lay down as a principle that no amending Act was necessary. He agreed that the sixth schedule in a great measure, if not entirely, was not wanted. In the first place, because of the exceeding amount of labour it gave them for a very small result; and, secondly, because the whole tone of it was that of a detective, an attempt as it were to cause them to furnish material out of which they might be brought to book, and the result of the investigation of those details put against their own statement under Schedule 5. He agreed with Mr. Hardy as to making the responsibility of those who prepared the statement under Schedule 5 as great as possible, but, having done that, let it suffice until they had some better reason than they had at the present moment for going behind it. The two words which were the watch-words of this question as brought before them by Mr. King were those which he adopted to the full—liberty and publicity. He was the enemy of all that would unduly hamper them in that freedom with which they conducted their business, but, on the other hand, he was the friend of all that would give every possible publicity in respect to those essential particulars which he thought the assured had a right to know. He regretted that in one respect he was at variance with many of his friends. He alluded to the question of stating, in whatever detail the particular office might be enabled to afford, the manner in which its money was invested. Great pains were taken to exhibit clearly in the fifth schedule the result of the actuarial investigation. Surely, having shown the liabilities, the other side of the account, the assets, was of equal consequence to those who wished to know whether the company was or was not stable. He admitted, of course, that to some extent a detailed account was not possible: large portions of their money were invested in forms with regard to which only a general statement could be made, but he was not without hope that when the Council of the Institute came to consider this question in detail, some means might be devised whereby the soundness of those investments which did not present their soundness on their face might be made abundantly clear. The latest failure in the insurance world—that of the Sovereign Office—was due mainly to the defect of the assets side of the account. There were two other points on which he should like to express his own personal opinion—first, that the Board of Trade should have the power, where there was manifest evidence of insolvency, to itself raise the question in the Courts; and, secondly, that it should

have the power to insist on the present Act, or whatever Act might be substituted for it, being carried out in its entirety. At the present moment, if an insurance company which offended, or was thought to offend, against the intention of the Legislature as expressed by the Act, was willing to stand the racket of a correspondence with the Board of Trade, it could safely defy that department. That was a position which he did not think seemly. He did not at all agree with Mr. King as to the necessity for setting out in detail those matters with regard to the source of new business, but he did think they should know more clearly than at present what was the cost of new business. No gentleman in that room would pretend to say that that was a question which did not concern the assured. It was at this moment probably the most serious question which concerned them. There was this peculiarity with regard to it. Some of the greatest offenders at the present moment with regard to the cost of new business were, he was sorry to say, the older offices. Now, owing to the fact that so much of the business of the older offices carried no commission whatever, it was quite possible to preserve in those offices a very low rate of expenditure for commission, although an enormous rate might, as a matter of fact, be paid as commission on new business. He would like some means to be taken whereby that fact might be clearly brought out. He hoped that, when they came to consider the question in detail, they would find themselves more in agreement than would appear as the result of this discussion. They had one wish, namely, to maintain to the full that integrity which, notwithstanding failures, had been in the main the characteristic of British insurance business. And if by close examination they found defects in the present machinery which could be remedied, he was quite sure that their own personal feelings and labour, and the fear that holes might be picked in their coats by some competitors, would not stand in the way of their doing what would be a great public benefit.

Mr. George King (in reply) said that his paper was not put forward as unassailable or with the intention that he should stand or fall by it, but it was presented for discussion, and the fact that different opinions had been expressed was an advantage and not a misfortune. Mr. Strachan would find that the recommendations it contained relating to the accounts of the companies were of a very mild character, and that, so far from asking for any sweeping amendments of the Act, he only suggested simple changes in detail He did not think the Government or the public would allow the sixth schedule to be abolished. They might easily take the revenue account he had proposed, and dissect it into several in a few minutes. For the sake of convenience, he proposed a single revenue account for all business other than life, and in that he was backed by no less an authority than Mr. Sprague, whose model form of revenue account agreed very closely with that which he (Mr. King) had suggested. His proposal was not to alter the present law, but merely to embody all the particulars in one account instead of two. The principal point of his recommendations was that the shareholders' and policyholders' funds should be kept altogether separate, and he still ventured to submit that that was a very important principle. In conclusion, he

would say that the suggestions made in his paper were based upon a careful study of the accounts in the Blue Books, and he could give chapter and verse for each one.

Value of the Life Interest of a Lady in a Reversion expectant on her own Death.

CASE.

Miss T., by her will, dated 4 April 1889, bequeathed the residue of her property upon trust for sale and conversion, and investment of the proceeds; and to pay the income of the proceeds of such sale and conversion, and the actual income of her property, until converted, to her mother, Mrs. T., during her life.

Miss T. died on 28 June 1889, leaving her mother surviving.

Mrs. T. died on 20 January 1891.

Miss T.'s property included a vested reversionary interest in a sum of £1,500 Consols, expectant on the death of the above mentioned Mrs. T. This reversionary interest was not sold.

The executors of Mrs. T. now claim that the reversionary interest ought to have been sold, and the proceeds invested, so that Mrs. T. might have received the income of that investment during her life; and the executors of Miss T. admit that the contention so raised by Mrs. T.'s executors is well founded.

Mrs. T.'s executors further contend that, as the reversion was not realized and the proceeds invested, they are now entitled to receive out of the proceeds of the £1,500 Consols a sum equal to the value, at the time of Miss T.'s death, of Mrs. T.'s life interest in the said proceeds. The executors of Miss T., on the other hand, contend that Mrs. T.'s executors are only entitled to receive the actual dividends which Mrs. T. would have during her life received, if the reversion had been realized and the proceeds invested in Consols, immediately after the proof of Miss T.'s will.

OPINION OF COUNSEL.

According to the authorities now in force, I think that the equities between the estates of Miss T. and Mrs. T. must be adjusted on the following principle, namely:—Miss T.'s executors should realize the Consols. They should then ascertain the sum which, put out at interest at 4 per-cent per annum on 28 June 1889, being the date of Miss T.'s death, and accumulating at compound interest, calculated also at 4 per-cent per annum, with yearly rests and deducting income tax, would, with the accumulations of interest, amount, on the day on which the reversionary interest is realized, to the sum actually received. The amount so ascertained is to be deemed capital; and the difference between that amount and the sum actually realized, is to be deemed income, and should be paid to Mrs. T.'s executors. This

mode of computation was laid down as correct by Mr. Justice Chitty *in re* Earl of Chesterfield's Trusts, 24 C.D., 643, following Beavan *v.* Beavan, cited in a note at 24 C.D., page 649; and was acted upon by Mr. Justice Kay *in re* Hobson, 34 W.R., 70, though His Lordship did not form an independent judgment on the question.

We are indebted to Mr. T. B. Sprague for the foregoing Case and Opinion regarding a very curious question. He says:

"It is clear that neither of the courses proposed in the case would be fair as between the parties. It would not be fair that Mrs. T.'s executors should receive a sum equal to the value, at the time of Miss T.'s death, of Mrs. T.'s life interest in the proceeds of sale of the Consols; for regard must now be had to the fact that Mrs. T. died about 18 months after her daughter. Nor would it be fair that Mrs. T.'s executors should receive the actual dividends which Mrs. T. would have received during her life, if the reversion had been sold and the proceeds invested, immediately after the proof of Miss T.'s will; for this course would not take into account in any way the price which the £1,500 Consols actually realized when sold. It would also have the effect of giving to the daughter's executors the whole of the profit which has arisen from their omission to sell the reversion as directed, and it is clearly right that the mother's estate should share in this profit. Another course that might be proposed would be that Mrs. T.'s executors should receive compound interest on the value of the reversion as at 28 June 1889; but this would be open to the same objections. The course laid down in the opinion is not open to any of these objections, and seems to be the best that could be adopted; except that the sum held to be income should be apportioned between the two estates, the mother's executors receiving only the proportion which corresponds to the time between the deaths of her daughter and herself." while the daughters executors should get proportion which corresponds to the time between the dea[th] of her mother & the sale of the Consols.

The Mortality amongst Europeans in certain Unhealthy Districts. *By* Thos. Glover Lyon, M.A., M.D., *Medical Officer to the Mutual Life Assurance Society.*

THE accompanying table is prepared from information kindly furnished to me by the Colonial and India Offices. In placing it before the members of the Institute of Actuaries it would not become me to add comments of any length.

It is often stated that sanitary science has reduced considerably

the risk incurred by Europeans residing at the places mentioned. The figures here given do not support such an opinion, except, perhaps, in the case of Sierra Leone, as the mortality showed no sign of diminishing during the period under observation.

On looking through the causes of death I found that diseases of the brain and abdominal organs, and fevers, occur in large proportions. This agrees with what Messrs. G. F. Hardy and H. J. Rothery deduced with respect to the mortality of Barbados. (*J.I.A.*, xvii, 161.)

Unfortunately, it was found impossible to follow the course of those who left the districts under notice. The ages of Europeans resident in these places have not been recorded, but they are mostly between 20 and 45, so that I believe we should not be far wrong in assuming the corresponding death-rate in England to be between 8 and 15 per thousand, or an average of about 11 per thousand.

With regard to Gambia and Sierra Leone, the population has not been recorded from year to year, and it has therefore been assumed that it was constant throughout at the latest known authentic figures.

Classing the whole of the West Coast of Africa together, the annual death-rate has been 56 per thousand, so that it appears to me that the offices would not cover themselves for the extra risk for much less than £5 per-cent.

Perhaps the fairest way, however, would be to have a scale of extra premium for each settlement, based upon the observed experience in that settlement, after making due allowance for fluctuations and possible deterioration in health of those who survive and leave the district.

Mortality Table of European Residents in Lagos, Gold Coast, Sierra Leone, Gambia, Calcutta, Madras, and Bombay, prepared from Government Reports.

Years of Life	Deaths	Deaths per mille per annum	Districts		1878	1879	1880	1881	1882	1883	1884	1885	1886	1887	1888	1889	1890
1080	76	70·4	Lagos	Residents	...	114	111	117	112	110	95	102	109	101	106	...	...
				Deaths	...	6	12	7	3	8	10	5	7	7	11	...	...
661	31	51·4	Gold Coast	Residents (Official)	...	14	48	61	62	70	76	78	65	75	82	...	...
				Deaths ,,	...	2	3	3	3	5	1	1	2	5	6	...	...
1264	106	84·7	,, ,,	Residents (Non-official)	...	90	115	149	125	115	122	131	142	130	145	...	...
				Deaths ,,	...	8	8	12	12	11	9	10	15	14	7	...	...
1925	140	72·7	,, ,,	Residents (Total)	...	134	163	210	187	185	198	209	207	205	227	...	...
				Deaths ,,	...	10	11	15	15	16	10	14	17	19	13	...	...
702*	37	52·7	Gambia	Deaths	6	1	1	2	1	7	2	4	4	3	3	2	1
1630†	44	27·0	Sierra Leone	Deaths (Residents)	...	6	4	3	7	8	4	3	2	2	5	...	...
1080	31	28·7	,, ,,	Deaths (Floating Population)	...	4	2	1	1	1	12	3	2	4	1	...	...
Average of Death-rates		14·4	Calcutta	Death-rates (per mille)	...	...	...	...	15·5	17·5	18·4	12·5	12·7	11·1	14·2	...	...
,,	,,	24·9	Madras	Death-rates ,,	...	...	28·2	22·1	21·3	24·6	30·9	21·5	23·1	23·7	28·0	25·9	...
,,	,,	19·3	Bombay	Death-rates ,,	...	...	26·9	22·7	17·7	18·9	19·2	19·2	17·9	17·2	19·0	20·7	...

* "At the present moment there are 54 in the Colony, and it is thought that not many in excess of this number were residing in Bathurst during the years mentioned."

† "At the Census of 1881 the whole population was 271, of whom 108 were floating population, *i.e.*, belonging to ships in harbour. Within the last 3 or 4 years the number of Europeans has increased."—*Report*, 1889.

CORRESPONDENCE.

THE ACTUARIAL SOCIETY OF AMERICA.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—No doubt all of your readers are aware of the fact that there is now in successful existence an association of American and Canadian actuaries, known as the Actuarial Society of America. The details regarding the formation and growth of a new and vigorous organization, with kindred aims and using the same language, can hardly fail to be interesting to the members of the old Institute. It has, therefore, occurred to me that a few words on these points, from one who enjoys the privilege of belonging to both societies, may not be out of place.

Early in 1889 several American actuaries, after informal consultations, decided that the time was opportune for the formation of a society on this side of the Atlantic, which would occupy a place somewhat similar to that of the Institute in Great Britain. The result was that in March of that year, a circular was addressed by Mr. David Parks Fackler, consulting actuary, of New York, to the actuaries of all the leading companies in the United States and Canada, suggesting the formation of such an institution. This circular met with a most favourable reception, and was followed by one calling a meeting at the Astor House, New York, for 25 April, for the completion of details. The result was that on the date named the organization was effected, a constitution drawn up, and the Actuarial Society of America fairly launched with a membership of 38—34 from the United States and 4 from Canada. Most of these charter members were present in person, and much enthusiasm was manifested. The importance of the step which was being taken was fully realized, and the greatest care exercised that the foundations of the new Society should be well and solidly laid, so as to allow of the building of a structure which would not only be permanent and useful, but would add honour and dignity to the profession, as the Institute has done in the old land. The original idea of the promoters had been to call the Society the "American Actuarial Association" (A.A.A.), but in deference to the Canadian members, and in order to emphasize the International idea, it was baptized the "Actuarial Society of America", the noun "America" not being limited to the United States, as the adjective generally is. In view of past experience in the "Chamber of Life Insurance", the membership was clearly defined to be a personal one, and not one of representatives of companies. Precautions were carefully taken to prevent the admission of undesirable or unqualified members, the constitution providing that two adverse votes from members of the council (11 in all) shall disqualify any applicant, and that even if recommended by the council, a three-fourths vote of the members present shall be necessary for election. There is a strong desire to

keep out all company rivalries and business controversies. An indication of this is seen in a clause of the constitution, which declares that no resolution expressive of opinion can be entertained at any meeting of the Society. The keenness of the competition between some of the offices renders these safeguards very necessary. The actuaries as a class are, however, freer from business jealousies than the presidents or managers, and one happy result which has already followed the establishment of the Society is, that persons who previously knew each other only by name are now intimately acquainted and, in many cases, warm friends. The effect which this will have in toning down business rivalries can easily be imagined.

The distance which many members have to come renders monthly meetings impossible. The Society assembles, therefore, but twice a year. The annual meeting is held in April, and the fall meeting usually in October, and each session lasts, at present, two days. The April meeting is always held in New York, but the other may be elsewhere. That for October 1890 was held in the Senate Chamber of the Capitol in Hartford, Connecticut, by the invitation of the Hartford members and the Governor of the State. The fall meeting this year will be held on 30 September and 1 October in Toronto, at the invitation of the Canadian members.

The programme followed on these occasions is admirable. The Society gathers at, perhaps, two o'clock on the first day, and after routine business the real work is proceeded with. The papers read at the previous meeting are first taken up. As a rule, no discussion has as yet taken place on these, the plan being to postpone criticisms until next meeting. The members have now had these papers before them in print for six months, and are ready to discuss them intelligently. This system may have its disadvantages, but the benefits far outweigh them. Off-hand criticisms are almost of necessity superficial, and any plan by which they can be replaced by deliberate and carefully-thought-out remarks is certainly a move in the right direction. Natural evolution is, moreover, carrying this idea still further, for in the circulars announcing the approaching meeting at Toronto, the council request that as far as possible the criticisms be written out, and copies sent to the secretary for printing 10 days before the meeting is held. When the discussions close, the members adjourn for dinner, and, after the good things have been disposed of, the evening is spent in social intercourse. At the last April meeting the time passed very pleasantly, while those present narrated personal reminiscences in answer to the question, "How I became an Actuary." Next morning at, say, 9.30 business is resumed. The discussions of the previous day are continued. The reading of original papers is then begun. These have been previously submitted to the council, and are already in print. If any are unusually long, the whole or part may be taken as read. The end is generally reached some time in the afternoon, and the members then adjourn, after completing any routine business which may remain.

The constitution provides that the president and vice-presidents shall not be eligible for the same offices for more than two years in succession. Notice has, moreover, been given of an amendment (since carried), that ex-presidents as such be hereafter members of

council in addition to the ordinary number. The desire is to have a steady influx of new blood into the management of the Society. Mr. Sheppard Homans has occupied the chair for two years, with great credit to himself and much satisfaction to the members. He has been succeeded by Mr. D. Parks Fackler, to whom the credit of founding the Society is chiefly due.

The membership has increased steadily, until it now embraces nearly every actuary of prominence in North America, besides some from abroad. The exact figures are: 56 from the United States, 7 from Canada, 2 from New Zealand, and 1 from Australia—in all 66. The indications are that future progress in numbers will be slower, as none but actuaries of improved standing will be admitted to full membership, and most of these on the continent have already joined. There is, as yet, but one class of members, that corresponding to fellows, the fee for all being $10.00 per annum. The council are, however, taking into consideration the question of establishing another class, more like associates, as by this means the influence and numbers of the Society would be considerably increased.

The papers which have been read are, as a class, briefer than those to which the Institute is accustomed, but this by no means implies that they are lacking in importance or depth. The following list speaks for itself:

"History of Dividend Systems in America"—by D. Parks Fackler, New York.

"Are our Mortality Tables reliable"—by William Hendry, Waterloo, Canada.

"Accumulation Formulæ"—by Edward B. Smith, Virginia (since deceased).

"The Value of New Blood in Life Insurance"—by Bloomfield J. Miller, Newark, New Jersey.

"Continued Life and Growth dependent upon Right Principles"—by Robert P. Field, Philadelphia.

"The Proper Treatment of a Life Insurance Company, in which the Reserve has become Impaired"—by Sheppard Homans, New York.

"Adverse Selection by Withdrawal"—by H. W. St. John, Hartford, Connecticut.

"The Just Apportionment of the Expenses of Mutual Life Insurance Companies"—by W. E. Starr, Worcester, Massachusetts.

"The Actuarial Elements involved in Fire Insurance"—by Walter S. Nichols, New York.

"Policy-Values: One of their Relations"—by Joseph H. Sprague, Hartford, Connecticut.

"Analysis and Disposition of Gains over Assumed Liabilities"—by Asa S. Wing, Philadelphia.

"Formulæ for ascertaining Contributions to Surplus"—by Emory McClintock, New York.

"Concerning Extra Premiums"—by D. Parks Fackler, New York.

"Some Thoughts regarding Margins for Expenses and Contingencies, and Surrender Charges"—by Sheppard Homans, New York.

"On the Provision for and Assessment of Life Insurance Expenses, &c."—by Walter C. Wright, Boston.

"Observations on Impaired Lives"—by J. M. Craig, New York.

"Insurance Values as Bases for Surrender Charges"—by Sheppard Homans, New York.

"Insurance Values and Distribution of Surplus"—by William Hendry, Waterloo, Canada.

"Some Thoughts on the Principles involved in Graduation"—by Walter S. Nichols, New York.

"Tate's Arithmometer"—by Max H. Peiler, Hartford, Connecticut.

"A Standard for measuring the Effects of Selection among Insured Lives"—by W. D. Whiting, New York.

"A Method of Measuring the Maximum Amount which an Insurance Company may properly assume on a Single Risk"—by Clayton C. Hall, Baltimore.

"Weight and Longevity"—by T. B. Macaulay, Montreal, Canada.

"Double Endowments"—by I. C. Pierson, New York.

The papers read during the first two years, and the discussions thereon, have been bound into a handy volume of about 200 pages, the first of a series. Volume ii will be for the third year, and will probably be larger than its predecessor. The wonderful stimulus which the Society has already given to all actuarial studies on this side of the Atlantic is bearing good fruit, in the quality and number of papers submitted, and it is hoped and believed that the transactions of the Actuarial Society of America will hereafter rank as one of the standard publications, which no progressive actuary can afford to have absent from his library.

Yours truly,

T. B. MACAULAY.

Montreal, Canada,
22 *September* 1891.

P.S.—Since the above was written, the meeting at Toronto, Canada, has been held. It was eminently successful, members attending from all the insurance centres of both the United States and Canada, and in many cases bringing their wives and daughters. Several important papers were read, the principal one being an elaborate comparison between American and Australian mortality, by Mr. Richard Teece, of Sydney, New South Wales. The discussions on the papers read at the preceding meeting were lengthy and unusually interesting. Most of the more important criticisms were in writing, and carefully prepared.

The publications of the Society can be obtained from the Secretary, Mr. Israel C. Pierson, 21 Cortlandt Street, New York.

AMOUNT OF INSURANCES AGAINST ISSUE.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—In continuation of the statements I have contributed to your pages on three former occasions, I now give, in the annext table, certain particulars of the insurances against issue granted by the various British Life Offices, as stated in the Board of Trade Returns for the five years 1886 to 1890 inclusive. As the Blue-Books for those years contain no valuation return of the *Edinburgh* Office (which makes a septennial valuation), I have taken the figures for that office from the Blue-Book for 1885.

Combining the figures with those I formerly gave, we have the following results:

Years	No. of Policies	Net Sums Assured	Net Premiums Received	Average Premium per-cent
		£	£	
1871–1875	261	875,558	62,238	6·75
1876–1880	383	1,264,166	97,495	7·51
1881–1885	539	1,696,747	116,704	6·82
1886–1890	698	2,076,859	133,200	6·08

From these figures it appears that this class of business has continued to increase steadily during the last five years, but not quite so fast as during the previous five years; also that both the average sum assured and the average premium per-cent have continued to decrease.

I am, Sir,

Your obedient servant,

T. B. SPRAGUE.

Edinburgh,
5 *October* 1891.

Table giving certain particulars of the Issue Insurances granted by British Life Offices, and remaining in force during the Years 1886 to 1890.

Name	Date of Valuation	Number of Policies	Net Amounts, deducting Reassurances		Gross Amounts			
			Sums Insured	Premiums Received	Sums Insured	Premiums Received	Reserve	Average Premium per cent
			£	£	£	£	£	
Eagle . . .	31 Dec. 1887	58	284,510	29,094	461,305	40,008	14,191	8·67
Guardian . . .	31 Dec. 1889	89	222,289	13,146	312,832	20,363	17,823	5·94
Equity and Law .	31 Dec. 1889	73	221,914	15,779	291,125	20,402	20,402	7·01
Law Union . .	30 Nov. 1889	68	116,075	5,454	275,075	10,085	10,085	3·67
Scottish Equitable .	1 Mar. 1888	46	106,700	6,384	113,050	6,651	6,075	5·88
North British & Merc.	31 Dec. 1885	23	104,176	6,345	117,176	7,133	6,676	6·09
Legal and General .	31 Dec. 1886	28	100,838	5,211	140,038	7,041	7,041	5·03
Law . . .	31 Dec. 1889	32	88,446	5,781	97,446	6,254	6,254	6·42
Standard . . .	15 Nov. 1885	22	81,784	5,300	187,784	11,790	4,964	6·28
Crown . . .	25 Mar. 1890	20	62,442	2,693	83,442	4,132	3,767	4·95
Pelican . . .	31 Dec. 1885	9	54,015	3,151	72,015	3,340	3,339	4·64
National . . .	31 Dec. 1889	22	50,150	2,312	50,150	2,312	1,849	4·61
Reliance . . .	31 Dec. 1887	9	45,437	3,558	45,437	3,558	3,100	7·83
Scottish Union & Nat.	31 Dec. 1889	10	43,808	2,010	53,808	2,460	2,460	4·57
Norwich Union .	30 June 1886	8	37,582	2,200	37,582	2,200	1,100	5·85
Rock . . .	19 Aug. 1889	8	37,300	3,491	92,800	6,988	5,412	7·53
London Assurance .	31 Dec. 1885	10	30,000	1,607	36,000	1,796	1,706	4·99
Universal . . .	31 Dec. 1888	8	28,800	1,608*	30,800	1,720	2,257	5·58
Royal Exchange .	31 Dec. 1885	6	28,400	645	28,400	645	516	2·27
Commercial Union .	31 Dec. 1887	12	25,018	1,138	25,018	1,138	1,138	4·55
Alliance . . .	31 Dec. 1888	8	25,000	923	25,000	923	923	3·69
Clerical, Med. & Gen.	30 June 1886	5	25,000	1,363	25,000	1,363	1,363	5·45
Atlas . . .	31 Dec. 1889	9	24,090	1,789	26,390	2,019	2,202	7·65
Caledonian . .	31 Dec. 1885	9	23,350	983	37,850	1,292	969	3·41
Imperial . . .	31 Jan. 1886	9	21,865	767	21,865	767	767	3·51
Union . . .	30 June 1887	6	21,500	1,339	21,500	1,339	715	6·23
Northern . . .	31 Dec. 1885	13	21,050	948	27,200	1,217	1,217	4·47
Liv. & London & Globe	31 Dec. 1888	8	20,700	885	20,700	885	520	4·28
West of England .	31 Dec. 1887	10	13,750	823	13,750	823	363	5·99
British Empire .	31 Dec. 1887	5	13,500	600	13,500	600	569	4·44
Edinburgh . .	31 Mar. 1885	5	13,425	455	13,425	455	409	3·39
General . . .	31 Dec. 1887	7	13,000	778	13,000	778	389	5·99
Mutual . . .	31 Dec. 1888	7	11,500	435	11,500	435	435	3·78
University . .	1 May 1890	5	10,300	1,083	10,300	1,083	1,083	10·52
City of Glasgow .	20 Jan. 1889	8	9,783	570	9,783	570	570	5·83
Scottish Metropolitan	31 Dec. 1888	6	7,500	356	15,000	834	300	5·56
Scottish Imperial .	31 Dec. 1885	4	7,000	403	7,000	403	341	5·76
Scottish . . .	31 May 1886	2	6,500	140	6,500	140	97	2·15
Sovereign . . .	31 Dec. 1885	2	5,000	284	5,000	284	142	5·68
Westminster . .	31 Dec. 1886	4	4,362	213	4,362	213	171	4·88
Patriotic . . .	31 July 1889	2	4,000	480	4,000	480	100	12·00
Provident Clerks .	31 Dec. 1887	1	2,000	126	2,000	126	126	6·30
Hand-in-Hand . .	31 Dec. 1889	1	2,000	525	2,000	525	525	26·25
Marine and General .	31 Dec. 1889	1	1,000	25	1,000	25	25	2·50
44 Companies .	...	698	2,076,859	133,200	2,918,908	177,595	134,779	6·08

* Approximated.

THE INSTITUTE OF ACTUARIES.

EXAMINATION OF THE INSTITUTE, OCTOBER 1891.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE (PART I).

Examiners—F. T. M. BYERS, Esq.; F. E. COLENSO, Esq., M.A.; G. F. HARDY, Esq.; A. G. HEMMING, Esq.

First Paper.

1. Explain the methods of short multiplication and short division of decimal fractions, and employ these methods to find the value of $(3{\cdot}8571431)^3$ and of $\cdot80754167 \div \cdot0092197$, making your results, in each case, true to the 6th place of decimals.

2. A small trader commences business with £200. Placing £160 in a bank, he purchases 8 hogsheads of sugar at £15, and gives a bill of exchange in settlement. He also sells 7 hogsheads at a profit of 20 per-cent, taking from the purchaser an acceptance, which he discounts at 6*s*. 2*d*. discount with his bankers, with whom he lodges the proceeds. Exhibit these transactions in the form of journal entries.

3. What are meant by "real", "personal", and "fictitious" accounts? Classify under these heads the accounts to which the transactions described in the last question would be carried, and indicate those which would appear in a balance sheet of the business.

4. Solve the following equations:—

(*a*) $$\frac{x^2+3x-7}{x+2+\dfrac{18}{x}}=1$$ (*b*) $$\sqrt{x^4-1}+\sqrt{x^2-1}=x^3$$

(*c*) $$\begin{cases} x^2+x+y=18-y^2 \\ xy=6 \end{cases}$$ (*d*) $$\begin{cases} x^2+xy=12 \\ xy-2y^2=1 \end{cases}$$

5. A bag contains 7 white, 5 red, 9 blue, and 14 black balls. These are withdrawn at random, two by two. Find the chance that a pair consisting of a red and a white ball will be withdrawn before a pair consisting of a black and a blue ball, assuming:—

(1) That the balls are replaced after each draw;

(2) That they are not replaced.

6. A wine merchant sold 7 dozen of sherry and 12 dozen of claret for £50. He sold 3 dozen more of sherry for £10 than he did of claret for £6. Find the price of each kind of wine.

7. A ship started on a voyage with a sufficient supply of water for all on board, numbering 175 souls. After 30 days there was a daily loss of 3 lives from scurvy. A storm delayed the ship 3 weeks, and it got to port just as the supply of water ran out. What was the length of the voyage?

8. Show how to obtain the *common* from the *hyperbolic* logarithm of a number.

If log 7 to the base $2{\cdot}71828 = 1{\cdot}94591$ and $2{\cdot}71828 = 10^{\cdot 43429}$, find log 7 to the base 10, and log 10 to the base 7.

9. If $\Delta x = a$, and $\Delta u_x = u_{x+a} - u_x$, prove that:—

$$u_{x+n} = u_x + \frac{n}{a} \cdot \Delta u_x + \frac{n(n-a)}{2 \, . \, a^2} \cdot \Delta^2 u_x$$

$$+ \frac{n(n-a)(n-2a)}{2 \, . \, 3 \, . \, a^3} \Delta^3 u_x + \&c.$$

10. Demonstrate Lagrange's formula of interpolation, and employ it to find the number of which the log is $2\frac{1}{3}$, having given:—

$$\begin{aligned} \log 200 &= 2{\cdot}30103 \\ 210 &= 2{\cdot}32222 \\ 220 &= 2{\cdot}34242 \\ 230 &= 2{\cdot}36173. \end{aligned}$$

11. Find the eccentricity of the ellipse:—

$$2 \, . \, x^2 + 3 \, . \, y^2 = c^2.$$

12. The opposite angles of any quadrilateral figure described in a circle are together equal to two right angles.

Second Paper.

13. Prove the rule for the conversion of a recurring decimal into a vulgar fraction.

14. In a geometric series, if the $(p+q)$th term $= m$, and the $(p-q)$th term $= n$, prove that the pth term $= \sqrt{m \, . \, n}$, and the qth term $= m \cdot \left(\frac{n}{m}\right)^{\frac{p}{2q}}$.

15. Prove that the difference between the coefficients of x^{r+1} and x^r, in the expansion of $(1+x)^{n+1}$, is equal to the difference between the coefficients of x^{r+1} and x^{r-1} in the expansion of $(1+x)^n$.

16. A and B play a match of $2n$ games, in each of which their respective chances of winning are as $4:5$. What is the probability

that the match will end in a tie? and what should be the relative amounts of the stakes?

17. In two throws with a pair of dice, what are the probabilities of the following events:—

α. 8 the first throw, followed by 9 the second;

β. 9 the first throw, or, *if not*, then 8 the second?

18. 10 Russian, 12 French, and 14 English ships are expected in port. Find the value of the expectation of a merchant, who will gain £2,100 if one of the two which first arrive is a Russian, and the other a French, ship.

19. Two persons, A and B, entered into a speculation to which B subscribed £15 more than A. After 4 months, C was admitted, who added £50 to the stock, and at the end of 12 months from C's admission they found that the total gain from the commencement was £159. A then withdrew, and received for principal and gain £88. What did A originally subscribe?

20. Bronze contains 91 per-cent of copper, 6 per-cent of zinc, and 3 per-cent of tin. A mass of bell-metal (consisting of copper and tin only) and bronze fused together is found to contain 88 per-cent of copper, 4·875 per-cent of zinc, and 7·125 of tin. Find the proportion of copper and tin in bell-metal.

21. If $S=\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}$ *ad inf.*, and $S'=\frac{1}{2.4}+\frac{1}{4.6}+\frac{1}{6.8}$ *ad inf.*, prove that $S-S'=\frac{1}{4}$.

22. Determine the number of ways in which it is possible to pay £20 in half-guineas and half-crowns only.

23. Demonstrate Mr. Woolhouse's formula for interpolation by central differences,

$$u_x = u_0 + \left(a_0 - \frac{c_0}{3}\right) \cdot x + \left(\frac{b_0}{2} - \frac{d_0}{4}\right) \cdot x^2 + \frac{c_0}{3} \cdot x^3 + \frac{d_0}{4} \cdot x^4,$$

assuming 4th differences constant.

24. Describe an isosceles triangle, having each of the angles at the base double of the third angle.

Hence find the side of a regular decagon inscribed in a circle of given radius.

25. Obtain the equations connecting the rectangular and polar co-ordinates of a point.

INDEX TO VOL. XXIX.

END OF VOL. XXIX.

Printed by C. & E. Layton, Farringdon Street.